QCD calculations at NLO:
doing integrals numerically

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Work with

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D. E. Soper, Phys. Rev. Lett. **81**, 2638 (1998) [arXiv:hep-ph/9804454]; Phys. Rev. D **62**, 014009 (2000) [arXiv:hep-ph/9910292]; Phys. Rev. D **64**, 034018 (2001) [arXiv:hep-ph/0103262]; M. Krämer and D. E. Soper, [arXiv:hep-ph/0306222]; D. E. Soper, *beowulf* Version 3.0, http://zebu.uoregon.edu/~soper/beowulf/.

Points to take home

- For flexibility, do the virtual loop integrations numerically.
- Real/virtual cancellations can happen without subtractions.
- You may want subtractions, nevertheless.
- Numerical virtual loop integrals need contour deformation.

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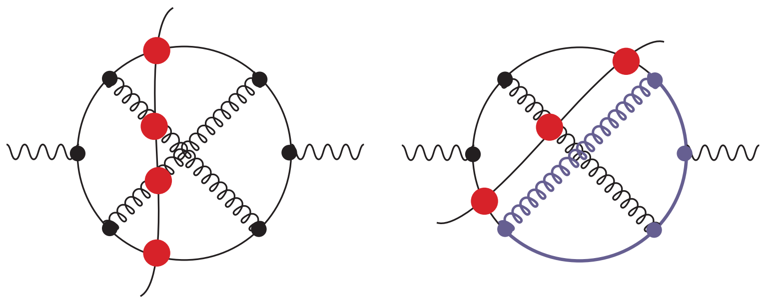
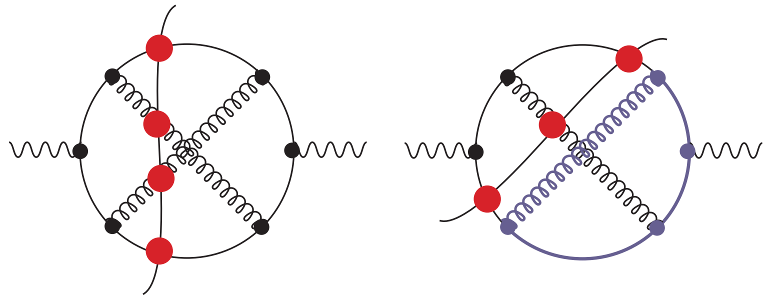


- You don't need to compute complicated integrals analytically.
- You do need a few simple integrals for renormalization.
- Feynman rules $\rightarrow$ integrand.
- Code exists for $e^+ + e^- \rightarrow$ 3 jets.
- Example of flexibility: Coulomb gauge (with M. Krämer).

Real/virtual cancellations can happen without subtractions.

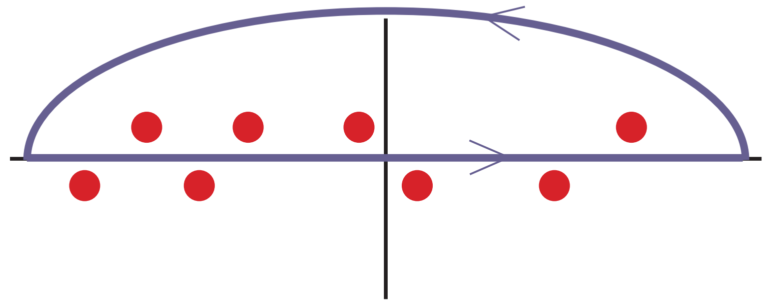
- Insert $\int d\sqrt{s}\; h(\sqrt{s})$.
- Real graph becomes

$\int d\vec{l}_1\; d\vec{l}_2\; d\vec{l}_3\; f_R(l)$.



- In loop, do $\int dE$.
- Loop graph becomes

$\int d\vec{l}_1\; d\vec{l}_2\; d\vec{l}_3\; f_V(l)$.



$$\int d\vec{l}_1\; d\vec{l}_2\; d\vec{l}_3\; [f_R(l) + f_V(l)].$$

- Leading singularities cancel between f_R and f_V.

You may want subtractions, nevertheless.

- Customarily, one subtracts for collinear and soft singularities separately in real and virtual graphs.

- Nothing prevents this.

- If you do subtract, you can have $\int d^4 l$ for a loop.

- This may be simpler for n-point functions with $n \geq 5$.

- There is an advantage with respect to adding showers.

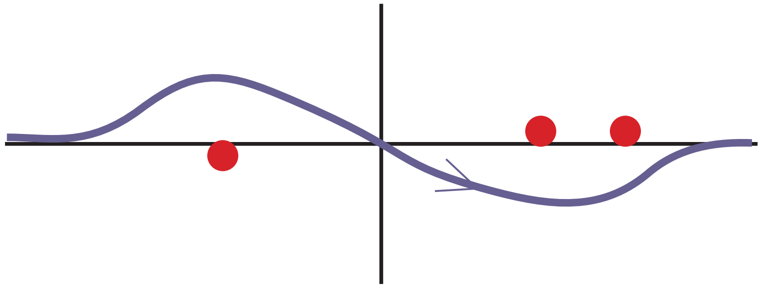
Numerical virtual loop integrals need contour deformation.

