
MUON G-2 MEASUREMENT

Jim Brau, University of Oregon
UO Quarknet - June 30, 2021

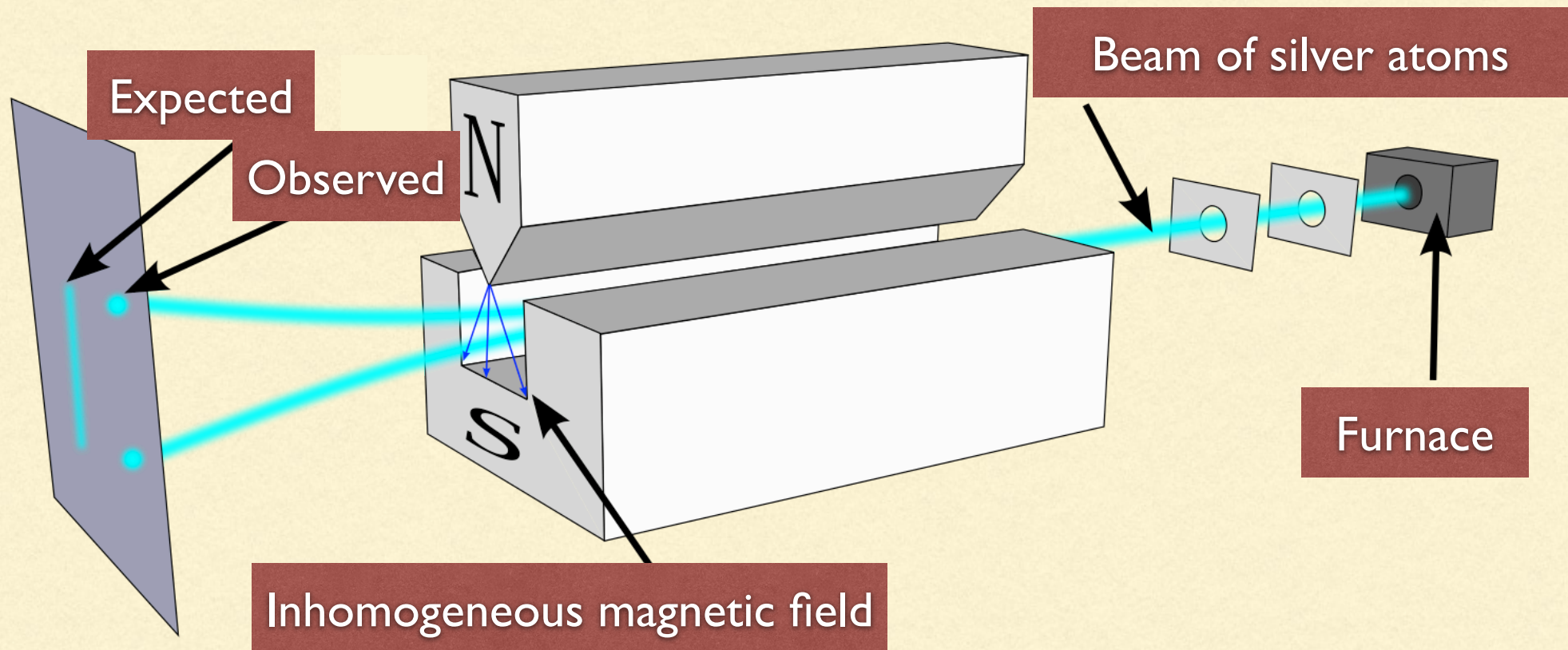
Muon $g-2$

- Introduction
 - Spin and magnetic moment of leptons
 - History of the muon
 - Why is muon $g-2$ interesting?
- History of muon measurements (1960-today)
 - Brookhaven (2004)
 - Fermilab (today)
- Future: J-PARC, tau leptons
- Diving into the theory to interpret experiment - Graham

Stern-Gerlach Experiment

1921-1922

Demonstrated spatial orientation of angular momentum is **quantized**.



Note: Spin $1/2$ not yet realized.

Uhlenbeck–Goudsmit model

1925

- Uhlenbeck & Goudsmit explained **quantization of emission spectra** of alkali metals based on assumption electrons are **spinning** around axis, with magnetic moment $\mu = q\hbar/2m$
 - Pauli had proposed (1924) two quantum states for electron to explain emission spectra of alkali atoms, but at first poo-poo'd idea that particles could be **spinning** since he thought the motion would exceed the speed of light.
 - 1927 - Pauli formalizes theory of **spin**.
- **Spin** ($\frac{+\hbar}{2}, \frac{-\hbar}{2}$) was important element in Dirac's theory of the electron, relativistic quantum mechanics (1928). - suggests anti-matter, also!

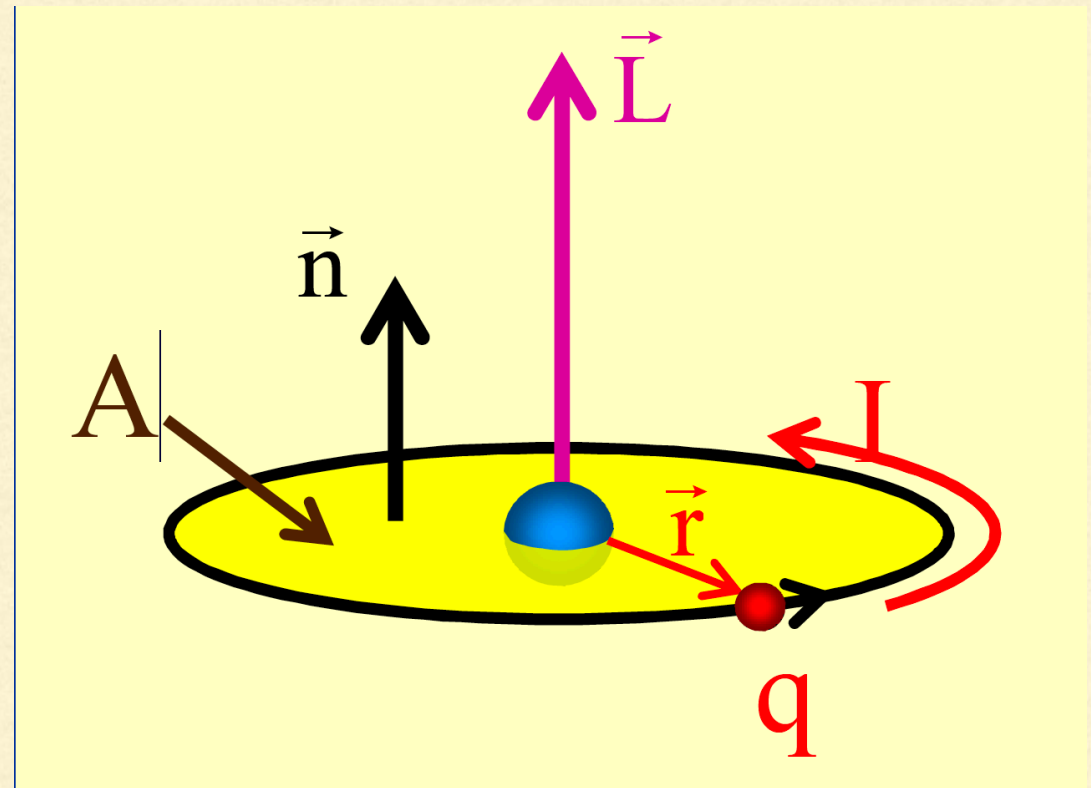
Magnetic moment - current loop

- Torque on a current loop:

- $\vec{\tau} = \vec{\mu} \times \vec{B}$

- Magnetic moment of current loop

- $\vec{\mu} = IA\hat{n}$



Magnetic moment - current loop

- Magnetic moment of current loop

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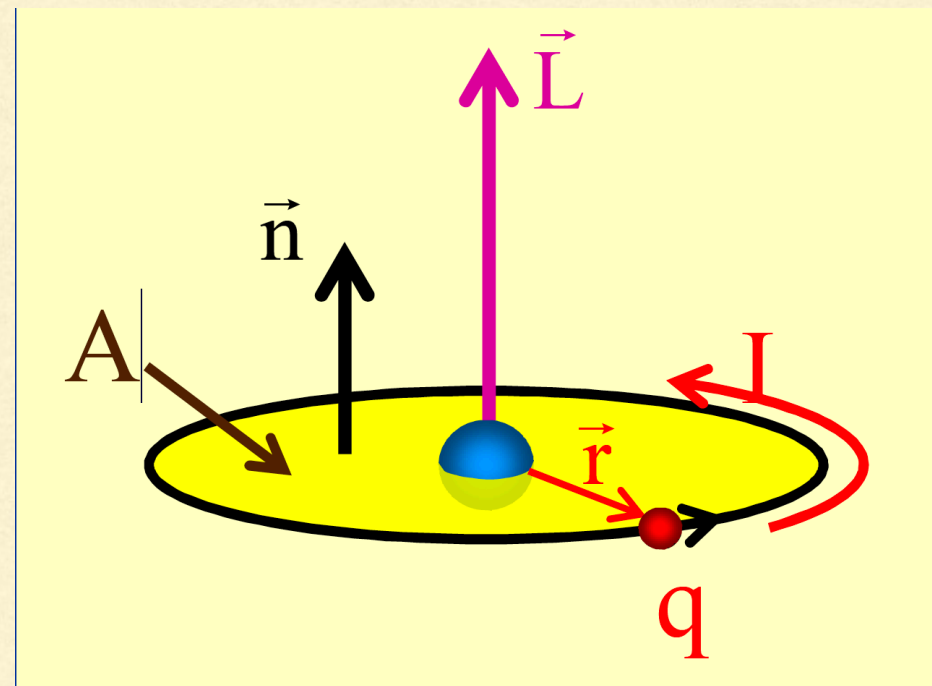
- or

- $\vec{\mu} = \left(\frac{q}{2\pi/\omega}\right) (\pi r^2) \hat{n}$

- $= \frac{q}{2m} mr^2\omega \hat{n}$

- Since $\vec{L} = mvr \hat{n} = mr^2\omega \hat{n}$:

- $\vec{\mu} = \frac{q}{2m} \vec{L}$



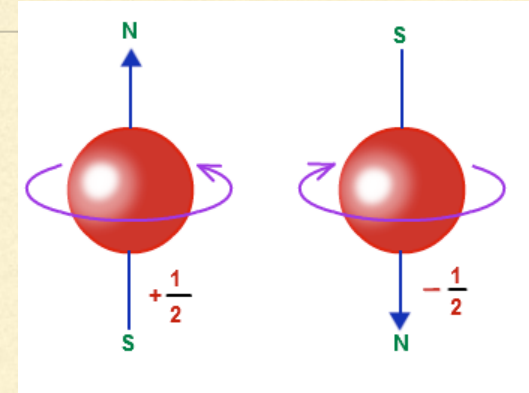
ω = angular frequency

m = mass

$v = r\omega$ = speed

Magnetic moment of electron (or muon)

- Magnetic moment of current loop
 - $\vec{\mu} = q/2m \vec{L}$
- Magnetic moment of electron (muon)



- $\mu = q\hbar/2m$
- So if $|\vec{L}| = |\vec{s}| = \hbar/2$,
 - $\vec{\mu} = g (q/2m) \vec{s}, \quad g = 2!!$
- 1928. $g=2$ explained in Dirac theory of electron.
 - "an unexpected bonus for me, completely unexpected." quoted by A. Pais
- Therefore, $g-2$ expresses deviation from Dirac theory.

The Quantum Theory of the Electron.

By P. A. M. DIRAC, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

Dirac explains $g=2$, but is g exactly 2?

