

MATH 307 SPRING 2025
LIST OF USEFUL TAUTOLOGIES, APRIL 2 VERSION
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Do not memorize this list, especially the names. I will give you a copy of this sheet on exams. (Exception: modus ponens. Remember it.)

- Conjunctive inference: $P \implies (Q \implies (P \wedge Q))$.
- Conjunctive simplification: $(P \wedge Q) \implies P$, $(P \wedge Q) \implies Q$.
- Syllogistic Inference: $((P \implies Q) \wedge (Q \implies R)) \implies (P \implies R)$.
- Modus Tollens: $((P \implies Q) \wedge \neg Q) \implies \neg P$.
- Inference by cases: $((P \implies Q) \wedge (S \implies Q)) \implies ((P \vee S) \implies Q)$.
- Disjunctive inference: $((P \vee Q) \wedge \neg P) \implies Q$, $((P \vee Q) \wedge \neg Q) \implies P$.
- Meaning of implication: $(P \implies Q) \iff (\neg P \vee Q)$. (Recall: $P \iff Q$ means $(P \implies Q) \wedge (Q \implies P)$.)
- Commutativity of and, or: $(P \wedge Q) \implies (Q \wedge P)$, $(P \vee Q) \implies (Q \vee P)$.
- Associativity of and, or: $((P \wedge Q) \wedge R) \iff (P \wedge (Q \wedge R))$ and similarly for $\vee$.
- De Morgan's law: $\neg(P \vee Q) \iff (\neg P \wedge \neg Q)$. And $\neg(P \wedge Q) \iff (\neg P \vee \neg Q)$.
- Double negation: $P \iff \neg(\neg P)$.
- Absurdity: $(P \wedge (\neg P)) \implies Q$.

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