In

$$\int d\vec{l}\, \frac{i}{E(\vec{l})^2 - \vec{l}^{\,2} + i\epsilon} \cdots$$

the $+i\epsilon$ instructs you to deform the integration contour in $\vec{l}$,

$$\vec{l} \to \vec{l} + i\vec{\kappa}(\vec{l}).$$

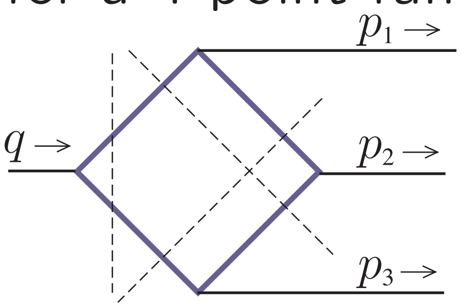


• The result is independent of the contour choice (as long as you don't cross the singularity).

• This works.

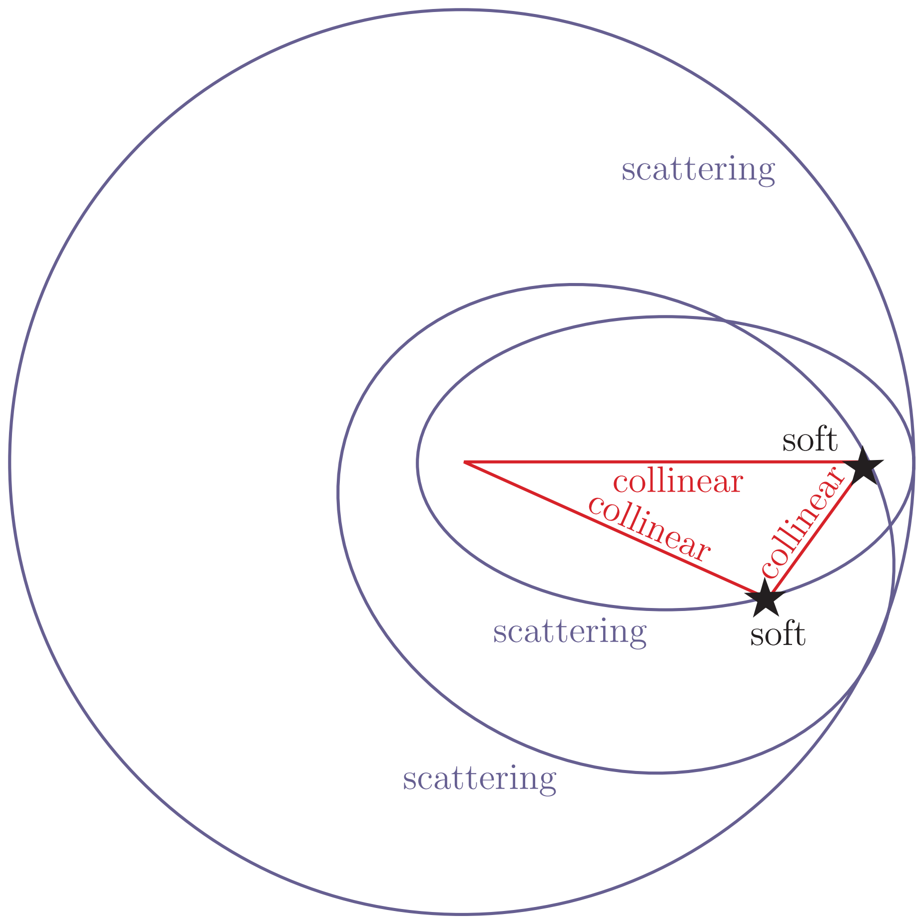
Singularities and deformations

Where the singularities are for a 4 point function.

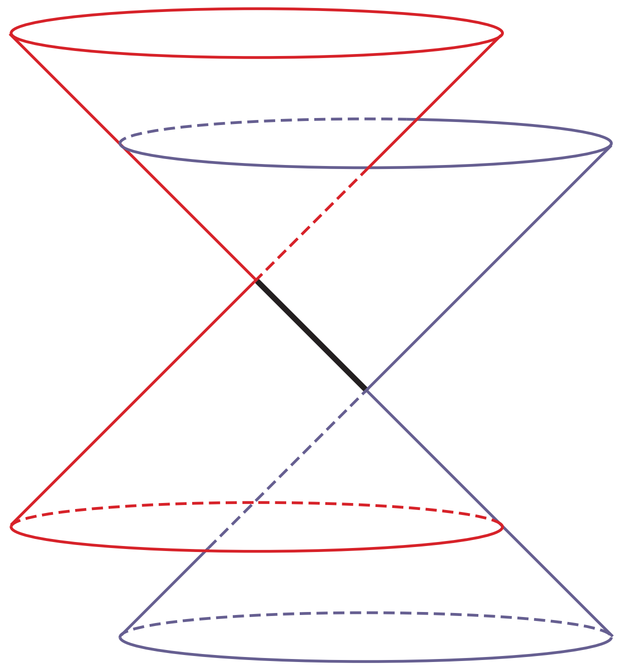


• Deform past the scattering singularities.

• Don't deform at the collinear or soft singularities.



Singularities and deformations for 4 dimensions



• In 4 dimensions, singularities to be deformed around are on cones.
• The soft singularities are at the tips of the cones.
• The collinear singularities are at points where two cones are tangent.

• Starting at five point functions there are some difficulties with the picture in 3 dimensions

• This 4-dimensional version may be more powerful.

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