MATH 636 HOMEWORK 9
DUE MAY 29, 2019.

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Problems to turn in:

(P-0) Deduce the bundle homotopy lemma for fibrations as stated in class from Hatcher, Proposition 4.62.

(P-0) The point of this exercise is to illustrate that inverse limits can be somewhat bonkers, even for nice maps between nice spaces. Consider the inverse system

$$S^1 \leftarrow S^1 \leftarrow S^1 \leftarrow \cdots,$$

where each map in the sequence is the double cover $z \mapsto z^2$. Let S^1_n denote the n^{th} term in this sequence and $p_n : S^1_{n+1} \to S^1_n$ the n^{th} map. Let $S^1_\infty = \varprojlim S^1_n$.

(a) Let $(z_1, z_2, \dots) \in S^1_\infty$. Show that the universal covering maps $q_n : \mathbb{R} \to S^1_n$, $q_n(t) = z_n^{-1} e^{2\pi i t/2^n}$, induce a continuous map $q_\infty : \mathbb{R} \to S^1_\infty$.

(b) Show that the image of q_∞ is a path component of S^1_∞.

(c) Show that q_∞ is not surjective.

(d) Show that the image of q_∞ is dense.

(e) Conclude that S^1_∞ is not locally path connected.

(P-0) (a) Suppose $\pi_i(X) = 0$ for $i < n$, for some $n \geq 2$. Show that there is a map $X \to K(\pi_n(X), n)$ inducing an isomorphism on π_n.

(b) Given a simply-connected space X and an integer n, by taking iterated homotopy fibers, build another space Y and a map $f : Y \to X$ so that $\pi_i(Y) = 0$ for $i < n$ and $f_* : \pi_i(Y) \to \pi_i(X)$ is an isomorphism for $i \geq n$. (Do not use the existence of Moore-Postnikov towers.)

(P-0) Hatcher 4.3.16.

(P-0) Hatcher 4.3.24.

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