

MATH 341
WRITTEN HOMEWORK 7
DUE MARCH 4, 2020.
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Required textbook problems (hand these in):

- (1) §4.1: 1, 2, 5, 6, 7, 8, 12, 25, 26, 27, 28, 29, 30.
- (2) §4.2: 7, 8, 9, 10, 16, 22, 31.
- (3) Recall that $\mathbb{P}_3$ is the vector space of polynomials $p(t)$ of degree ≤ 3 . Consider the map $F: \mathbb{P}_3 \rightarrow \mathbb{P}_3$ defined $F(p(t)) = p'(t) + p''(t)$.
 - (a) Compute $F(t^3 + 2t + 3)$.
 - (b) Verify that F is a linear transformation.
 - (c) Is F one-to-one (injective)? Justify your answer.
 - (d) Is F onto (surjective)? Justify your answer.
 - (e) Describe the kernel (null space) of F .
 - (f) Describe the image (what the book calls the “range”) of F .
 - (g) Find one solution $p(t)$ to the equation $F(p(t)) = 2t^2 + 3t + 4$.
 - (h) Find all solutions $p(t)$ to the equation $F(p(t)) = 2t^2 + 3t + 4$.
- (4) Consider the map $G: \mathbb{P}_2 \rightarrow \mathbb{P}_3$ defined $G(p(t)) = \int_0^t p(x)dx$.
 - (a) Compute $G(t^2 + t + 5)$.
 - (b) Verify that F is a linear transformation.
 - (c) Is G one-to-one (injective)? Justify your answer.
 - (d) Is G onto (surjective)? Justify your answer.
 - (e) Describe the kernel (null space) of F .
 - (f) Describe the image (what the book calls the “range”) of G .
 - (g) Find one solution $p(t)$ to the equation $G(p(t)) = 2t^2 + 3t$.
 - (h) Find all solutions $p(t)$ to the equation $G(p(t)) = 2t^2 + 3t$.
- (5) Consider the map $H: \mathbb{P}_3 \rightarrow \mathbb{P}_3$ defined by $H(p(t)) = p'(t)p''(t)$.
 - Compute $H(t^3 + t + 3)$.
 - Is H a linear transformation? Justify (prove) your answer.
- (6) Consider the map $K: \mathbb{P}_3 \rightarrow \mathbb{P}_3$ defined by $K(p(t)) = t^3p(0) + 2p'(t)$.
 - Compute $K(t^3 + 2t + 3)$.
 - Verify that K is a linear transformation.
 - Is K one-to-one (injective)? Justify your answer.
 - Is K onto (surjective)? Justify your answer.
 - Compute $(F \circ K)(t^3 + 2t + 3) = F(K(t^3 + 2t + 3))$.
 - Compute $(K \circ F)(t^3 + 2t + 3) = K(F(t^3 + 2t + 3))$.

Suggested practice (don't hand these in):

- Please read and make sure you can do the practice problems in section 4.1, 4.2.
- Please read and use for review problems 4.1.23, 4.1.24, 4.2.25, 4.2.26.
- Some more nice practice with the definitions: 4.1.19, 4.1.31, 4.1.32, 4.1.33, 4.1.34
- If you had trouble or got help with any of the assigned problems, solve another, similar problem (or two).

Bonus points. As usual, bonus points for learning Sage.

- (1) Follow the post “Random matrices for practice”. Print out your worksheet and turn it in.

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