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PHYSICAL REVIEW

VOLUME 74, NUMBER 3

AUGUST 1, 1948

The Magnetic Moment of the Electron†

P. KUSCH AND H. M. FOLEY

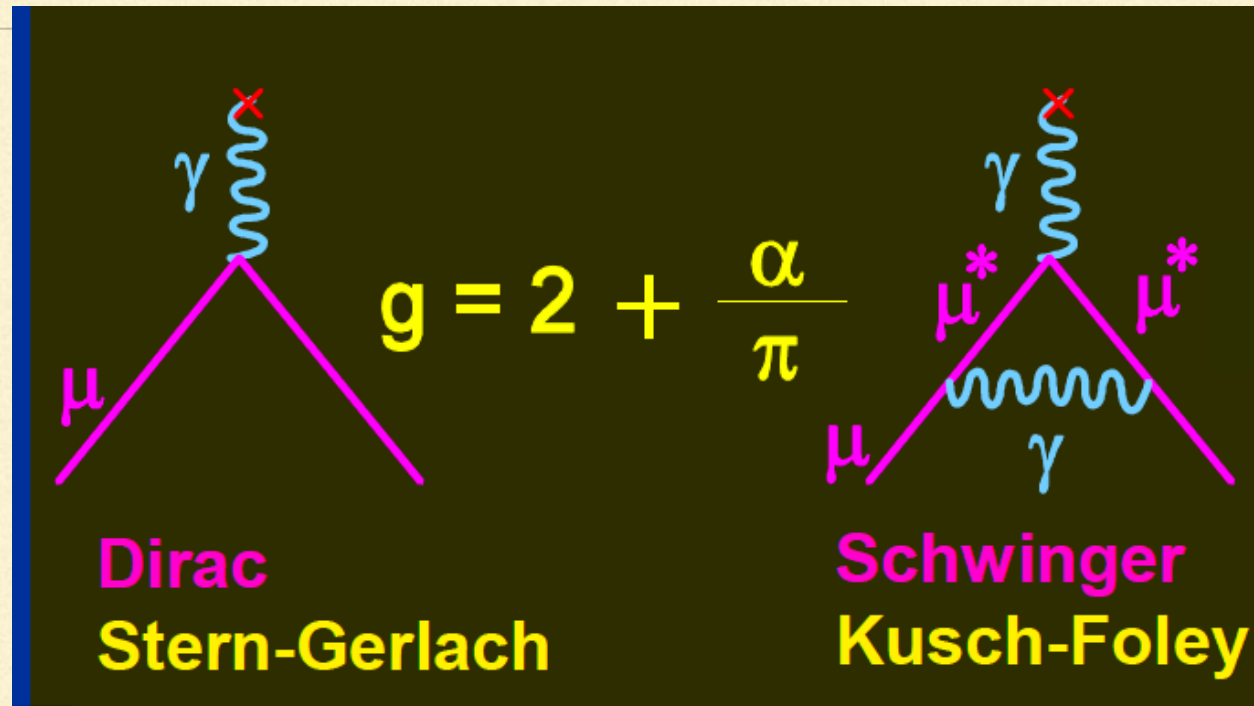
Department of Physics, Columbia University, New York, New York

(Received April 19, 1948)

A comparison of the g_J values of Ga in the $^2P_{3/2}$ and $^2P_{1/2}$ states, In in the $^2P_{1/2}$ state, and Na in the $^2S_{1/2}$ state has been made by a measurement of the frequencies of lines in the hfs spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. The results can be described by the statement that $g_L = 1$ and $g_S = 2(1.00119 \pm 0.00005)$. The observed effects may be explained by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

- $g \neq 2$. $g = 2 + 0.00238 \pm 0.00010$. - too large! - why?

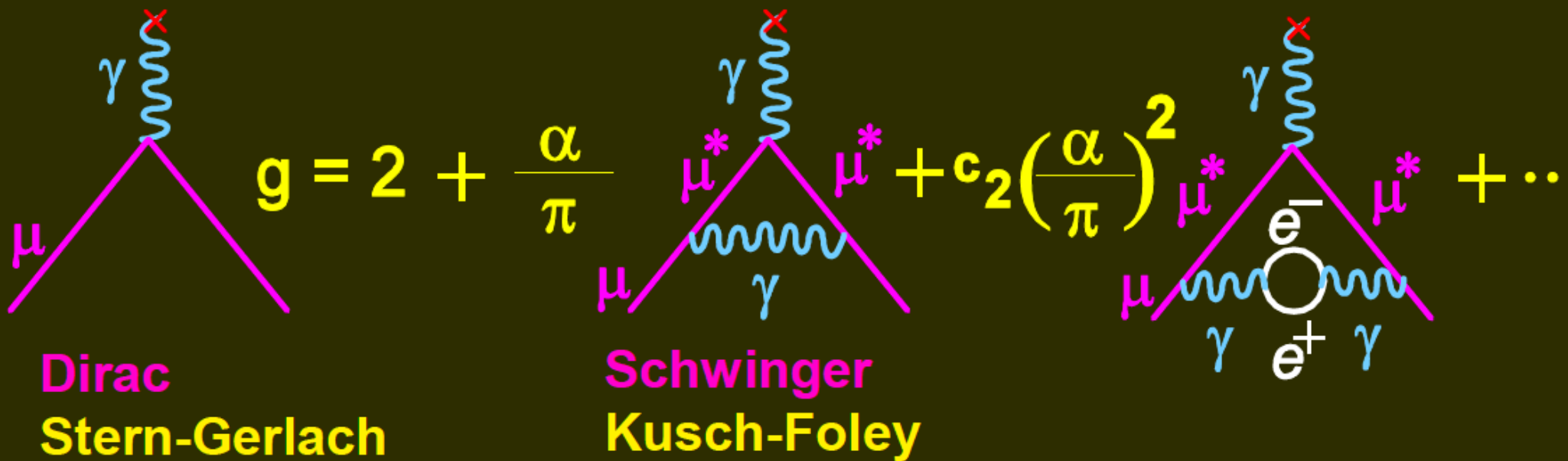
Modern theoretical understanding



$$g = 2 + \alpha/\pi = 2(1.00116), \quad \alpha = 1/137$$

$$g_s = 2(1.00119 \pm 0.00005).$$

Modern theoretical understanding



The image displays a series of Feynman diagrams representing the theoretical calculation of the muon's anomalous magnetic moment, $g-2$. The diagrams are arranged in a sequence separated by plus signs, indicating a perturbative expansion. The first diagram, labeled 'Dirac Stern-Gerlach', shows a muon line (pink) with a photon (blue wavy line) attached, marked with a red 'x'. The second diagram, labeled 'Schwinger Kusch-Foley', shows a muon line with a photon loop (blue wavy lines) and a muon-antimuon pair (pink lines with asterisks). The third diagram shows a muon line with a photon loop and a positronium loop (white circle with e^- and e^+). The expansion is given by the equation:
$$g = 2 + \frac{\alpha}{\pi} + c_2 \left(\frac{\alpha}{\pi} \right)^2 + \dots$$

Dirac Stern-Gerlach

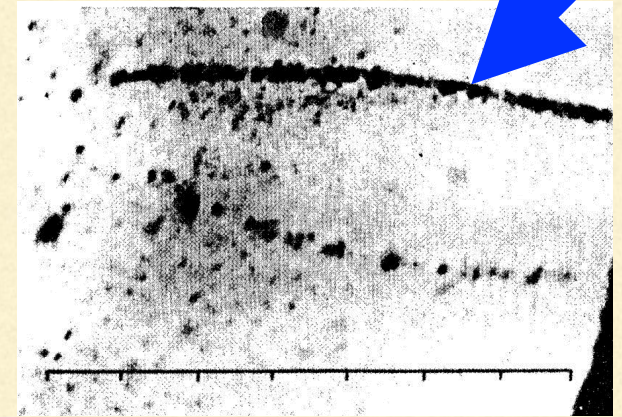
Schwinger Kusch-Foley

- There's much more - Graham will get into that in the next talk, especially for the muon.

Discovery of the muon

1932-1947

- 1932 - Paul Kunze observes traces of an unknown, positively charged particle (too weak for proton & too strong for positron).
- 1937 - Carl Anderson & Seth Neddermeyer publish discovery of muon. (suspected to be pion of Yukawa theory).
- 1947 - pi plus is discovered, revealing muon as another fundamental particle. "Who ordered that?" Isidor I. Rabi



MAY 15, 1937

PHYSICAL REVIEW

VOLUME 51

Note on the Nature of Cosmic-Ray Particles

SETH H. NEDDERMEYER AND CARL D. ANDERSON
California Institute of Technology, Pasadena, California
(Received March 30, 1937)

MEASUREMENTS¹ of the energy loss of massive than protons but more penetrating than particles occurring in the cosmic-ray showers have shown that this loss is proportional to the energy of the particles. In contrast to electrons obeying the Bethe-Heitler theory, we have taken about 6000 counter-tripped photo-

Muon magnetic moment

- $\vec{\mu}_\mu = g \left(\frac{q}{2m_\mu} \right) \vec{s} \qquad g \approx 2$

Muon magnetic moment

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- Define anomaly:

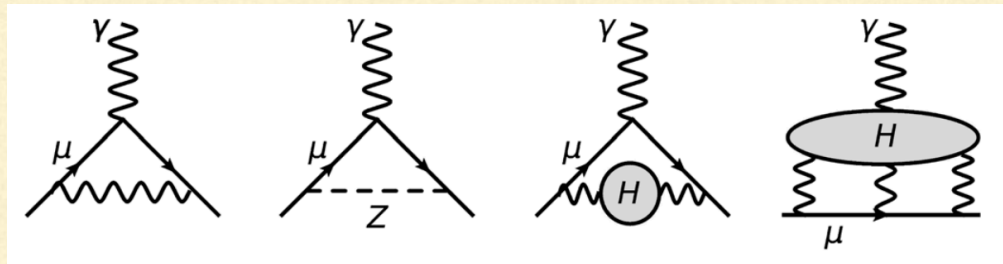
- $a_\mu = \frac{(g - 2)}{2}$

Muon magnetic moment

- $\vec{\mu}_\mu = g \left(\frac{q}{2m_\mu} \right) \vec{s} \quad g \approx 2$

- Define anomaly:

- $a_\mu = \frac{(g - 2)}{2}$



- Effects of virtual processes larger than for electron due to larger mass, and therefore larger phase space and bigger potential for impact of beyond SM physics.

