CORRECTION TO "A STEENROD SQUARE ON KHOVANOV HOMOLOGY"

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The problem. Formula (3.1) defines a map $\mathfrak{r}\colon \mathbb{R}^m \to \mathbb{R}^m$ or by

$$\mathfrak{r}(x_1, \dots, x_m) = (-x_1, x_2, \dots, x_m).$$

Lemma 3.3 then asserts that the map $\mathfrak{r}$ on frames takes standard frame paths between oppositely-oriented frames to nonstandard ones. There are two problems here:

1. A frame is an n-tuple of vectors in $\mathbb{R}^m$. So, it is ambiguous how $\mathfrak{r}$ defines a map from frames to frames.
2. If one extends $\mathfrak{r}$ to frames by defining $\mathfrak{r}(\vec{v}_1, \dots, \vec{v}_m) = (\mathfrak{r}(\vec{v}_1), \dots, \mathfrak{r}(\vec{v}_m))$ then the result does not take the first kind of standard frame paths to non-standard frame paths.

The correction. The correct definition of $\mathfrak{r}$, for a frame in $\mathbb{R}^{m+1}$ consisting of m vectors $\vec{v}_1, \dots, \vec{v}_m$, is

$$\mathfrak{r}(\vec{v}_1, \dots, \vec{v}_m) = (-\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m). \tag{1}$$

This is not induced by applying the same map to each vector.

By contrast, the map $\mathfrak{s}$ on frames is defined by

$$\mathfrak{s}(\vec{v}_1, \dots, \vec{v}_m) = (\mathfrak{s}(\vec{v}_1), \mathfrak{s}(\vec{v}_2), \dots, \mathfrak{s}(\vec{v}_m)) \tag{2}$$

where $\mathfrak{s}(\vec{v})$ is as defined in Formula (3.2), that is,

$$\mathfrak{s}(x_0, x_1, \dots, x_m) = (x_0, x_1, \dots, x_{m_1}, -x_{m_1+1}, x_{m_1+2}, \dots, x_m). \tag{3}$$

(Formula (3) corrects an off-by-one error in the indexing in Formula (3.2).)

There are a few places later in the paper where the map $\mathfrak{r}$ defined in Formula (3.1) actually does arise, such as when defining Ξ on page 20, when defining the map Ψ on the bottom of page 26, and again at the top of page 27. More generally, if the paper refers to a map of spaces instead of a map of frames, it means the map from Formula (3.1). When it refers to a map of frames (the main usage), though, the map $\mathfrak{r}$ should be the map on frames defined in Equation (1) here.

Note that if we write a vector $\vec{w} = (c_1, \dots, c_m)^T \in \mathbb{R}^m = \{0\} \times \mathbb{R}^m$ as $c_1\vec{e}_1 + \cdots + c_m\vec{e_m}$, using the standard basis, applying the map $\mathfrak{r}$ from Formula (3.1) to $\vec{w}$ is the same as applying

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the map of frames here to the frame $(\vec{e}_1, \dots, \vec{e_m})$ and then taking the linear combination. That is,

$$\mathfrak{r}\left(\begin{bmatrix}\vec{e}_1 & \cdots & \vec{e}_m\end{bmatrix}\begin{bmatrix}c_1\\ \vdots\\ c_m\end{bmatrix}\right) = \mathfrak{r}\left(\begin{bmatrix}\vec{e}_1 & \cdots & \vec{e}_m\end{bmatrix}\right)\begin{bmatrix}c_1\\ \vdots\\ c_m\end{bmatrix}$$

where on the left $\mathfrak{r}$ means the map from Formula (3.1) and on the right it means the map of frames in Formula (1) here. This is only true for the standard basis, but helps with checking some of the formulas in the paper.

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