MATH 607 WINTER 2017
HOMEWORK 3

INSTRUCTOR: ROBERT LIPSHITZ

(1) Choose a pair of curves $L_0, L_1 \subset \mathbb{C}$ which intersect transversely, and give an explicit bounded, injective holomorphic map

$$u\colon (\mathbb{R} \times [0,1], \mathbb{R} \times \{0\}, \mathbb{R} \times \{1\}) \to (\mathbb{C}, L_0, L_1).$$

Verify that u converges exponentially to points in $L_0 \cap L_1$ as $s \to \pm\infty$.

(2) In class, we stated the following corollary:

Corollary. If $u\colon \mathbb{R} \times [0,1] \to (M, L_0, L_1)$ is J-holomorphic and has finite energy then u converges exponentially to points in $L_0 \cap L_1$ as $s \to \pm\infty$.

(a) Give a precise statement of this corollary.

(b) Prove it (assuming the technical theorem from class that preceded it—a relative version of Theorem 8.9.1 in Audin-Damian.

(3) Prove that $\bar{\partial}_J u \in \Omega^{0,1}(u^*TM)$.

(4) Let M, N, and P be smooth manifolds, $F\colon P \times M \to N$ a map and $n \in N$ a regular value of F. Write $f_p(m) = F(p, m)$.

(a) Suppose M, N, and P are finite-dimensional and F is smooth. Prove that for generic $p \in P$, n is a regular value of f_p, so $f_p^{-1}(n)$ is a smooth manifold of dimension $\dim(M) - \dim(N)$.

(b) Suppose instead that M, N, and P are Banach manifolds, Df_p is Fredholm for $p \in P$, and F is C^q where $q > \max\{\mathrm{ind}(Df_p), 0\}$. Prove that for generic $p \in P$, $f_p^{-1}(n)$ is a finite-dimensional manifold.

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