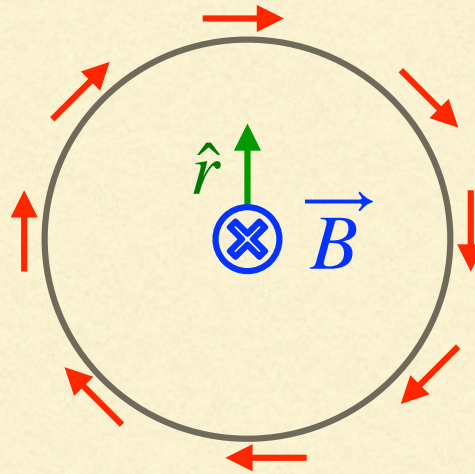
How to measure muon g-2: Storage ring

$$\vec{F} = Qe(\vec{v} \times \vec{B})$$

Lorentz force

$$\vec{F} = \frac{-\gamma m v^2}{r} \hat{r}$$

Centripetal force



μ^-
Momentum

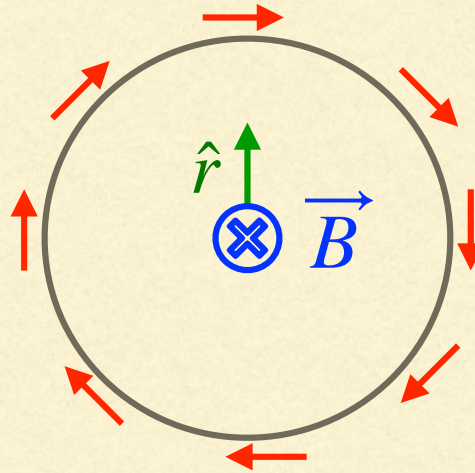
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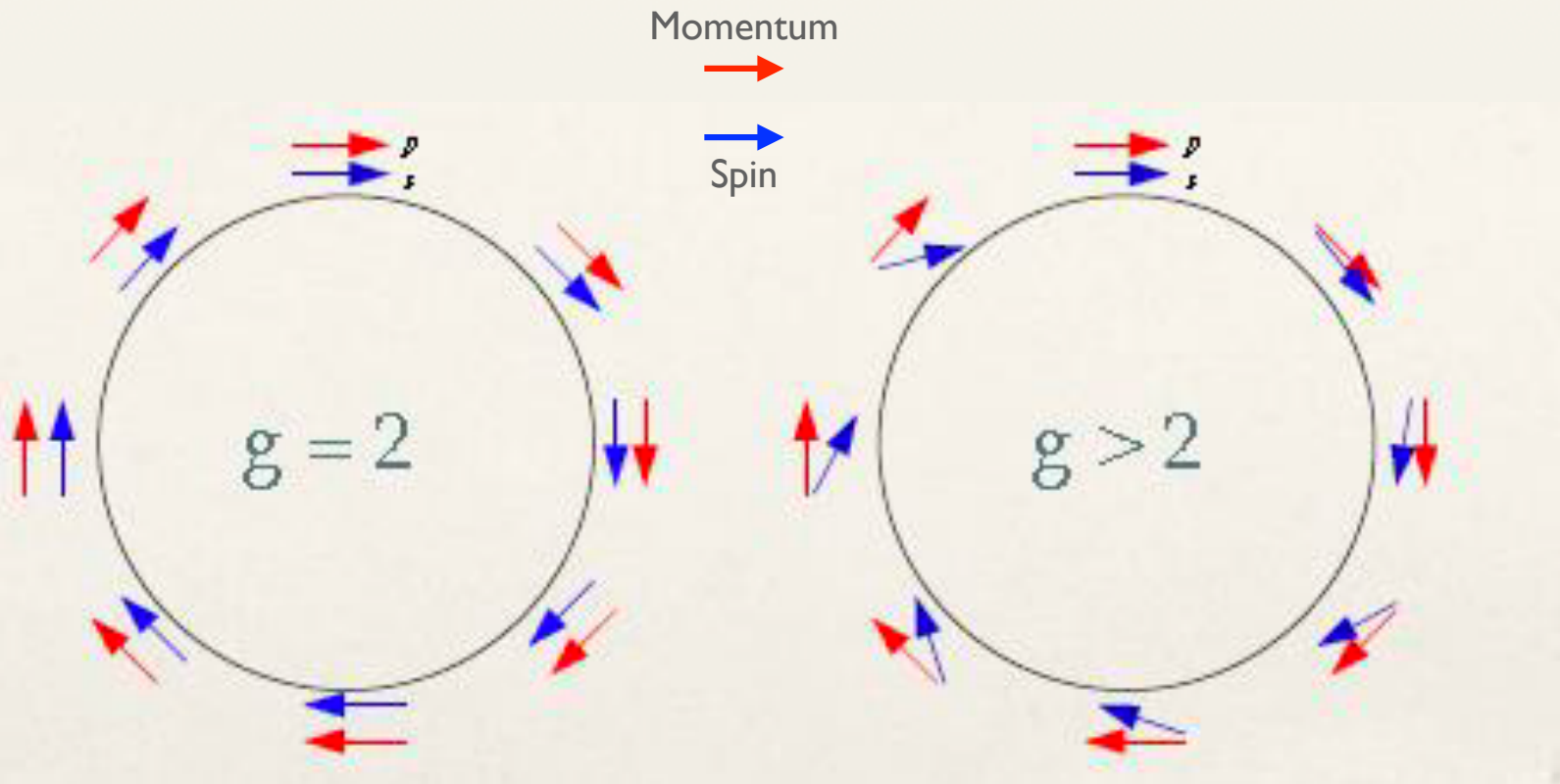


μ^-
Momentum

$$\square \quad \frac{\gamma m v^2}{r} = -QeBv$$

$$\square \quad \frac{v}{r} = \omega_c = \frac{-QeB}{m\gamma} \quad \text{(Relativistic angular cyclotron frequency)}$$

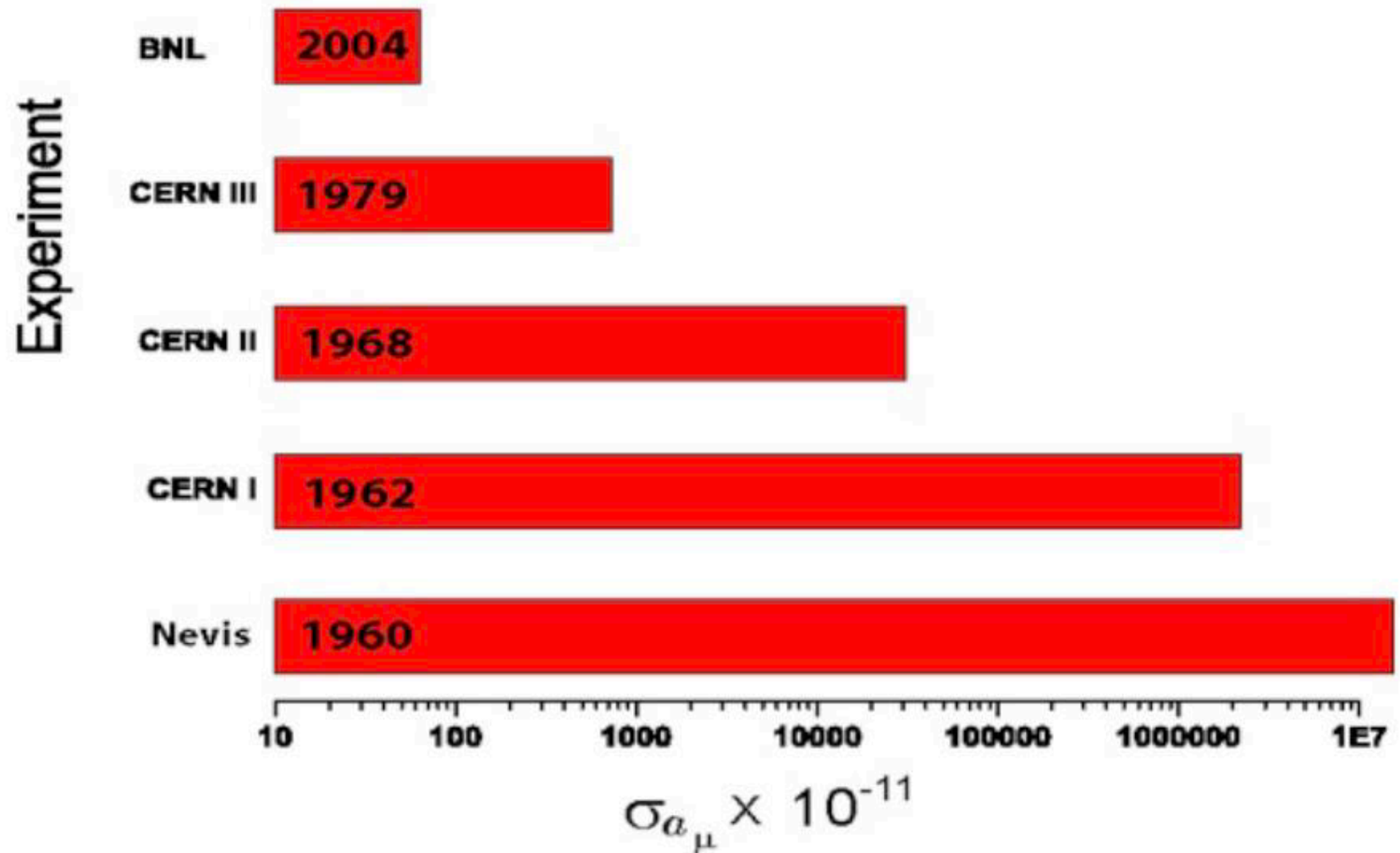
Spin in storage ring



For $g=2$, spin is synchronized
with momentum

Measure spin - momentum mismatch
to determine $g-2$

Measurements of Muon g-2



Measurement at Brookhaven

2004

$$a_{\mu}^{exp} = 116\,592\,089\,(63) \times 10^{-11}$$

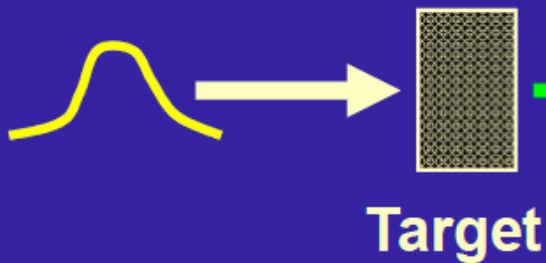
$$a_{\mu}^{SM} = 116\,591\,802\,(49) \times 10^{-11}$$

$$a_{\mu}^{exp} - a_{\mu}^{SM} = 287\,(80) \times 10^{-11}$$

Phys. Rev.D 73(2006) 072003 [hep-ex/0602035].

Experimental Technique AGS experiment

Proton bunches
from accelerator



Pions

$p=3.1\text{GeV}/c$

π^-

$\mu^- \bar{\nu}_\mu$

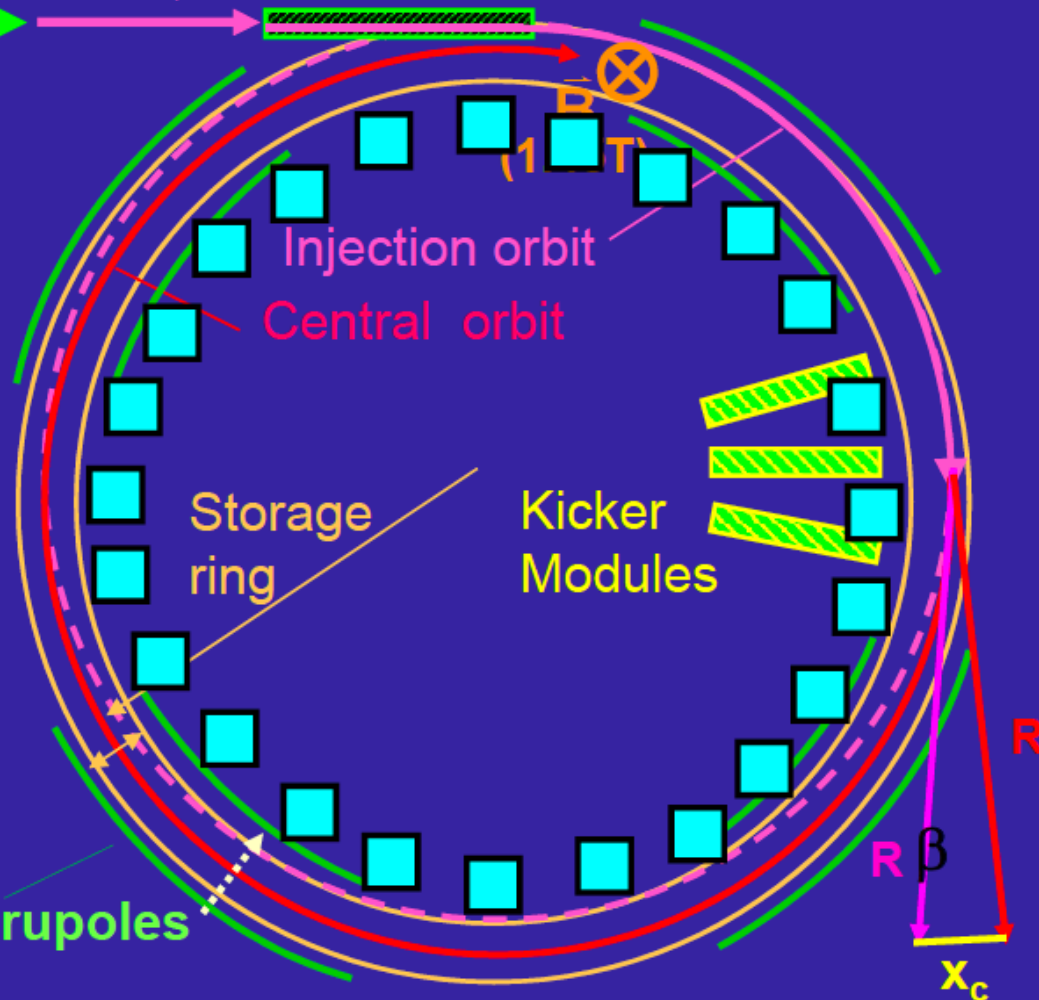
Inflector

$x_c \approx 77\text{ mm}$
 $\beta \approx 10\text{ mrad}$
 $B \cdot dl \approx 0.1\text{ Tm}$

