

MATH 342
WRITTEN HOMEWORK 4
DUE APRIL 27, 2020.
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Required problems (hand these in):

- (1) Let V be a vector space and $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ an ordered basis for V . Suppose that $T: V \rightarrow \mathbb{R}^2$ is a linear transformation so that

$$T(x_1\vec{b}_1 + x_2\vec{b}_2 + x_3\vec{b}_3) = \begin{bmatrix} x_1 - x_2 + 2x_3 \\ 3x_1 - 5x_2 \end{bmatrix}.$$

Find the matrix for T with respect to $\mathcal{B}$ and the standard basis for $\mathbb{R}^2$.

- (2) Let $T: \mathbb{P}_2 \rightarrow \mathbb{P}_3$ be the transformation

$$T(p(t)) = (t+3)p(t) + p'(t).$$

- (a) Compute $T(2 + 3t + 4t^2)$.
(b) Show that T is a linear transformation.
(c) Find the matrix for T with respect to the ordered bases $\{1, t, t^2\}$ for $\mathbb{P}_2$ and $\{1, t, t^2, t^3\}$ for $\mathbb{P}_3$.
(3) Define $T: \mathbb{P}_1 \rightarrow \mathbb{R}^3$ by

$$T(p(t)) = \begin{bmatrix} p(0) \\ p(1) \\ p(2) \end{bmatrix}.$$

- (a) Compute $T(2 + 3t)$.
(b) Show that T is a linear transformation.
(c) Find the matrix for T with respect to the ordered basis $\{1, t\}$ for $\mathbb{P}_1$ and the standard basis for $\mathbb{R}^3$.
(4) Define $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by

$$T(\vec{x}) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \vec{x}.$$

Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}.$$

Find the matrix $[T]_{\mathcal{B}}$ for T with respect to $\mathcal{B}$.

- (5) Define $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by

$$T(\vec{x}) = \begin{bmatrix} -7 & 18 \\ -3 & 8 \end{bmatrix} \vec{x}.$$

Find a basis $\mathcal{B}$ so that $[T]_{\mathcal{B}}$ is diagonal (and find $[T]_{\mathcal{B}}$).

- (6) Let $V = \{a_0 + a_1e^x + a_2e^{2x} + a_3e^{3x} \mid a_0, a_1, a_2, a_3 \in \mathbb{R}\}$, which is a subspace of the vector space of functions $\mathbb{R} \rightarrow \mathbb{R}$. Let $T: V \rightarrow V$ be the linear transformation $T(f(x)) = f''(x)$. What are the eigenvalues of T ? Find an eigenvector corresponding to each eigenvalue.
- (7) Consider the linear transformation $T: \mathbb{P}_2 \rightarrow \mathbb{P}_2$ given by $T(p(x)) = p(0) + p(1) + p'(x) + 3x^2p''(x)$. Let $\mathcal{B}$ be the basis $\{1, x, x^2\}$ for $\mathbb{P}_2$.
- Find the matrix A for T with respect to the basis $\mathcal{B}$.
 - Find the eigenvalues of A , and a basis for $\mathbb{R}^3$ consisting of eigenvectors of A .
 - Find a basis for $\mathbb{P}_2$ consisting of eigenvectors for T .
- (8) Let

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}.$$

Find the eigenvalues of A (which may be complex numbers) and a basis for each eigenspace in $\mathbb{C}^2$.

- (9) Let

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}.$$

Find the eigenvalues of A (which may be complex numbers) and a basis for each eigenspace in $\mathbb{C}^2$.

- (10) The matrix

$$\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

corresponds to the composition of a rotation and a scaling. Give the angle ϕ of the rotation and the scale factor r .

- (11) Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$. Compute A^{20} (by hand).
- (12) Plot the following complex numbers in the complex plane, on one graph: $1 + i$, $2 - 3i$, $2e^{\frac{\pi}{4}i}$, $2e^{\frac{\pi}{2}i}$, $2e^{\frac{3\pi}{4}i}$, $2\cos(\pi/3) + 2i\sin(\pi/3)$.
- (13) Plot the following complex numbers in the plane, on one graph: $(1 + i\sqrt{3})$, $(1 + i\sqrt{3})^2$, $(1 + i\sqrt{3})^3$, $(1 + i\sqrt{3})^4$, $e^{\frac{\pi}{3}i}$, $e^{\frac{2\pi}{3}i}$, $e^{\frac{3\pi}{3}i}$, $e^{\frac{4\pi}{3}i}$, $e^{\frac{5\pi}{3}i}$.

Suggested practice (don't hand these in):

- Please read and make sure you can do the practice problems in Sections 5.4 and 5.5.
- If you have taken a course on writing proofs, try problems 5.4.25, 5.4.26, 5.4.19, 5.4.20, 5.4.21, 5.4.23, 5.5.23, and 5.5.24.
- If you had trouble or got help with any of the assigned problems, solve another, similar problem (or two).

Similar problems:

HW Problems	Similar textbook problems
1–3	5.4.1–10
4	5.4.11–12
5	5.4.13–16
8–9	5.5.1–6
10	5.5.7–12

Blog. Optional:

- Work through the blog post “Complex numbers”.
- Use CoCalc to check your answers to problems 4, 8, 9, 12, and 13.

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