

MATH 690 HOMEWORK 6

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- (1) Compute $H^*(BSO(n); \mathbb{F}_2)$ via a different method from class (e.g., inductively using the Gysin sequence).
- (2) Prove:
 - (a) For a real vector bundle E , $E \otimes \mathbb{C} \cong \overline{E \otimes \mathbb{C}}$ as complex bundles.
 - (b) For an n -dimensional, oriented vector bundle E , $E \otimes \mathbb{C} \cong E \oplus E$ as real bundles, and this multiplies the orientation by $(-1)^{n(n+1)/2}$.
 - (c) For a complex vector bundle E , $E \otimes_{\mathbb{R}\mathbb{C}} \cong E \oplus \overline{E}$ as complex vector bundles.
- (3) Compute the Pontrjagin classes of the universal bundle over infinite-dimensional quaternionic projective space.
- (4) Give another example of a (nontrivial) immersion obstruction coming from Pontrjagin classes.
- (5) Show that the unit sphere bundle in the universal vector bundle over $BSO(n)$ is homotopy equivalent to $BSO(n-1)$.
- (6) Milnor-Stasheff, Problem 7-B.
- (7) Milnor-Stasheff, Problem 15-A.

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