MATH 431/531 FALL 2021
HOMEWORK 9
DUE NOVEMBER 24, 2021.

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Note: there are a lot of good problems in Chapter 4. At least read through all the problems (starting on page 122).

Revised: removed 4-27 and 4-32, because we probably will not talk about these topics in class, and half of the first problem because we did prove that in class.

Problems:

(1) Prove Lee, Theorem 4.47.
(2) Lee, Exercise 4.70.
(3) Lee, Problem 4-23.

Challenge problems (required for Math 531, optional for 431):

(4) Let X be a topological space. An *end* of X is a function f from $\{$compact subsets of $X\}$ to $\{$subsets of $X\}$ with the following properties:
   (a) For each compact $K \subset X$, $f(K)$ is a connected component of $X \setminus K$.
   (b) If $K \subset L$ then $f(L) \subset f(K)$.

   Prove that $\mathbb{R}$ has two ends and $\mathbb{R}^n$ has one end for $n > 1$. Give an example of a subspace of $\mathbb{R}^n$ with infinitely many ends, and prove your subspace has infinitely many ends.

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