

MATH 431/531 FALL 2016
HOMEWORK 9
DUE NOVEMBER 23, 2016.
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Problems:

- (1) Prove Lee, Theorems 4.46 and 4.47.
- (2) Lee, Exercise 4.70.
- (3) Lee, Problem 4-23.
- (4) Lee, Problem 4-27.
- (5) Lee, Problem 4-29.
- (6) Lee, Problem 4-32.

Challenge problems (required for Math 531, optional for 431):

- (7) Let X be a topological space. An *end* of X is a function f from $\{\text{compact subsets of } X\}$ to $\{\text{subsets of } X\}$ with the following properties:
 - (a) For each compact $K \subset X$, $f(K)$ is a connected component of $X \setminus K$.
 - (b) If $K \subset L$ then $f(L) \subset f(K)$.

Prove that $\mathbb{R}$ has two ends and $\mathbb{R}^n$ has one end for $n > 1$. Give an example of a subspace of $\mathbb{R}^n$ with infinitely many ends, and prove your subspace has infinitely many ends.

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