

MATH 636 HOMEWORK 9
DUE MAY 28, 2021.
 INSTRUCTOR: ROBERT LIPSHITZ

- (1) Hatcher 4.3.12 (p. 420).
- (2) Hatcher 4.3.16 (p. 420).
- (3) (a) Suppose $\pi_i(X) = 0$ for $i < n$, for some $n \geq 2$. Show that there is a map $X \rightarrow K(\pi_n(X), n)$ inducing an isomorphism on π_n .
 (b) Given a simply-connected space X and an integer n , by taking iterated homotopy fibers, build another space Y and a map $f: Y \rightarrow X$ so that $\pi_i(Y) = 0$ for $i < n$ and $f_*: \pi_i(Y) \rightarrow \pi_i(X)$ is an isomorphism for $i \geq n$. (Do not use the existence of Moore-Postnikov towers.)
- (4) Up to homotopy equivalence, how many spaces X are there with $\pi_2(X) \cong \mathbb{Z}$, $\pi_3(X) \cong \mathbb{Z}/2\mathbb{Z}$, and all other homotopy groups trivial? Describe each of these spaces as explicitly as you can.
- (5) Fix $n \geq 2$. Describe explicitly the first n stages of a Postnikov tower for $\mathbb{C}P^n$ consisting of principal fibrations (up through what Hatcher labels X_n), and compute the corresponding k -invariants $k_1, \dots, k_{2n-1}$. Show further that k_{2n} is nontrivial.
- (6) (a) Let V be a finite-dimensional vector space over a field k , and F a filtration on V . Prove that V is isomorphic (as vector spaces) to the associated graded vector space (with respect to this filtration).
 (b) Give a counterexample if, instead, V is a free abelian group.

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Suggested review / qualifying exam practice (not to turn in):

- (1) Hatcher 4.3.11, 4.3.15, 4.3.17, 4.3.20, 4.3.21, 4.3.22.

More problems to think about but not turn in:

- (1) Deduce the bundle homotopy lemma for fibrations as stated in class from Hatcher, Proposition 4.62.
- (2) The point of this exercise is to illustrate that inverse limits can be somewhat bonkers, even for nice maps between nice spaces. Consider the inverse system

$$S^1 \leftarrow S^1 \leftarrow S^1 \leftarrow \dots,$$

where each map in the sequence is the double cover $z \mapsto z^2$. Let S_n^1 denote the n^{th} term in this sequence and $p_n: S_{n+1}^1 \rightarrow S_n^1$ the n^{th} map. Let $S_\infty^1 = \varprojlim S_n^1$.

- (a) Let $(z_1, z_2, \dots) \in S_\infty^1$. Show that the universal covering maps $q_n: \mathbb{R} \rightarrow S_n^1$, $q_n(t) = z_n^{-1} e^{2\pi i t/2^n}$, induce a continuous map $q_\infty: \mathbb{R} \rightarrow S_\infty^1$.
- (b) Show that the image of q_∞ is a path component of S_∞^1 .
- (c) Show that q_∞ is not surjective.
- (d) Show that the image of q_∞ is dense.
- (e) Conclude that S_∞^1 is not locally path connected.

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