MATH 432/532 SPRING 2017
HOMEWORK 2
DUE JANUARY 18, 2017.

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Reminder: homework is due at the *beginning* of class, handed to me or in my mailbox on the second floor of Fenton.

Problems:

(1) (Rudin, Exercise 9.8) Suppose that U is an open subset of $\mathbb{R}^n$ and $f\colon U \to \mathbb{R}$ is differentiable on U and has a local maximum at a point $a \in U$. Prove that $DF(a) = 0$.

(2) Another version of the contraction mapping principle is the following: Fix a compact metric space X and a continuous function $F\colon X \to X$ with the following property: if $x, y \in X$ and $x \neq y$ then $d(F(x), F(y)) < d(x, y)$. Then F has a unique fixed point.

Prove this. (Hint: since X is compact, any continuous function $X \to \mathbb{R}$ takes on a minimum value. Construct a function $g\colon X \to \mathbb{R}$ so that $g(x) = 0$ if and only if x is a fixed point of F.)

(3) (Rudin, Exercise 9.9) Suppose that U is a connected, open subset of $\mathbb{R}^n$ and $F\colon U \to \mathbb{R}^n$ is a differentiable function such that $DF(x) = 0$ for all $x \in U$. Prove that F is constant.

(4) (Rudin, Exercise 9.17) Define $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ by $F(x, y) = (e^x \cos(y), e^x \sin(y))$.
  (a) What is the image of F?
  (b) Use the inverse function theorem to show that every point in $\mathbb{R}^2$ has a neighborhood U so that $F|_U$ is injective.
  (c) Show that F is not injective.
  (d) Let $a = (0, \pi/3)$ and $b = F(a)$. Find a local inverse for F near b, compute its derivative, and compare your result with what the inverse function theorem tells you.

Challenge problems (required for Math 532, optional for 432):

(5) (Rudin, Exercise 9.13) Suppose that $F\colon \mathbb{R} \to \mathbb{R}^3$ is a differentiable function such that $\|F(t)\| = 1$ for all $t \in \mathbb{R}$. Prove that $F'(t) \cdot F(t) = 0$. (Justify each step!)

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