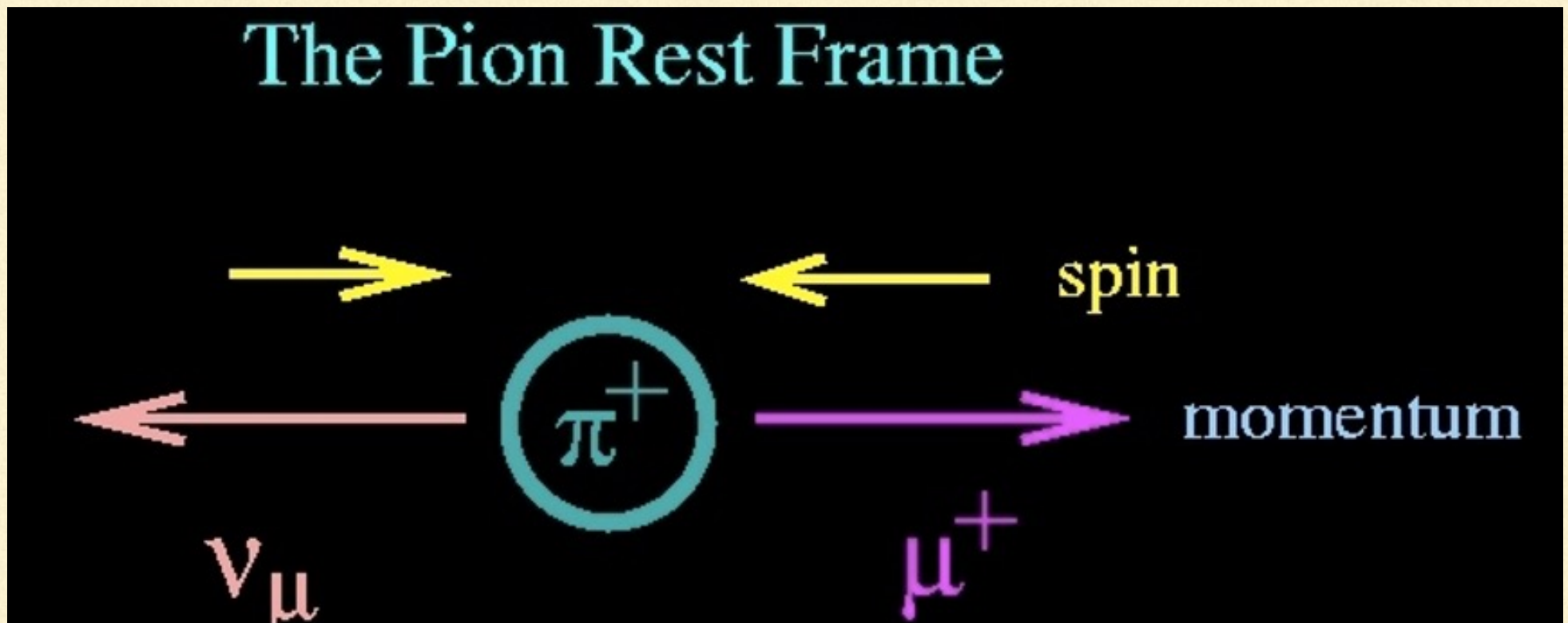
- Muon polarization
- Muon storage ring
- injection & kicking
- focus with Electric Quadrupoles
- 24 electron calorimeters

$$\vec{\omega}_a = -\frac{e}{m} a_\mu \vec{B}$$

Electric Quadrupoles



Spin Zero Pion decay produces polarized muons



Neutrinos are left-handed. -
so. - μ^+ must be left-handed, since pion has no spin.
(Opposite for π^- , anti-neutrinos and μ^-)

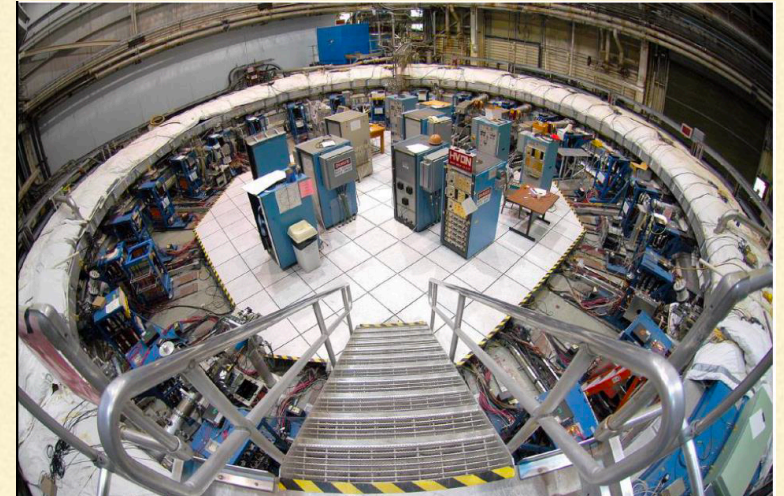
Muons in storage ring

- Inject polarized muons into a very uniform magnetic field (storage ring).
- Muon momentum turns with cyclotron frequency:

- $$\omega_c = \frac{-QeB}{m\gamma}$$

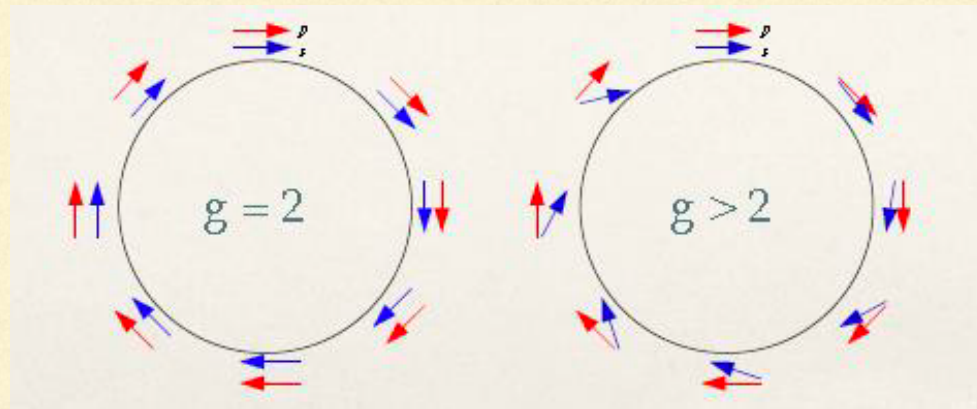
- Spin turns with a different frequency:

- $$\omega_s = g \frac{-QeB}{2m} - (1 - \gamma) \frac{QeB}{\gamma m}$$

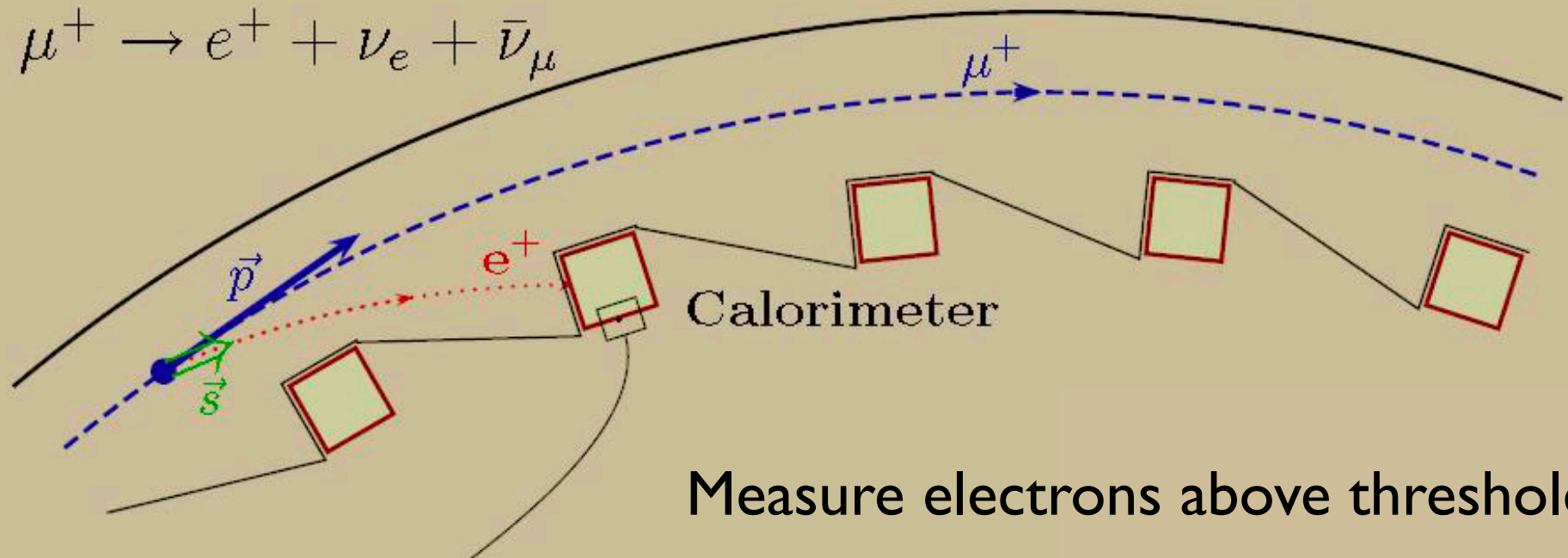
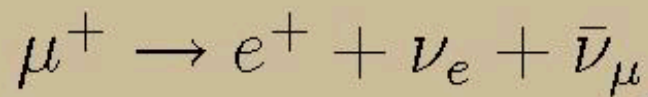
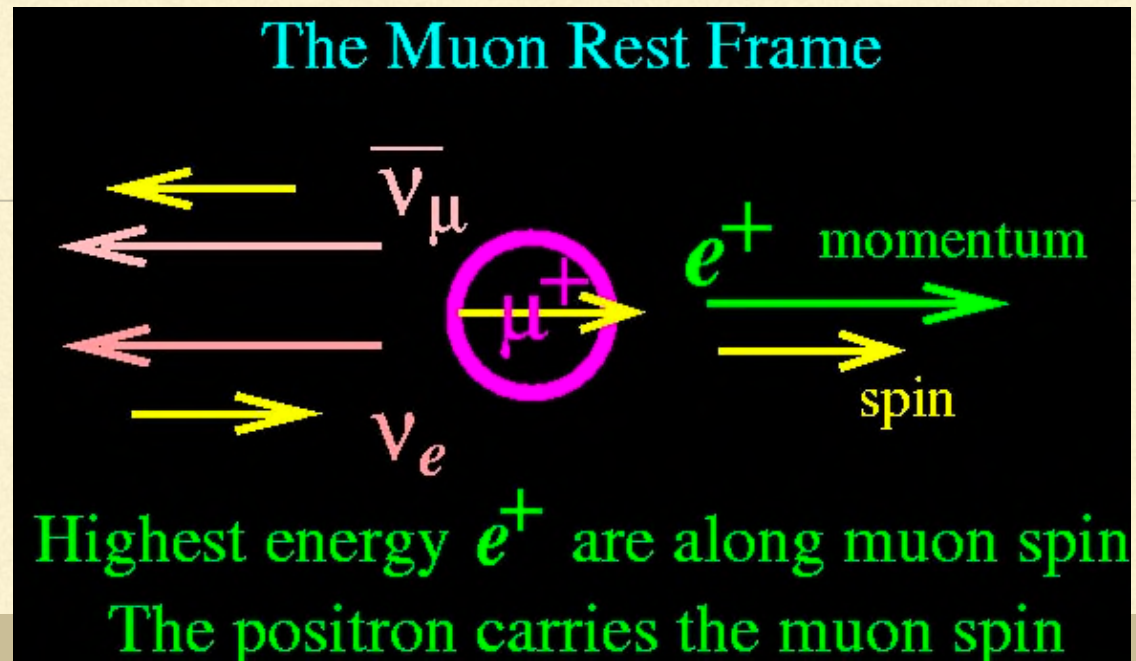
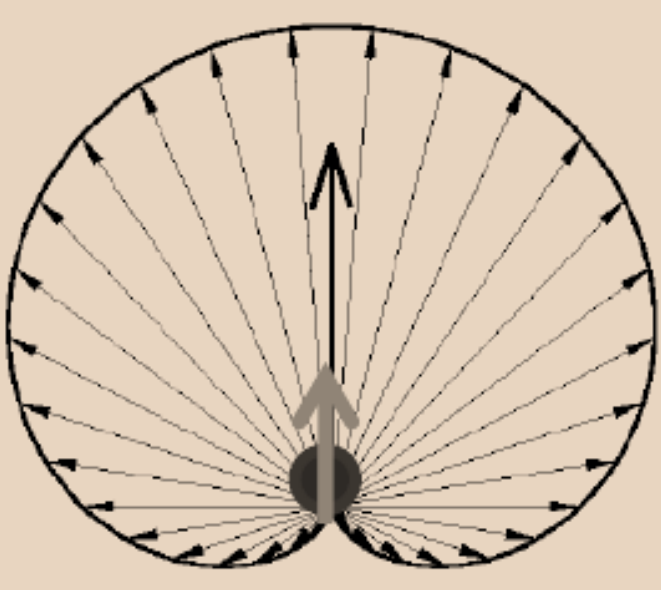


