

**MATH 636 HOMEWORK 4**  
**DUE APRIL 23, 2021.**  
INSTRUCTOR: ROBERT LIPSHITZ

- (1) Hatcher 4.1.16 (p. 359). (Hint: build  $Z$  by attaching cells to  $X$ .)
- (2) Hatcher 4.1.17 (p. 359).
- (3) Hatcher 4.1.19 (p. 359).
- (4) Hatcher 4.1.20 (p. 359).
- (5) Hatcher 4.2.5 (p. 389).

Suggested review / qualifying exam practice (not to turn in):

- (1) Adapt the proof from class to prove the injectivity statement in the excision theorem for homotopy groups.

More problems to think about but not turn in:

- (1) Suppose that  $M$  is a smooth,  $n$ -dimensional manifold, with tangent bundle  $TM$ . Define a vector bundle  $\Lambda^n TM$  whose fiber over a point  $x \in M$  is  $\Lambda^n T_x M$ , the top exterior power of the tangent space at  $x$ . (“Define” means you should write down a precise definition of such a bundle, not that the previous sentence is a precise definition.) There is an associated covering space  $(\Lambda^n TM \setminus 0)/\mathbb{R}_+$  obtained by deleting the 0-section and quotienting by scaling by positive real numbers. Show that  $(\Lambda^n TM \setminus 0)/\mathbb{R}_+$  is the orientation double cover of  $M$ .
- (2) Given a manifold  $M$ , give a natural definition of an element  $w \in H^1(M; \mathbb{F}_2)$  so that  $w = 0$  if and only if  $M$  is orientable. (If you want “natural” to be precise,  $w$  is natural with respect to codimension-0 embeddings of (open) submanifolds.)

*Email address:* lipshitz@uoregon.edu