# MATH 690 HOMEWORK 5
# NOVEMBER 9, 2017.

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(1) Let $\pi\colon E \to X$ be an $n$-dimensional vector bundle. Show that if $E$ is not orientable, the obstruction theory definition of the Euler class gives a well-defined element of $H^n(X; \underline{Z})$ for an appropriate local coefficient system $\underline{Z}$.
(2) Prove: If there is a bilinear map $p\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ without zero divisors then $T\mathbb{R}P^{n-1}$ is trivial.
(3) Prove that $\mathbb{R}P^{10}$ does not immerse in $\mathbb{R}^n$ for $n < 15$.
(4) Milnor-Stasheff, Exercise 4-C.
(5) Milnor-Stasheff, Exercise 4-D.
(6) Fill in the definition of the projectivization as a bundle. What is its structure group?
(7) Prove that given line bundles $L, L'$ over $X$, $c_1(L \otimes L') = c_1(L) + c_1(L')$ and $c_1(L^*) = -c_1(L)$.
(8) Prove that the Stiefel-Whitney classes of a complex bundle are the mod-2 reductions of the Chern classes. (Hint: use splitting principle and a direct computation for canonical bundle over $\mathbb{C}P^\infty$.)
(9) Let $E$ be an $n$-dimensional vector bundle over $S^n$. Consider the fiber bundle $S^{n-1} \to SE \to S^n$. Prove that the connecting homomorphism in the long exact sequence

$$\cdots \to \pi_n(S^n) \xrightarrow{\delta} \pi_{n-1}(S^{n-1}) \to \pi_{n-1}(SE) \to \pi_{n-1}(S^n) = 0$$

is multiplication by $\pm\langle e(E), [S^n]\rangle$.
(10) Milnor-Stasheff, Exercise 12-B.
(11) In class, we gave an inductive construction of the Stiefel-Whitney classes of a smooth vector bundle over a smooth manifold by intersecting certain families of sections with the 0-section. Prove this definition agrees with some other definition of Stiefel-Whitney classes.
(12) Give a similar definition of the Steenrod squares of a cohomology class represented by a smooth submanifold.

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