MATH 342 SECTION 33650
WRITTEN HOMEWORK 4
DUE APRIL 30, 2018.

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Required textbook problems (hand these in):

(1) §5.4: 6, 9, 12, 16.
(2) Let $V = \{a_0 + a_1 e^x + a_2 e^{2x} + a_3 e^{3x} \mid a_1, a_2, a_3 \in \mathbb{R}\}$, which is a subspace of the vector space of functions $\mathbb{R} \to \mathbb{R}$. Let $T\colon V \to V$ be the linear transformation $T(f(x)) = f''(x)$. What are the eigenvalues of T? Find an eigenvector corresponding to each eigenvalue.
(3) Consider the linear transformation $T\colon \mathbb{P}_2 \to \mathbb{P}_2$ given by $T(p(x)) = p(0) + p(1) + p'(x) + 3x^2 p''(x)$. Let $\mathcal{B}$ be the basis $\{1, x, x^2\}$ for $\mathbb{P}_2$.
   (a) Find the matrix A for T with respect to the basis $\mathcal{B}$.
   (b) Find the eigenvalues of A, and a basis for $\mathbb{R}^3$ consisting of eigenvectors of A.
   (c) Find a basis for $\mathbb{P}_2$ consisting of eigenvectors for T.
(4) §5.5: 3, 6, 8
(5) Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$. Compute A^{20} (by hand).
(6) Plot the following complex numbers in the complex plane, on one graph: $1 + i$, $2 - 3i$, $2e^{\frac{\pi}{4}i}$, $2e^{\frac{\pi}{2}i}$, $2e^{\frac{3\pi}{4}i}$, $2\cos(\pi/3) + 2i\sin(\pi/3)$.
(7) Plot the following complex numbers in the plane, on one graph: $(1 + i\sqrt{3})$, $(1 + i\sqrt{3})^2$, $(1 + i\sqrt{3})^3$, $(1 + i\sqrt{3})^4$, $(1 + i\sqrt{3})^5$, $e^{\frac{\pi}{3}i}$, $e^{\frac{2\pi}{3}i}$, $e^{\frac{3\pi}{3}i}$, $e^{\frac{4\pi}{3}i}$, $e^{\frac{5\pi}{3}i}$.

Suggested practice (don't hand these in):

- Please read and make sure you can do the practice problems in Sections 5.4 and 5.5.
- If you have taken a course on writing proofs, try problems 5.4.25, 5.4.26, 5.4.19, 5.4.20, 5.4.21, 5.4.23.
- If you had trouble or got help with any of the assigned problems, solve another, similar problem (or two).

Bonus points. Sage, as usual. For this week:

(1) Work through the blog post "complex numbers in Sage".
(2) Use Sage to check your answers to 5.4.12, 5.6.14, 5.5.3, and 5.5.6.

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