

MATH 635 WINTER 2024
HOMEWORK 5
DUE FEBRUARY 19, 2024
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(This was formerly named “Homework 4”. Also, one problem has been moved to HW6.)

Required problems:

- (1) Use the Universal Coefficient Theorems to confirm your computations in parts (b)–(e) in Problem (7) on Homework 1.
- (2) Hatcher 3.1.2 (p. 204).
- (3) Hatcher 3.1.3 (p. 204).
- (4) Let C_* be a bounded-below chain complex of free modules, and D_* any bounded-below chain complex, and $f: E_* \rightarrow D_*$ a free resolution of D_* . Let $g: C_* \rightarrow D_*$ be a chain map. Prove there is a lift $h: C_* \rightarrow E_*$ so that the diagram

$$\begin{array}{ccc} & & \nearrow h \\ q.iso \downarrow f & & \\ C_* & \xrightarrow{g} & \end{array}$$

commutes up to homotopy. Prove, moreover, that h is unique up to homotopy.

- (5) Hatcher 3.1.9 (p. 205). Use the Universal Coefficient Theorem.
- (6) Let K be a knot in S^3 , that is, a smoothly embedded circle. It follows from the implicit function theorem that K has a neighborhood U homeomorphic to $S^1 \times D^2$, so that K is identified with $S^1 \times 0$. Use the Mayer-Vietoris sequence to compute $H_1(S^3 \setminus K)$. (Hint: cover S^3 by U and $S^3 \setminus K$.)
- (7) Hatcher 2.2.33 (p. 158)

Optional problems:

- (8) The optional homological algebra problems from last week that I didn’t solve in class.
- (9) More homological algebra: Hatcher 3.1.1, 3.A.2, 3.A.3, 3.A.4, 3.A.5, 3.A.6.
- (10) More degree problems; these make good qual problems: Hatcher 2.2.2, 2.2.3, 2.2.4.
- (11) More Mayer-Vietoris practice; these also make qual problems: Hatcher 2.2.28, 2.2.29.
- (12) The *solid torus* is the space $S^1 \times D^2$; this is a donut. Use the Mayer-Vietoris sequence to compute the homology of the solid torus.

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