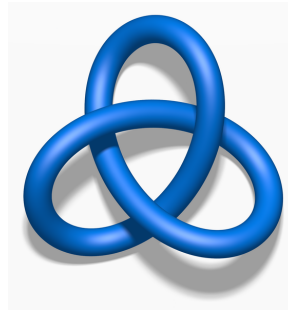


MATH 635 HOMEWORK 1
DUE JANUARY 16, 2019.
 INSTRUCTOR: ROBERT LIPSHITZ

Reminder: homework is due at the *beginning* of class, handed to me or in my mailbox on the second floor of Fenton.

- (1) Hatcher, 2.2.7, p. 155.
- (2) Hatcher, 2.2.9(a,b,c), p. 156.
- (3) The *solid torus* is the space $S^1 \times D^2$; this is a donut.
 - (a) Use cellular homology to compute the homology of the solid torus.
 - (b) Use the Mayer-Vietoris sequence to compute the homology of the solid torus.
- (4) Let K be the trefoil knot:



- (a) Use cellular homology to prove that $H_1(S^3 \setminus \text{nbdd}(K)) \cong \mathbb{Z}$ and $H_2(S^3 \setminus \text{nbdd}(K))$ is trivial. (Hint: remember Hatcher exercise 22, p. 55.)
- (b) Given a map $\gamma: S^1 \rightarrow S^3 \setminus K$, the *linking number* of γ with K is the degree of the map

$$\mathbb{Z} = H_1(S^1) \rightarrow H_1(S^3 \setminus K) \cong \mathbb{Z}.$$

(This is well-defined up to sign.)

Let $\phi: \partial(S^1 \times D^2) = S^1 \times S^1 \rightarrow \partial \text{nbdd}(K)$ be a homeomorphism. Compute the homology of

$$S_\phi^3(K) := (S^3 \setminus \text{nbdd}(K)) \cup_\phi (S^1 \times D^2)$$

in terms of the linking number of $\phi(\{pt\} \times S^1)$ and K . (The manifold $S_\phi^3(K)$ is called a *surgery on K* .) (Hint: you can solve this either using cellular homology or the Mayer-Vietoris theorem.)

- (5) Hatcher, 2.2.33, p. 158.

Review / qualifying exam practice (not to turn in):

- (1) Hatcher 2.2.2, 2.2.3, 2.2.4, 2.2.14, 2.2.15, 2.2.19, 2.2.28, 2.2.29, 2.2.30
- (2) With notation as in Problem 4, give two non-homotopic maps γ with linking number ± 1 with K . (Hint: think about π_1 .)

More problems to think about but not turn in:

- (1) Hatcher, 2.2.8, p. 155. Also, deduce the fundamental theorem of algebra.

- (2) Give an example of a surgery on the trefoil which has trivial H_1 but is not homeomorphic to S^3 .

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