1. Moving clock

moving clock $\quad s^2 = c^2 d\tau^2$

frame with clock at rest $\quad s^2 = c^2 dt^2 - v^2 dt^2$

$$c^2 d\tau^2 = c^2 dt^2 - v^2 dt^2$$

$$\therefore \boxed{\frac{dt}{d\tau} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

2. Moving meter sticks

In the rest frame

$$x^\mu = u^\mu \tau + n^\mu \sigma = (\tau, 0, 0, \sigma)$$

In the frame in which the meter stick moves along z axis.

$$t = \frac{\tau + \frac{v}{c^2}\sigma}{\sqrt{1-v^2/c^2}}$$

$$z = \frac{\sigma + v\tau}{\sqrt{1-v^2/c^2}} \quad , y = y' \ , \ z = z'$$

$\therefore$ in this frame
$$x^1 = \frac{\tau + \frac{v}{c^2}\sigma}{\sqrt{1-v^2/c^2}} = u^1\tau + n^1\sigma$$

$$x^4 = \frac{v\tau + \sigma}{\sqrt{1-v^2/c^2}} = u^4\tau + n^4\sigma$$

$$\boxed{\begin{aligned} u^\mu &= \left(\frac{1}{\sqrt{1-v^2/c^2}}, 0, 0, \frac{v}{\sqrt{1-v^2/c^2}}\right) = (u^0, 0, 0, u^3) \\ n^\mu &= \left(\frac{v/c^2}{\sqrt{1-v^2/c^2}}, 0, 0, \frac{1}{\sqrt{1-v^2/c^2}}\right) = (n^0, 0, 0, n^3) \end{aligned}}$$

$$\sigma = \frac{z - vt_1}{\sqrt{1-v^2/c^2}} \qquad 0 = \frac{z' - vt_2}{\sqrt{1-v^2/c^2}}$$

$$\frac{L'}{L} = \frac{z - z'}{\sigma - 0} = \sqrt{1-v^2/c^2} \qquad \therefore \boxed{L' = \sqrt{1-v^2/c^2}\, L}$$