MATH 432/532 SPRING 2017
HOMEWORK 10
DUE MARCH 17, 2017.

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Problems:

(1) Lee, Exercise 6.7.
(2) Lee, Problem 6-1.
(3) Prove that there is no smooth space-filling curve, i.e., no smooth, surjective map $f\colon [0,1] \to [0,1]^2$. (*Remark.* There are continuous, surjective maps $[0,1] \to [0,1]^2$.)
(4) Recall that a subset S of a topological space X is called *meager* if S is a countable union of nowhere dense sets. (Like "measure 0", "meager" is a notion of being small.) Prove that $\mathbb{R}$ is the union of a meager set and a measure 0 set.

Challenge problems (required for Math 532, optional for 432):

(5) Lee, Problem 6-2. (You may use the result from Problem 5-6 without proving it, though you could prove it for extra practice.)

Optional practice (not required for anyone):

(6) Deduce the Brouwer fixed point theorem for continuous maps and the hairy ball theorem for continuous vector fields from the smooth versions.
(7) Milnor, Problems (at the end of the book): 1, 2, 3, 4, 5 (start by proving it for smooth maps, using Sard's theorem and then approximate), 6, 7.

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