Anomalous frequency

$$\begin{aligned}\omega_a &= \omega_c - \omega_s = \frac{-QeB}{m\gamma} + g\frac{QeB}{2m} + (1 - \gamma)\frac{QeB}{\gamma m} \\ &= \left(\frac{g - 2}{2}\right)\frac{QeB}{m} = a_\mu \frac{QeB}{m}\end{aligned}$$



Detecting the Muon spin



Measure electrons above threshold

Brookhaven E821

Muon lifetime

$$t_{\mu} = 64.4 \mu\text{s}$$

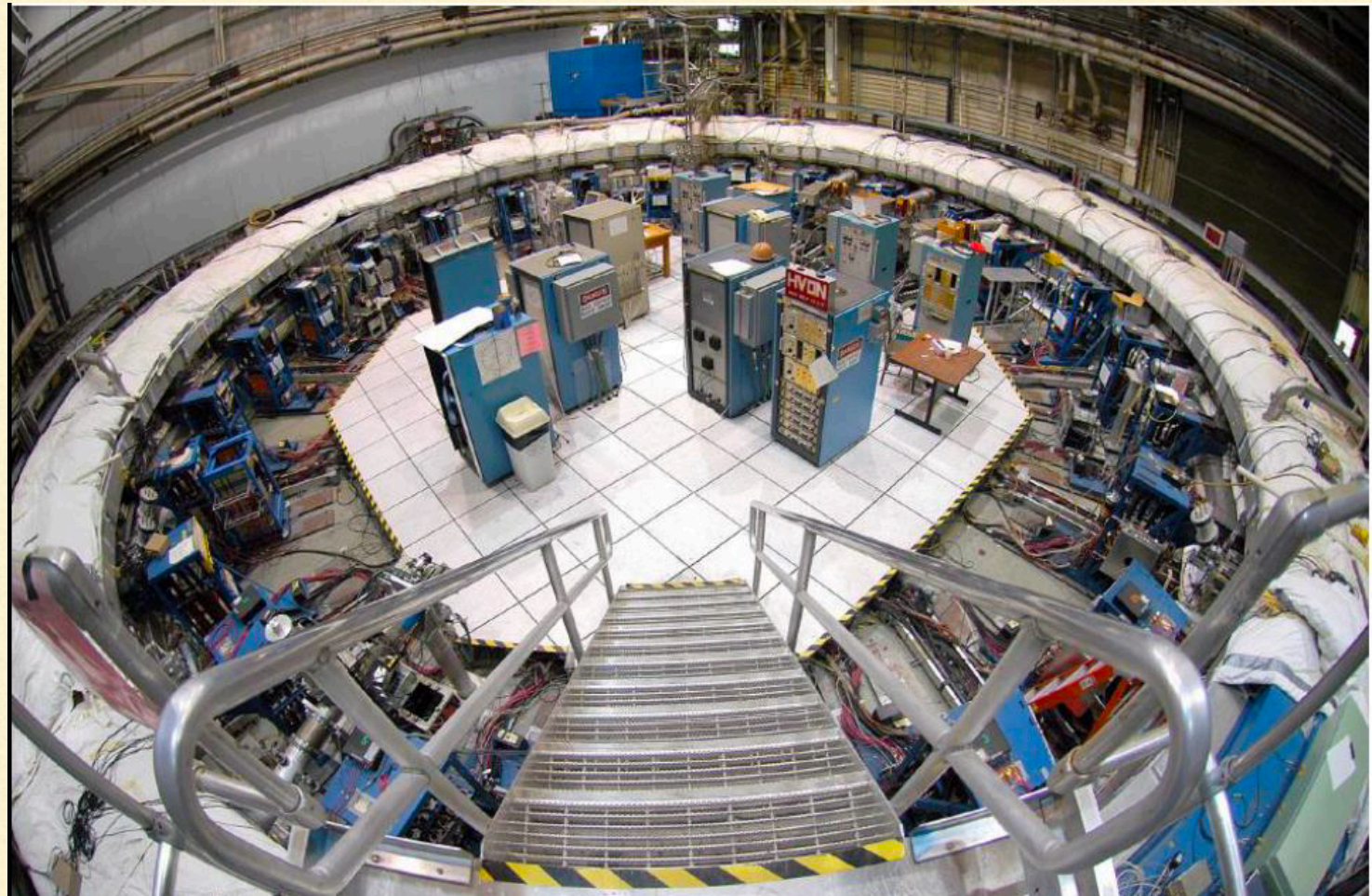
$$(p_{\mu} = 3.1 \text{ GeV}/c)$$

(g-2) period

$$t_a = 4.37 \mu\text{s}$$

Cyclotron period

$$t_c = 149 \text{ ns}$$



Brookhaven E821

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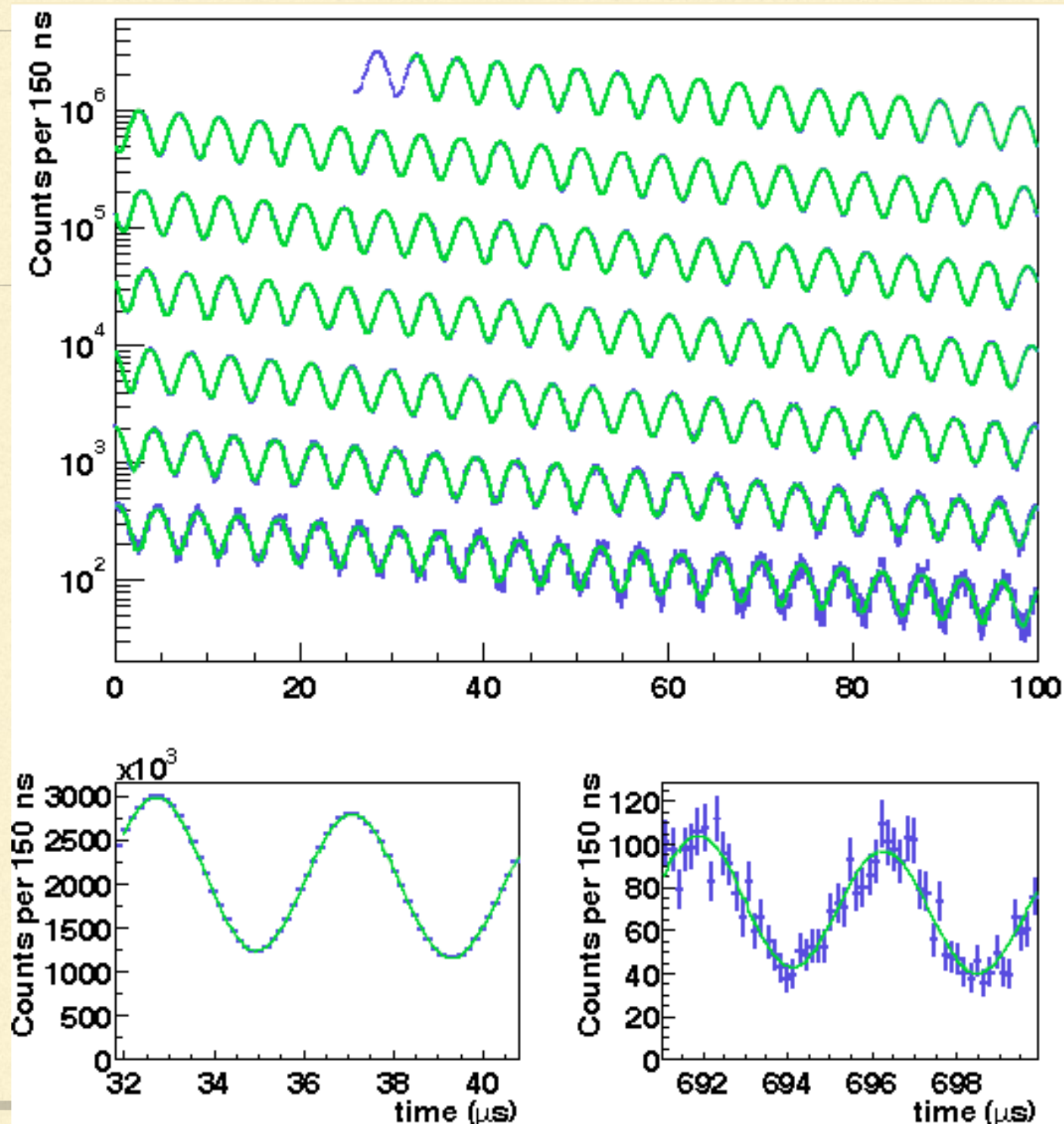
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Measurement at Brookhaven

2004

$$a_{\mu}^{exp} = 116\,592\,089\,(63) \times 10^{-11}$$

$$a_{\mu}^{SM} = 116\,591\,802\,(49) \times 10^{-11}$$

- $\sim 3.5 \sigma$ discrepancy. Is this new physics?
- Improve and repeat experiment.

