MATH 636 HOMEWORK 7
DUE MAY 14, 2021.

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(1) Prove that, for the category of based topological spaces, the reduced suspension functor is left adjoint to the based loop space functor. (See also Optional Problem (OP-1) for more discussion.)
(2) Hatcher 4.2.34 (p. 392).
(3) Hatcher 4.2.37 (p. 392). (You'll have to read the definitions of Whitehead products and H-spaces first.)
(4) Recall that $SO(3) \cong \mathbb{R}P^3$. Compute $\pi_i(SO(3))$ and $\pi_i(SO(4))$ for $i \leq 3$. Similarly, use the fact that $SU(2) \cong S^3$ to compute $\pi_i(SU(2))$ for $i \leq 5$, $\pi_i(SU(3))$ for $i \leq 3$, and prove $\pi_5(SU(3))$ is nontrivial. (You may take as given the stable homotopy groups of spheres listed in Hatcher, but be careful about what the stable range is.)

Suggested review / qualifying exam practice (not to turn in):

(1) Hatcher 4.2.14, 4.2.19, 4.2.30.

More problems to think about but not turn in:

(OP-0) In required problem 1, you prove that $\mathrm{Hom}(\Sigma(X, x_0), (Y, y_0)) \cong \mathrm{Hom}((X, x_0), \Omega(Y, y_0))$, naturally. Here, there are two ways you can interpret $\cong$: a bijection of sets, which is easier and what the required problem intended, or as a homeomorphism of topological spaces, where the mapping spaces have the compact-open topology. Prove the stronger result, for the category of based locally compact, Hausdorff spaces. In particular, this implies that $[\Sigma(X, x_0), (Y, y_0)] = [(X, x_0), \Omega(Y, y_0)]$ (for X, Y locally compact, Hausdorff).

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