# MATH 432/532 SPRING 2017
# HOMEWORK 4
# DUE FEBRUARY 1, 2017.

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*Note.* Graduate students are strongly encouraged to solve Problem 1 (Lee, Exercise 1.20) this week, even though it's somewhat painful. (Other than problem 1, this should be a fairly short assignment.)

**Problems**:

(1) Lee, Exercise 1.20.
(2) Lee, Problem (not exercise) 1-7.
(3) Lee, Problem (not exercise) 1-11.

**Challenge problems** (required for Math 532, optional for 432):

(4) The set $SL_n(\mathbb{R})$ is the set of $n \times n$ real matrices with determinant 1. Prove that $SL_n(\mathbb{R})$ is a smooth manifold in a natural way. (Hint: show that 1 is a regular value of the determinant function $\det\colon \mathbb{R}^{n\times n} \to \mathbb{R}$. To do so, let $A = [v_1 \ \cdots \ v_n]$ be an $n \times n$ matrix; the $v_i$ are the columns of $A$. Consider the matrix $B = [v_1 \ 0 \ \cdots \ 0]$, and compute the directional derivative $D_B \det(A)$.)
(5) For $A$ an arbitrary subset of $\mathbb{R}^n$, we had two definitions in class of a smooth map $F\colon A \to \mathbb{R}^m$:
   (a) There is an open subset $U \subset \mathbb{R}^n$ containing $A$ and a smooth map $G\colon U \to \mathbb{R}^m$ so that $G|_A = F$.
   (b) For each $x \in A$ there is a neighborhood $U_x \ni x$ in $\mathbb{R}^n$ and a smooth map $G_x\colon U_x \to \mathbb{R}^m$ so that $G_x|_{U_x \cap A} = F|_{U_x \cap A}$.

   Prove these two definitions are equivalent. (Hint: one direction is trivial. For the other, use a partition of unity subordinate to an appropriate cover.)

**Optional practice** (not required for anyone):

(6) Lee, Problem (not exercise) 1-8.
(7) Lee Problem (not exercise) 1-9.

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