MATH 307 SPRING 2025
WEEK 1 HOMEWORK
DUE APRIL 7

INSTRUCTOR: ROBERT LIPSHITZ

Solutions are due by Canvas upload at 11:59 p.m. on Monday, April 7. *Please make sure your solutions are easy to read.* Note on your solutions who you worked with, anyone else you got help from, and any materials other than the textbooks that you used. *Electronic resources (Google, ChatGPT, Chegg, etc.) except the textbooks are **not** allowed.*

Required problems (turn these in):

(1) In English, we have lots of different ways of expressing conditional statements. For each of the following, rewrite it using the "If–Then" form in a way that preserves the meaning:
  (a) Passing the test requires getting 70%.
  (b) I get mad whenever you do that.
  (c) Being first in line guarantees getting a ticket.
  (d) You will fail the class unless you hand in your exam.

(2) Recall the tautologies $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$, $\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$, and $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$. Use these to write English language versions for the negation of each sentence below:
  (a) If you wash the dishes, I will pay you \$10.
  (b) I will wash the car or I will mow the lawn.
  (c) If I go to the movies, then I will see Barbie or I will see Oppenheimer.
  (d) If it is hot and dry then my cat will be happy.

(3) Give the truth table for each of the following statements. Say which of the statements are tautologies, which are contradictions, and which are neither.
  (a) $(P \wedge Q) \Rightarrow (P \vee Q)$
  (b) $(P \wedge Q) \Rightarrow \neg P$
  (c) $(P \vee Q) \wedge \neg(P \wedge Q)$
  (d) $(P \vee \neg P) \Rightarrow (Q \vee \neg Q)$

For the proofs below, you can use the logic rules MP, CS, CI, CPI, MT, SI, and IC. See the list of rules on the course webpage. For exercises that call for filling in blanks, you need to copy the whole proof onto a sheet of your homework—don't just fill in the blanks on this exercise sheet.

When you use a tautology, say where you have seen it before: either cite it by name (e.g., DeMorgan's Law), say"in class", or refer to a specific homework problem where you checked it was a tautology. If you can't do any of these, include the truth table that shows it is a tautology.

(4) Prove that $P, P \Leftrightarrow Q \;\vdash\; Q$ by filling in the blanks below. This rule of logic is called "Modus Ponens for the Biconditional (MPB)". Once you have proven it, you can use it in the other problems.

	Statement	Explanation
1.	P	??
2.	$P \Leftrightarrow Q$	??
3.	$P \Rightarrow Q$	??
4.	Q	??

(5) Fill in the blanks to give a proof of $A \vee B,\ C \Rightarrow \neg A, B \Rightarrow D,\ C \Rightarrow \neg D\ \vdash\ \neg C$.

	Statement	Explanation
1.	$A \vee B$	??
2.	$C \Rightarrow \neg A$	??
3.	$B \Rightarrow D$	??
4.	??	Hyp
5.	$\neg(\neg A) \Rightarrow \neg C$	??
6.	$A \Rightarrow \neg(\neg A)$	Taut.
7.	$A \Rightarrow \neg C$	??
8.	$\neg(\neg D) \Rightarrow \neg C$	??
9.	$D \Rightarrow \neg(\neg D)$	??
10.	$D \Rightarrow \neg C$	??
11.	$B \Rightarrow \neg C$	??
12.	$A \vee B \Rightarrow \neg C$	??
13.	??	??

Also, check any tautologies on lines where you wrote "Taut."

(6) Show that $\neg Q \Rightarrow \neg P,\ \neg Q \wedge R\ \vdash\ \neg P$.

(7) Show that $R \Rightarrow S,\ R \wedge Q\ \vdash\ R \wedge S$.

(8) Show that $[P \vee R] \Rightarrow Q,\ P\ \vdash\ P \wedge Q$.

(9) Show that $(\neg P \wedge S) \Rightarrow R,\ \neg P \Rightarrow S,\ \neg P\ \vdash\ \neg(P \vee \neg R)$.

(10) Show that $Q,\ [P \Rightarrow Q] \Rightarrow [R \wedge S]\ \vdash\ S$. (This can be done in 6 lines if you find the right tautology. Try working backwards.)

(11) There are five people considering going to the beach: Alex, Boris, Chris, Dev, and Ellen. The following are true statements:
- If Alex goes to the beach, then Ellen will not go.
- If Dev goes (to the beach), then Chris will go too.
- If Dev does not go, then Ellen will go.
- If Ellen goes to the beach, then either Alex will go or Boris will not go.

This is all too complicated for Chris, who gets fed up and decides not to go to the beach. Determine who goes to the beach, and who stays home.