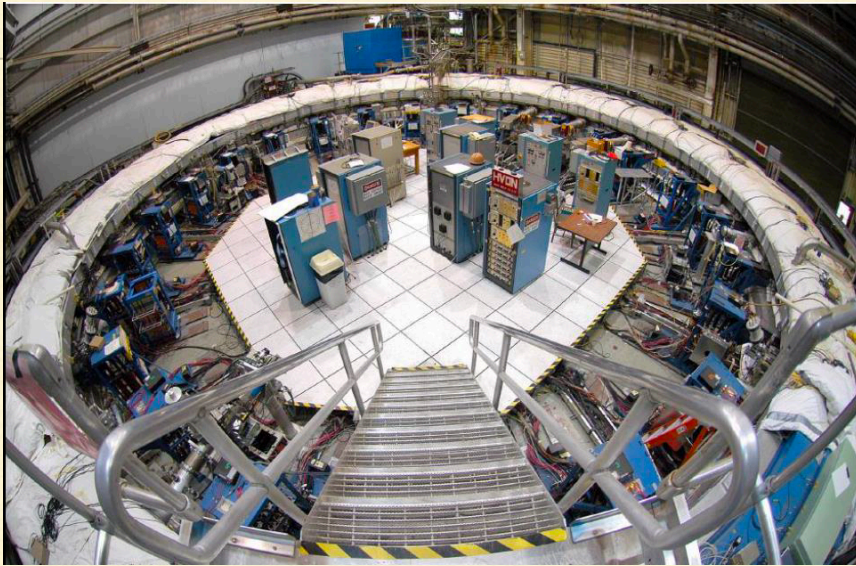
$$a_{\mu}^{exp} - a_{\mu}^{SM} = 287\,(80) \times 10^{-11}$$

Phys. Rev. Lett. 126, 141801 (2021).

arXiv:2104.03281 [hep-ex]

Phys. Rev.D 73(2006) 072003 [hep-ex/0602035].

Storage ring moved to Fermilab

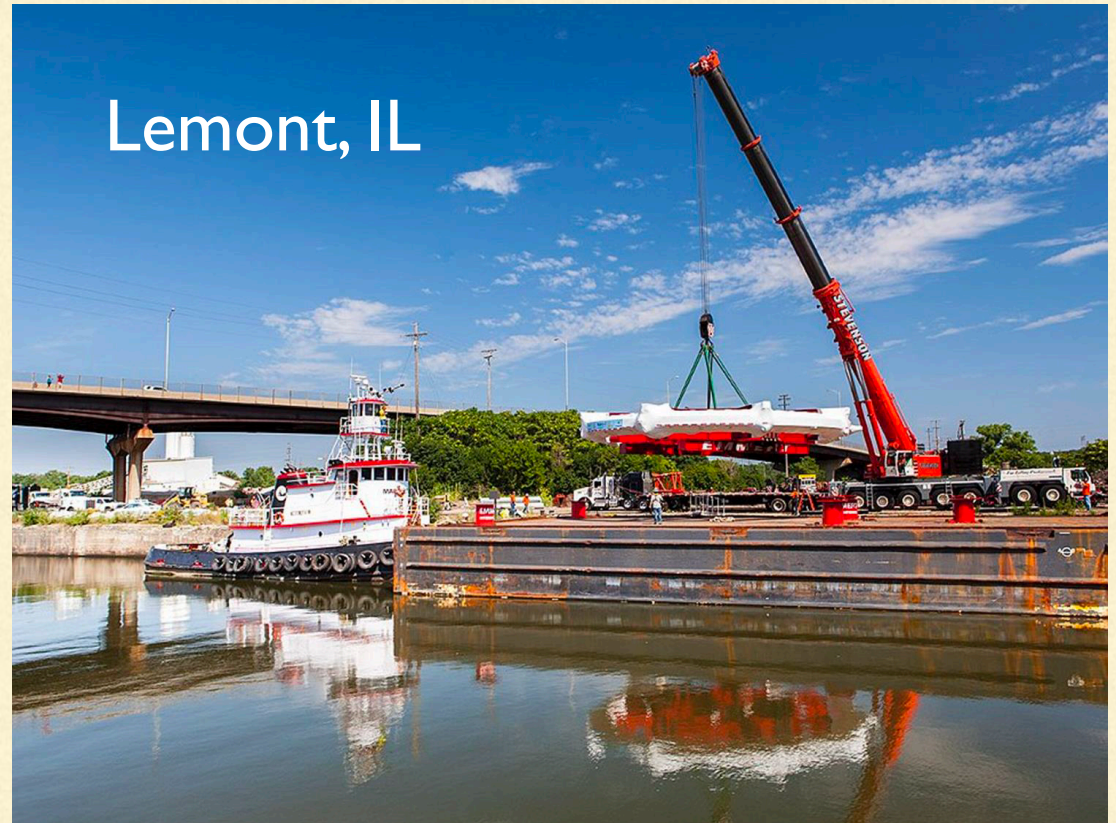


Moving to Fermilab



June 25 - July 20, 2013





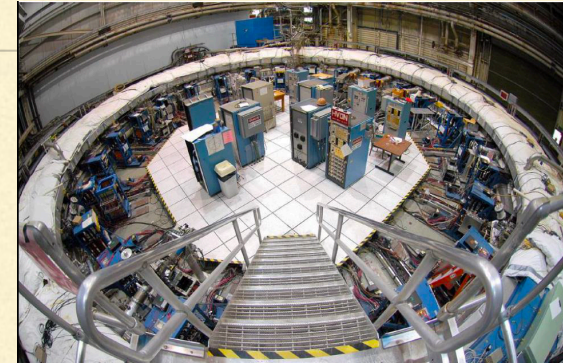
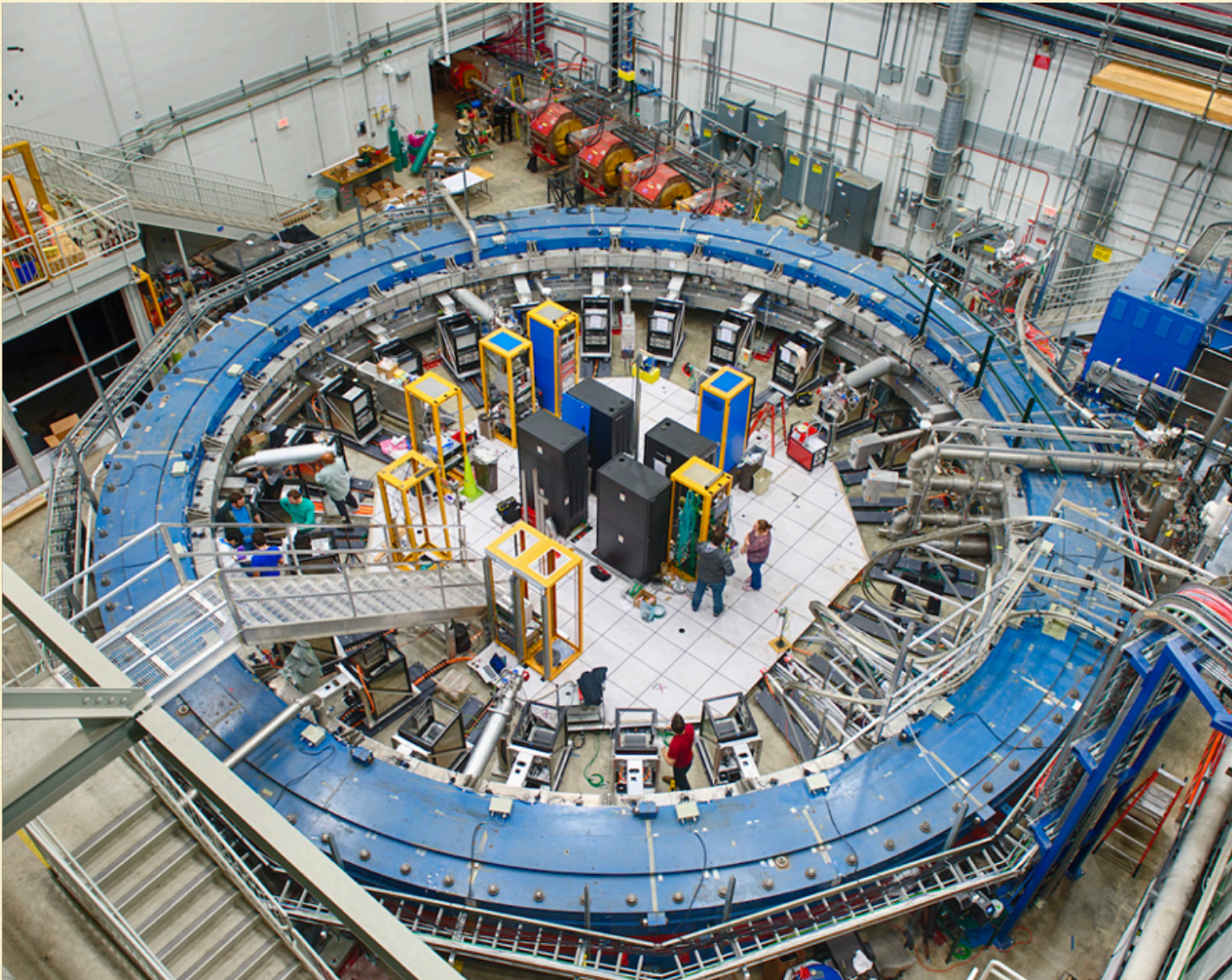
Lemont, IL



Arriving at Fermilab, July, 2013



Fermilab - now

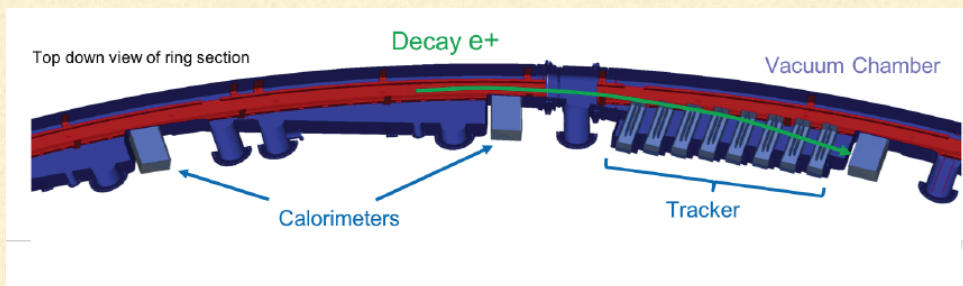
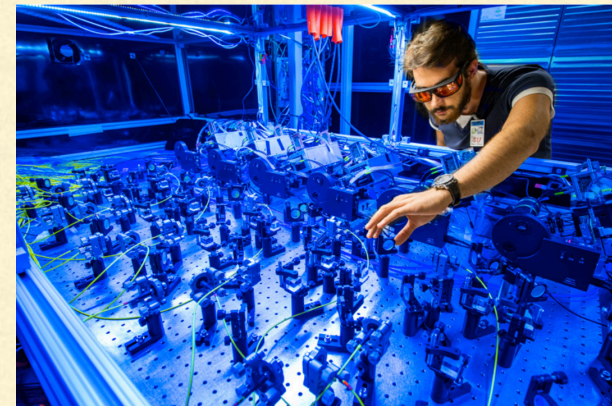
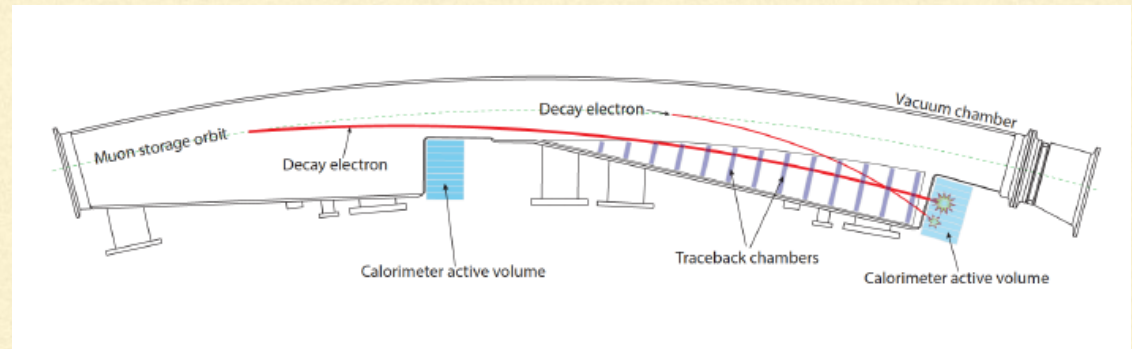


Brookhaven

- Many improvements at Fermilab

Many upgrades from Brookhaven effort

- PbF_2 crystals
 - Cherenkov detectors
 - 24 stations, 9x6 arrays
- Laser calibration system
 - Energy calibration of crystals
- Straw trackers for beam dynamics simulation studies.



