

(3)

write $t = \tan \frac{\theta}{2}$

then $d\theta = \frac{2}{1+t^2} dt$ $\cos \theta = \frac{1-t^2}{1+t^2}$

$$\begin{aligned} \frac{d\theta}{\cos^2 \theta} &= \frac{2dt}{1+t^2} \cdot \frac{(1+t^2)^2}{(1-t^2)^2} = \frac{2(1+t^2)dt}{(1-t^2)^2} \\ &= \frac{1}{(1-t)^2} dt + \frac{1}{(1+t)^2} dt \end{aligned}$$

$$\therefore \int \frac{d\theta}{\cos^2 \theta} = -1 - \frac{1}{t-1} + 1 - \frac{1}{t+1} = -\frac{1}{t-1} - \frac{1}{t+1}$$

$$\therefore \int \frac{dv}{(1-v^2/c^2)^{3/2}} = C \cdot \frac{2t}{1-t^2} = C \cdot \frac{v/c}{\sqrt{1-v^2/c^2}} = \frac{v}{\sqrt{1-v^2/c^2}}$$

$$\therefore \frac{v}{\sqrt{1-v^2/c^2}} = \frac{qE}{m} t$$

$$\therefore v(t) = \frac{\frac{qE}{mc} t \cdot C}{\sqrt{1 + \left(\frac{qE}{mc} t\right)^2}}$$

$$\therefore Z = \int v dt'$$

$$= \int_0^t C \frac{\frac{qE}{mc} t'}{\sqrt{1 + \left(\frac{qE}{mc} t'\right)^2}} dt'$$

$$= \frac{mc^2}{qE} \int_0^{\frac{qE}{mc} t} \frac{x dx}{\sqrt{1+x^2}} = \left(\sqrt{1 + \left(\frac{qE}{mc} t\right)^2} - 1 \right) \frac{mc^2}{qE}$$

$$\boxed{\therefore Z = \frac{mc^2}{qE} \left(\sqrt{1 + \left(\frac{qE}{mc} t\right)^2} - 1 \right)}$$