

**MATH 692 SPRING 2024**  
**MINI-PAPER TOPICS.**  
INSTRUCTOR: ROBERT LIPSHITZ

Here is a list of possible topics for mini-papers. You are welcome to suggest others, but topics should be cleared with me in advance. If you choose to write a paper on one of these, add “**Taken—RL**”, with “RL” replaced by your initials, next to it (in the git repository).

Last updated April 3, 2024. This list of topics will expand over time.

- (1) Use the classification of surfaces to prove that the mapping class group acts transitively on the space of smooth, non-separating simple closed curves in  $\Sigma$ .
- (2) **MA** Prove that the mapping class  $\tau_\gamma$  of a Dehn twist along a curve  $\gamma$  depends only on the isotopy class of  $\gamma$ .
- (3) **HH** Prove Smale’s Theorem, that  $\text{Diff}^+(I^2, \partial I^2)$  is contractible (giving a complete statements of results used along the way, like the Poincaré-Bendixson Theorem, but not re-proving them).
- (4) Prove that  $\text{Diff}^+(I^2, \partial I^2) \cong \text{Diff}^+(D^2, \partial D^2)$ .
- (5) **KB** Prove Macbeath’s Theorem.
- (6) Prove that every triangulated, orientable 3-manifold admits a Heegaard splitting.
- (7) Any problem marked with a star on Homework 1.
- (8) Prove that  $\text{Diff}^+(S^2) \simeq SO(3)$ .
- (9) Any problem marked with a star on Homework 2.
- (10) **HH** Prove that for any real numbers  $a, b, c > 0$  there is a unique right-angled, geodesic hexagon (in  $\mathbb{H}^2$ , say) so that the lengths of its first, third, and fifth sides are  $a, b, c$ , by fleshing out the proof in the caption of Figure 4.15 in Thurston’s book.

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