Fermilab g-2 results

Muon lifetime

$$t_{\mu} = 64.4 \mu\text{s}$$

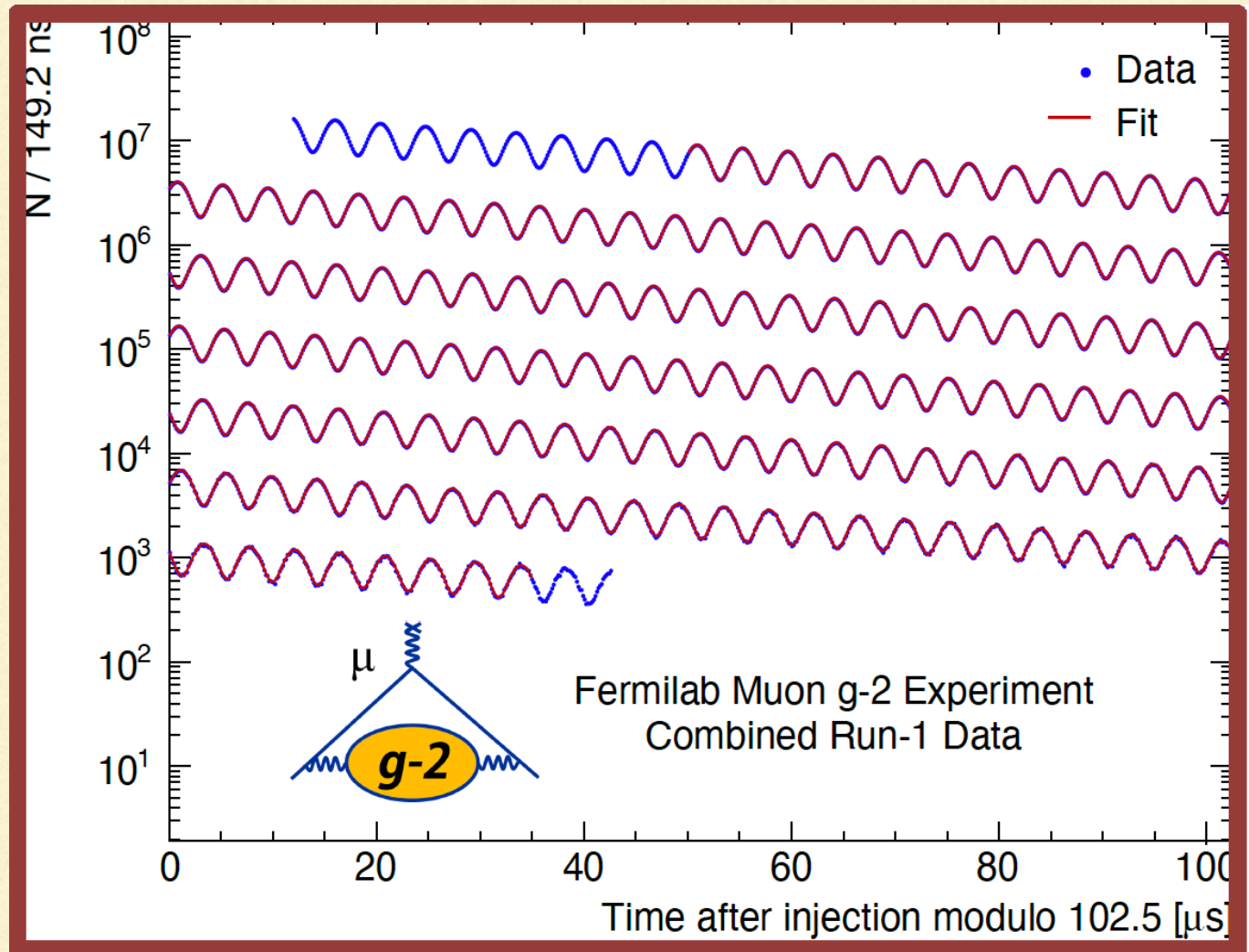
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(g-2) period

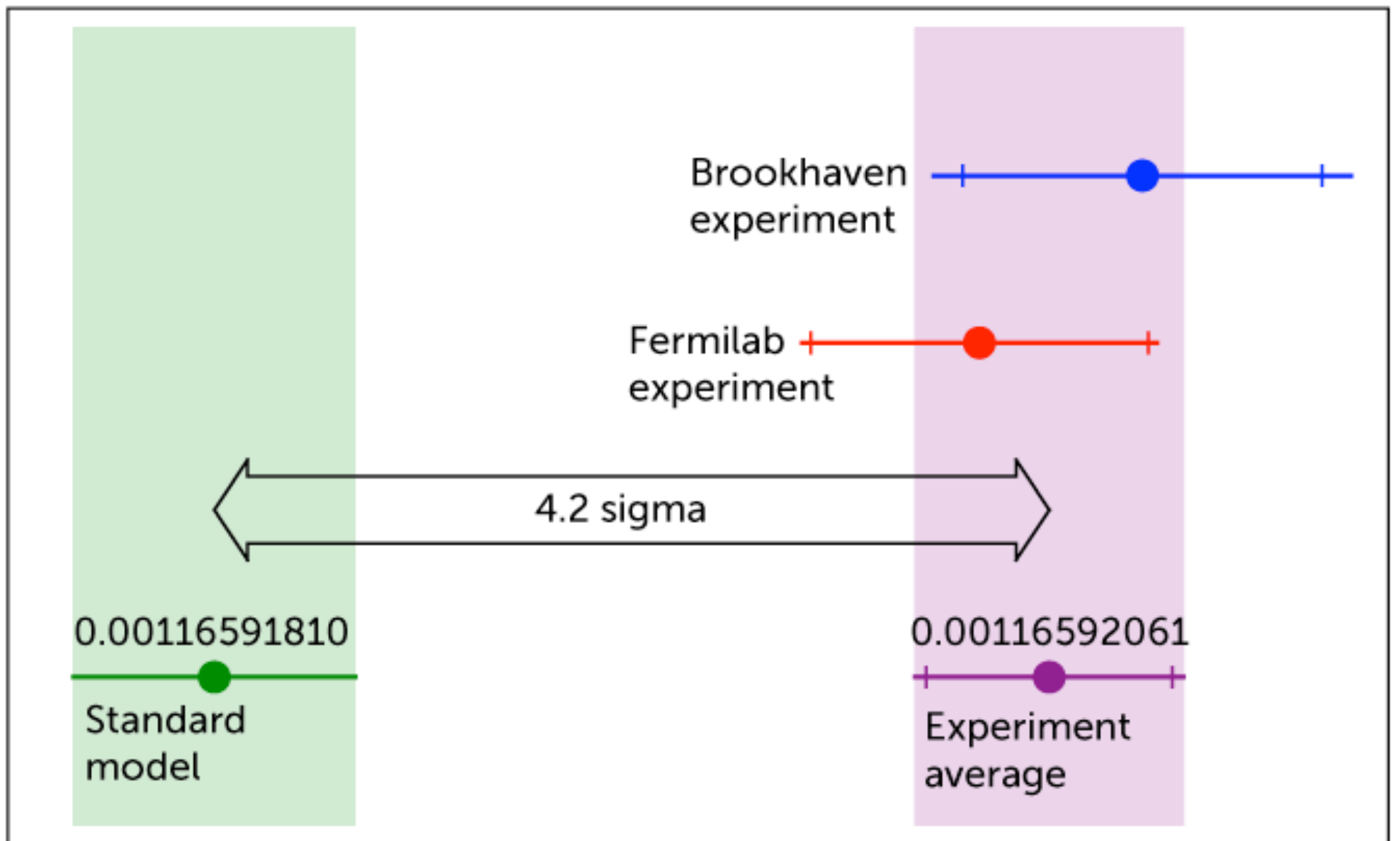
$$t_a = 4.37 \mu\text{s}$$

Cyclotron period

$$t_c = 149 \text{ ns}$$



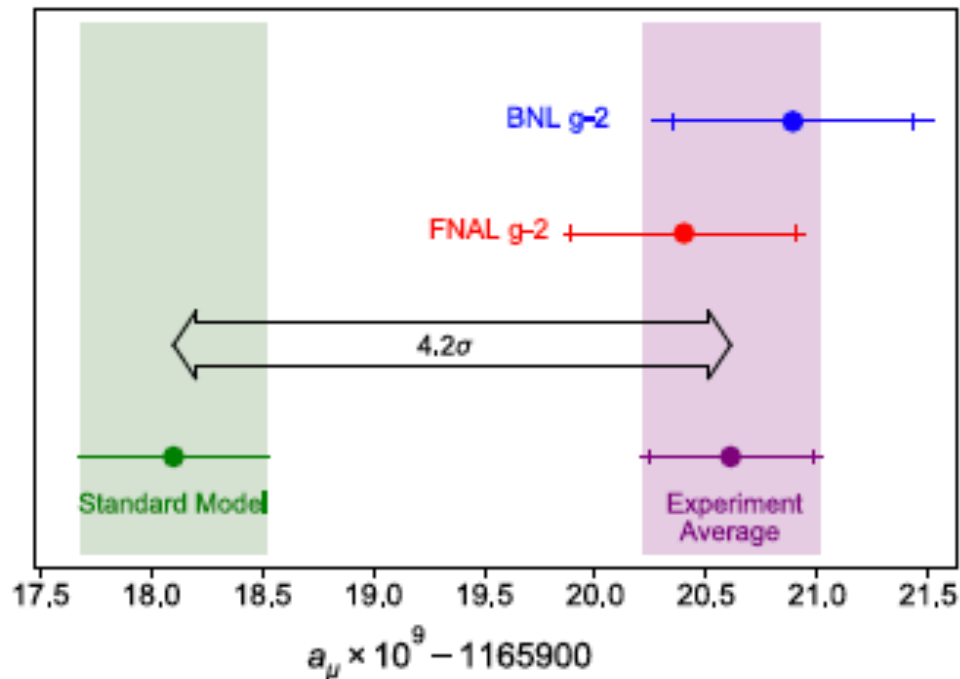
Phys. Rev. Lett. 126, 141801 (2021).
arXiv:2104.03281 [hep-ex]



Muon magnetic anomaly

Phys. Rev. Lett. 126, 141801 (2021).
arXiv:2104.03281 [hep-ex]

