

MATH 342
WRITTEN HOMEWORK 6
DUE MAY 11, 2020.

INSTRUCTOR: ROBERT LIPSHITZ

Updated: fixed a typo in Problem 4c.

Required problems (hand these in):

- (1) Determine whether each of the following sets of vectors is orthonormal, orthogonal but not orthonormal, or not orthogonal.

(a)

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}, \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

- (2) Compute the orthogonal projection of $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ onto the line L through $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$ and the origin. Then write $\vec{v}$ as the sum of a vector in L and a vector orthogonal to L .

- (3) Consider the pair of vectors

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

- (a) Normalize these vectors to obtain an orthonormal set.

- (b) Find the orthogonal projection of $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ onto the subspace

$$\text{Span}(\{\vec{u}_1, \vec{u}_2\}).$$

- (4) Let

$$\vec{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

- (a) Find the closest point to $\vec{y}$ in $\text{Span}(\{\vec{v}_1, \vec{v}_2\})$.

- (b) Write $\vec{y}$ as the sum of a vector in $\text{Span}(\{\vec{v}_1, \vec{v}_2\})$ and a vector orthogonal to $\text{Span}(\{\vec{v}_1, \vec{v}_2\})$.
- (c) Find the best approximation to $\vec{y}$ by vectors of the form $c_1\vec{v}_1 + c_2\vec{v}_2$. What are c_1 and c_2 ?
- (5) Consider the subspace

$$W = \text{Span} \left(\left\{ \begin{bmatrix} 4 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 7 \end{bmatrix} \right\} \right) \subset \mathbb{R}^3.$$

Apply the Gram-Schmidt process to find an orthonormal basis for W .

- (6) Find an orthonormal basis for the column space of the following matrix:

$$\begin{bmatrix} 4 & 3 & 5 \\ 2 & 4 & 5 \\ 2 & 0 & -4 \\ 1 & 5 & 3 \end{bmatrix}$$

- (7) Let $\{\vec{v}_1, \dots, \vec{v}_k\}$ be an orthonormal basis for a subspace W of $\mathbb{R}^n$. Define $T: \mathbb{R}^n \rightarrow \mathbb{R}^n$ to be $T(\vec{x}) = \text{proj}_W(\vec{x})$. Show that T is a linear transformation. (Hint: start by writing down the formula for T .)

Suggested practice (don't hand these in):

- Please read and make sure you can do the practice problems in Sections 6.2, 6.3, and 6.4.
- Use Exercises 6.2.23, 6.2.24, 6.3.21, 6.3.22, 6.4.17(a,b), 6.4.18(a,b) for review.
- If you have some experience writing proofs, try Exercises 6.2.28, 6.2.29, 6.2.33.
- If you had trouble or got help with any of the assigned problems, solve another, similar problem (or two).

Similar problems:

HW Problems	Similar textbook problems
1	6.2.1–6
2	6.2.11–16
3	6.2.17–22, 6.3.3–6
4	6.3.7–14
5	6.4.1–8
6	6.4.9–12
7	6.4.22

Blog. Optional:

- Work through the blog post “Dot Products and Orthogonality”.
- Use CoCalc to check your answers to the problems above (except for Problem 7).
- Write a function in CoCalc (Sage) which computes the orthogonal projection of $\vec{v}$ to $\text{Span}(\vec{w})$, where $\vec{v}$ and $\vec{w}$ are vectors in the same $\mathbb{R}^n$. (Your function should take two inputs, $\vec{v}$ and $\vec{w}$.) Test your function in some examples. Only slightly harder: write a function which computes the orthogonal projection of $\vec{v}$ onto $\text{Span}(\vec{w}_1, \dots, \vec{w}_k)$. (In the second case, start by applying Sage’s Gram-Schmidt to $\vec{w}_1, \dots, \vec{w}_k$.)

Email address: `lipshitz@uoregon.edu`