Convert this into a problem in symbolic logic. Use "A" to stand for the proposition "Alex goes to the beach". Give a symbolic logic proof, as we have been doing in the above exercises, where you use the rules of logic to deduce who does and does not go to the beach. You should have five hypotheses.

(12) Some number tabulation, for likely use later in the course:
  (a) Remember that a natural number $p > 1$ is *prime* if the only numbers that divide it are 1 and p. For example, 2, 3, 5, and 7 are prime, but 4, 6, and 8 aren't. List all prime numbers less than 50.
  (b) A *perfect square* is an integer that is the square of another one. For example, 0, 1, 4, and 9 are perfect squares. List all perfect squares less than 50.

(c) Which numbers up to 30 can be written as the sum of 2 perfect squares? Which primes? (To get you started: $0 = 0^2 + 0^2$, $1 = 1^2 + 0^2$, $2 = 1^2 + 1^2$, 3 can't be written this way, $4 = 2^2 + 0^2$, $5 = 2^2 + 1^2$,....)

(d) Which numbers of up 30 can be written as a sum of 3 perfect squares? Which primes? (Hint: the first that *can't be* is 7.) (Note: since 0 is a perfect square, I could have equivalently said "at most 3 perfect squares.")

(e) Which numbers up to 30 can be written as a sum of 4 perfect squares?

(13) Pascal's triangle is the array of numbers

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Each number in the triangle is the sum of the two above it. (Any space not filled with a number is 0.) List the first 10 rows of Pascal's triangle. Circle all the even numbers in it, and make another version where you circle all the numbers divisible by 3.

Compute it yourself—don't just look up a version online and copy it down. On the other hand, if you want to write your own Excel or Google Sheets spreadsheet to compute it, or a computer program, go for it.

Extra practice. Do not turn these in, but if you had trouble with some of the required problems, try these:

() Write in "If–Then" form:

(a) My cat gets scared when the phone rings.

(b) To get paid, you have to work.

(c) I like to sit outside, unless it's raining.

() Write the negation of each sentence below:

(a) I ate steak last night and I ate cereal this morning.

(b) If I am a monkey and you are a baboon, then pigs can fly.

(c) If it's hot and sunny, then I eat ice cream.

() Give the truth table for each of the following statements. Say which of the statements are tautologies.

(a) $\neg[\neg P \wedge \neg Q]$

(b) $(P \vee Q) \Leftrightarrow (\neg P \Rightarrow Q)$
(c) $(Q \wedge \neg Q) \Rightarrow P$
(d) $Q \wedge (P \Rightarrow Q)$
(e) $(P \wedge Q) \Rightarrow (P \vee Q)$
(f) $(P \wedge Q) \Leftrightarrow (\neg Q \Rightarrow P)$
(g) $(P \vee Q) \Rightarrow (\neg Q \vee P)$
(h) $[P \Rightarrow (Q \Rightarrow R)] \Leftrightarrow [(P \wedge Q) \Rightarrow R]$.

() Fill in the blanks to give a proof of $[P \wedge Q] \Rightarrow R, \neg R, P \vdash \neg Q$.

	Statement	Explanation
1.	$[P \wedge Q] \Rightarrow R$	??
2.	$\neg R$	??
3.	$\neg[P \wedge Q]$	??
4.	$\neg[P \wedge Q] \Leftrightarrow [\neg P \vee \neg Q]$	??
5.	??	MPB, For 4, For 3
6.	$[\neg P \vee \neg Q] \Leftrightarrow [P \Rightarrow \neg Q]$	??
7.	$P \Rightarrow \neg Q$	??
8.	??	hyp.
9.	??	??

() Show that $Q \vee R, \neg R \vdash Q$.

() Show that $\neg[P \wedge R] \Rightarrow \neg S, \neg S \Rightarrow Q, \neg[P \wedge R] \vdash \neg S \wedge Q$.

() Show that $R \wedge \neg Q, P \Rightarrow Q \vdash R \wedge \neg P$.

() Show that $Q \Rightarrow S, \neg S \vee R, \neg R \vdash \neg Q$.

() Four mathematicians, Arkady, Boris, Chris, and Dev, are suspected of murder. They testify as follows:

Arkady: If Boris is guilty, so is Dev.
Boris: Arkady is guilty, but Dev is not.
Chris: I'm not guilty, but either Arkady or Dev is guilty.
Dev: If Arkardy is not guilty, then Chris is guilty.

(a) Are the four statements consistent?
(b) If everyone is telling the truth, who is guilty?
(c) If the guilty people lie and the innocent tell the truth, who is guilty?

(Note: some problems written by others, and in particular some adapted from Jean Rubin, *Mathematical Logic: Applications and Theory*.)

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