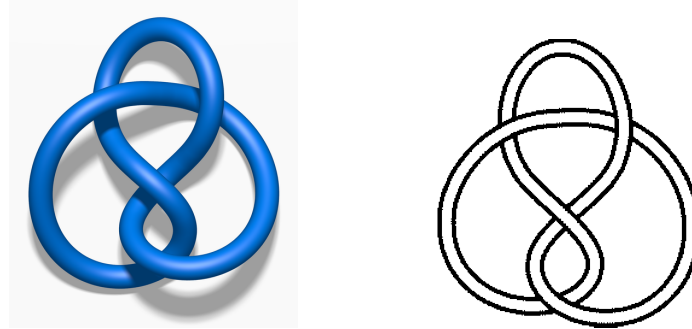


MATH 634 HOMEWORK 5
DUE OCTOBER 24, 2018.
 INSTRUCTOR: ROBERT LIPSHITZ

Required problems, to turn in:

- (1) Hatcher 1.2.4.
- (2) Hatcher 1.2.9.
- (3) Hatcher 1.2.16. Do this two ways. First, use Hatcher's version of Van Kampen's theorem where he allows covers by infinitely many open sets. Second, use the version of the Seifert-van Kampen theorem for two sets. (Hint for the second: $[0, 1]$ and $[0, 1] \times [0, 1]$ are compact.)
- (4) Hatcher 1.2.22. And:
 - (c) Let K denote Figure 8 Knot:



(Pictures from the Knot Atlas.)

Compute $\pi_1(\mathbb{R}^3 \setminus K)$.

- (d) For K the Figure 8 Knot, show that there is a surjective homomorphism from $\pi_1(\mathbb{R}^3 \setminus K)$ to D_5 , the group of symmetries of a regular pentagon. Use this to conclude that K is genuinely knotted.
- (5) Recall the universal mapping property for free products of groups: given groups G and H there is a group $G * H$ and maps $G \rightarrow G * H$, $H \rightarrow G * H$ so that for any other group L and maps $g: G \rightarrow L$, $h: H \rightarrow L$ there is a unique map $(g * h): G * H \rightarrow L$ so that

$$G * H @>> [dr]^{g*h} G[l][d]^g H[u][r]_h L$$

commutes.

What is the analogue for free products with amalgamation? Prove your claim.

To think about but not turn in:

- (1) Recall that we defined $H^1(X) = [X, S^1]$. What's the analogue of our weak van Kampen theorem (i.e., $\pi_1(X \vee Y) \cong \pi_1(X) * \pi_1(Y)$) for H^1 ? Of the full Van Kampen theorem?
- (2) How does Exercise 5 relate to wedge sums? What's the analogue of free products with amalgamation in other categories (e.g., sets, topological spaces, based topological spaces)? Can you give a very abstract statement of Van Kampen's theorem?
- (3) Read through the remaining problems in Section 1.2, and do any that seem difficult or surprising. In particular, there are a bunch of nice computation problems

(like 1.2.11–1.2.14) which would be good practice. There are also some conceptually interesting ones, like 1.2.1, 1.2.2, 1.2.17, and 1.2.19.

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