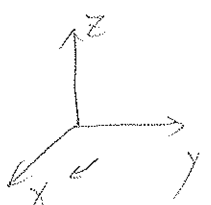
3. $\hat{z} \uparrow \vec{B}$

$$\frac{d\vec{p}}{dt} = q\vec{v}\times\vec{B}$$

$$\frac{dW}{dt} = q\vec{v}\times\vec{B}\cdot\vec{v} = 0. \Rightarrow W \text{ is a constant} \Rightarrow |\vec{v}| \text{ is a constant}$$

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{m}{\sqrt{1-v^2/c^2}}\frac{d\vec{v}}{dt} = q\,\vec{v}\times\vec{B}$$

$\vec{B} = B\hat{z}$ $\quad \vec{v}\perp\vec{B}$



$$\therefore \dot{\vec{v}} = \frac{\sqrt{1-v^2/c^2}}{m}\,q\,\vec{v}\times\vec{B} = \frac{q}{\gamma m}\vec{v}\times\vec{B}$$

$\therefore$ the particle moves in a circular orbit.

$$\frac{\gamma m v^2}{R} = qvB$$

$$\boxed{\therefore R = \frac{\gamma m v}{qB} \qquad \omega = \frac{v}{R} = \frac{qB}{\gamma m}}$$

4. $\vec{E}\parallel\hat{z}$, at $t=0$, $\vec{v}=\vec{0}$ $\therefore \vec{v}\parallel\hat{z}$

$$\frac{d\vec{p}}{dt} = \hat{z}\frac{d}{dt}\left(\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \hat{z}\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}\frac{dv}{dt} + mv\left(-\frac{1}{2}\right)\frac{-2\frac{v}{c^2}\frac{dv}{dt}}{\left(1-\frac{v^2}{c^2}\right)\sqrt{1-\frac{v^2}{c^2}}}\hat{z}$$

$$= \hat{z}\frac{m}{(1-v^2/c^2)^{3/2}}\frac{dv}{dt} = qE\hat{z}$$

$$\therefore \frac{dv}{(1-v^2/c^2)^{3/2}} = \frac{qE}{m}dt$$

write $\frac{v}{c} = \sin\theta$ then $dv = c\cdot\cos\theta\, d\theta$, when $v=0$, $\theta=0$.

$$\frac{dv}{(1-v^2/c^2)^{3/2}} = \frac{c\,d\theta}{\cos^2\theta}$$