MATH 307 SPRING 2025
LIST OF USEFUL TAUTOLOGIES, APRIL 4 VERSION

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Do not memorize this list, especially the names. I will give you a copy of this sheet on exams. (Exception: modus ponens. Remember it.)

Inference Rules

For each rule, we give the prototypical example of how it's used, and the tautology that leads to it. More general examples are obtained by substituting formulas for the propositional variables (P, Q, etc.).

Modus Ponens (MP): this is our only basic inference rule (not induced by MP and some tautologies).

(1)	P	
(2)	$P \Rightarrow Q$	
(3)	Q	MP (2), (1)

Conjunctive Inference (CI), from $P \Rightarrow (Q \Rightarrow (P \wedge Q))$:

(1)	P	
(2)	Q	
(3)	$P \wedge Q$	CI (1), (2)

Conjunctive Simplification (CS), from $(P \wedge Q) \Rightarrow P$ and $(P \wedge Q) \Rightarrow Q$:

(1)	$P \wedge Q$	
(2)	P	CS (1) (technically, RCS)
(3)	Q	CS (1) (technically, LCS)

Contrapositive Inference (CPI), from $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow \neg P$:

(1)	$P \Rightarrow Q$	
(2)	$\neg Q \Rightarrow \neg P$	CPI (1)

Modus Tollens (MT), from $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow \neg P$:

(1)	$P \Rightarrow Q$	
(2)	$\neg Q$	
(3)	$\neg P$	MT (1), (2)

Syllogistic Inference (SI), from $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$:

(1)	$P \Rightarrow Q$	
(2)	$Q \Rightarrow R$	
(3)	$P \Rightarrow R$	SI (1), (2)

Inference by Cases (IC), from $((P \Rightarrow Q) \wedge (S \Rightarrow Q)) \Rightarrow ((P \vee S) \Rightarrow Q)$:

(1)	$P \Rightarrow R$	
(2)	$Q \Rightarrow R$	
(3)	$(P \vee Q) \Rightarrow R$	IC (1), (2)

Other Tautologies

- Disjunctive inference: $((P \vee Q) \wedge \neg P) \Rightarrow Q$, $((P \vee Q) \wedge \neg Q) \Rightarrow P$.
- Meaning of implication: $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$. (Recall: $P \Leftrightarrow Q$ means $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$.)
- Commutativity of and, or: $(P \wedge Q) \Rightarrow (Q \wedge P)$, $(P \vee Q) \Rightarrow (Q \vee P)$.
- Associativity of and, or: $((P \wedge Q) \wedge R) \Leftrightarrow (P \wedge (Q \wedge R))$ and similarly for $\vee$.
- De Morgan's law: $\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$. And $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$.
- Double negation: $P \Leftrightarrow \neg(\neg P)$.
- Absurdity: $(P \wedge (\neg P)) \Rightarrow Q$.

Tautology Principle

If $\mathcal{F}$ is a tautology and $\mathcal{F}'$ is the result of substituting some formula $\mathcal{G}$ for all instances of one of the variables in $\mathcal{F}$ then $\mathcal{F}'$ is also a tautology.

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