

Three problems in special relativity¹

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1 Moving clock

A clock moves with velocity $\mathbf{v}$. What is the ratio $dt/d\tau$ between coordinate time t as seen by clocks at rest in the coordinate system and proper time τ as measured by the moving clock?

2 Moving meter sticks

We can describe a meter stick in a covariant fashion by saying that coordinate markers on the meter stick are given by a parameter σ with $0 < \sigma < L$ and that a clock attached to each coordinate marker measures proper time τ with $-\infty < \tau < \infty$. Then the space-time coordinates traced out by the points on the meter stick are

$$x^\mu = u^\mu \tau + n^\mu \sigma . \quad (1)$$

In the rest frame of the meter stick, $u = (1, 0, 0, 0)$ and $n = (0, 0, 0, 1)$. In any other frame, u and n are different. However we have the invariant relations $u^2 = 1$, $n^2 = -1$ and $n \cdot u = 0$.

Consider a frame in which the meter stick is aligned along the z axis and is moving in the z direction. Then $u = (u^0, 0, 0, u^3)$ and $n = (n^0, 0, 0, n^3)$. How are n^0 and n^3 related to u^0 and u^3 ? How is u^3 related to the velocity v of the meter stick? Use these relations to find the length L' of the meter stick as measured in this reference frame. That is, how is L' related to L and v .

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3 Motion in a magnetic field

A particle of mass m and charge q moves in a uniform magnetic field B directed in the $+z$ direction. The particle moves in the x - y plane. Suppose that the speed of the particle is v . Show that the particle moves in a circular orbit. Find the radius R and the angular velocity ω of the orbit as functions of v and B .

4 Motion in an electric field

A particle of mass m and charge q moves in a uniform electric field E directed in the $+z$ direction. At time $t = 0$, the particle is at rest. Find the subsequent motion of the particle.