Result from combined Run 1 datasets



$$a_\mu(\text{BNL}) = 0.00116592089(63) \rightarrow 540 \text{ ppb}$$

$$a_\mu(\text{FNAL, R1}) = 0.00116592040(54) \rightarrow 463 \text{ ppb}$$

Both experiments uncertainty dominated by statistics:

$$a_\mu(\text{Exp}) = 0.00116592061(41) \rightarrow 350 \text{ ppb}$$

$$a_\mu(\text{SM}) = 0.00116591810(43) \rightarrow 350 \text{ ppb}$$

4.2 σ discrepancy between experiment and community approved SM prediction

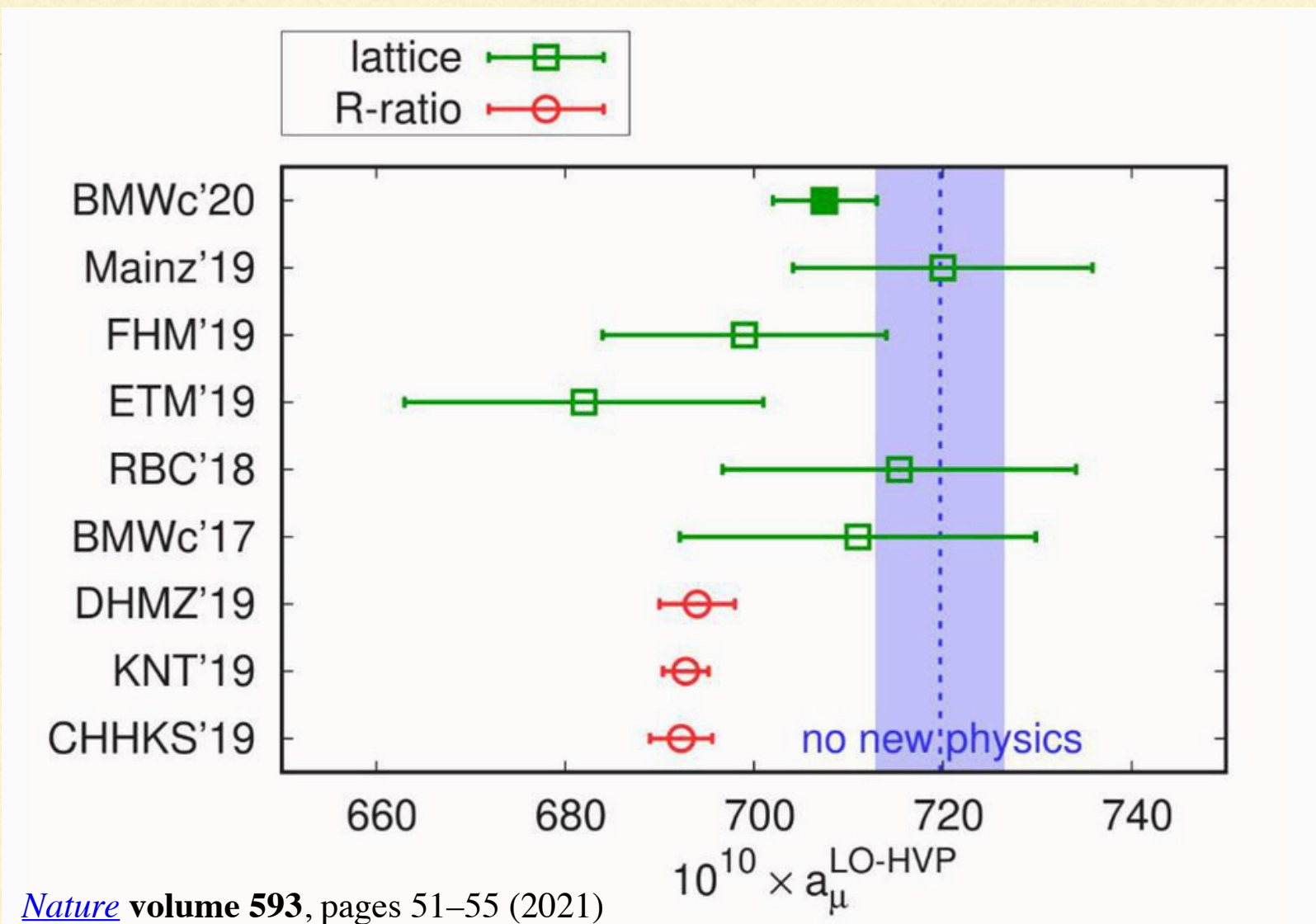
M. Fertil - Stavanger, April 22nd 2021

50



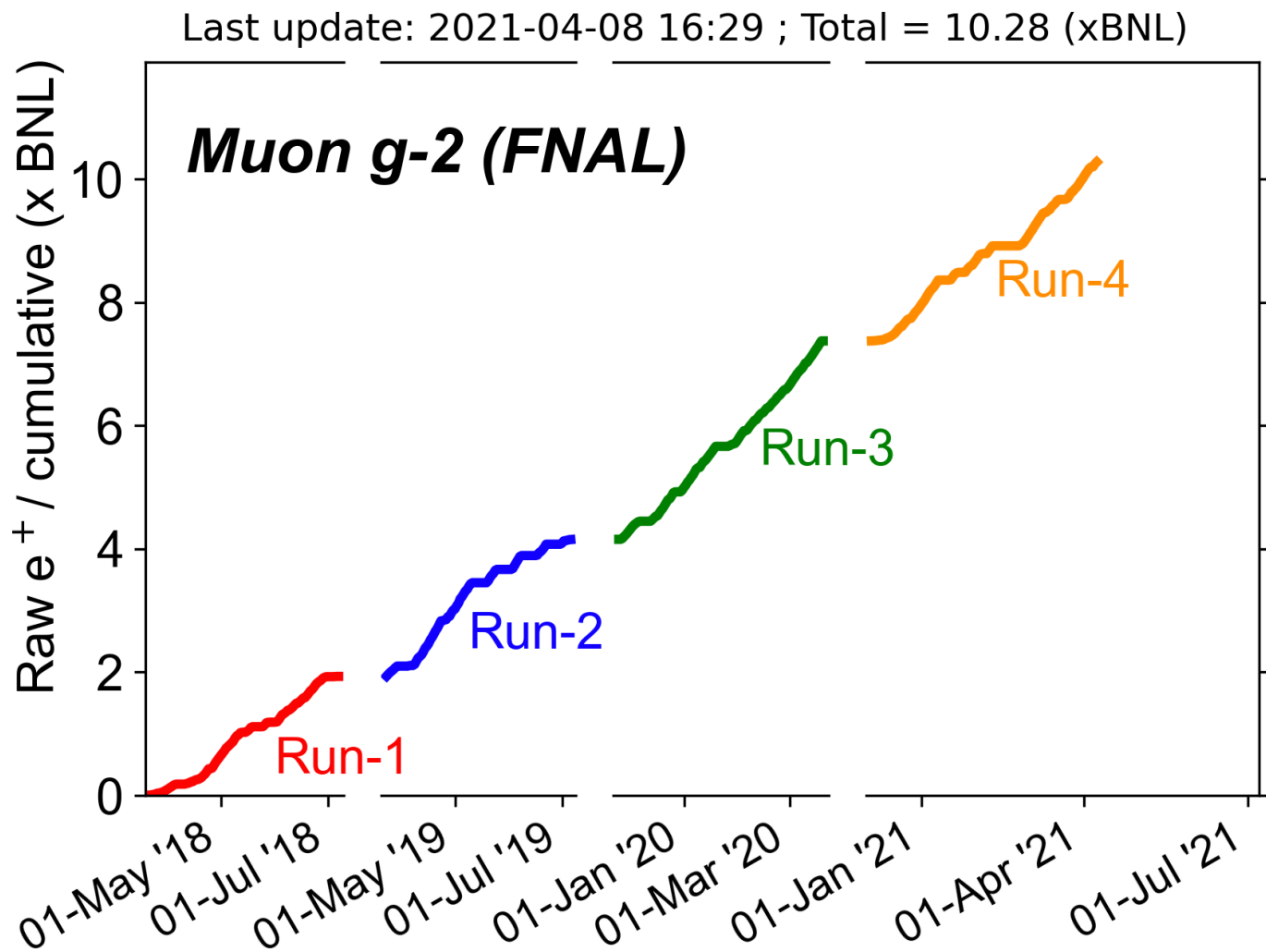
Phys. Rev. Lett. 126, 141801 (2021).
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There are two theoretical approaches



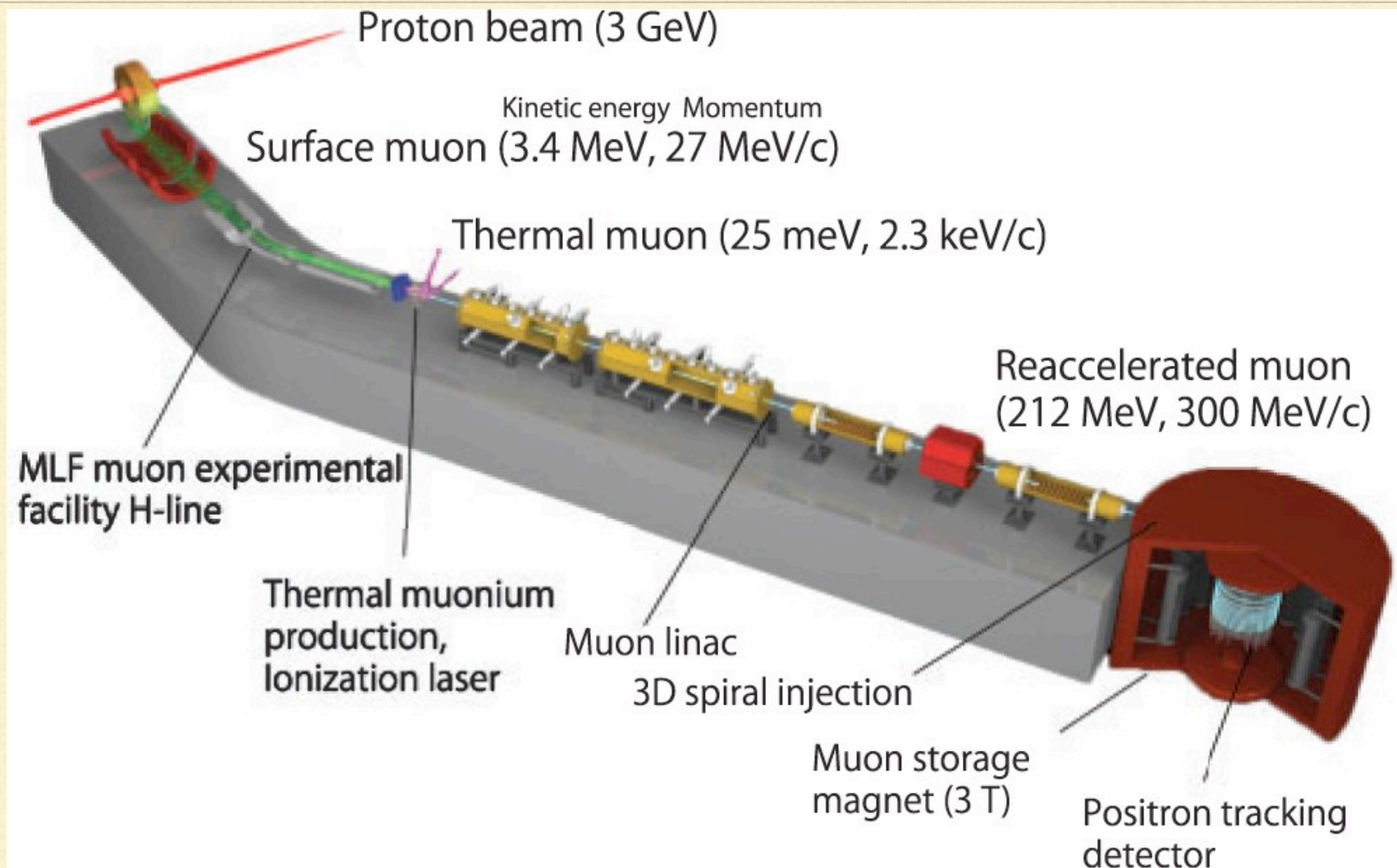
[Nature](#) volume 593, pages 51–55 (2021)

More data to analyze



- 5 times Run-1 data already acquired.
- Run-5 to start in Fall.

J-PARC Experiment



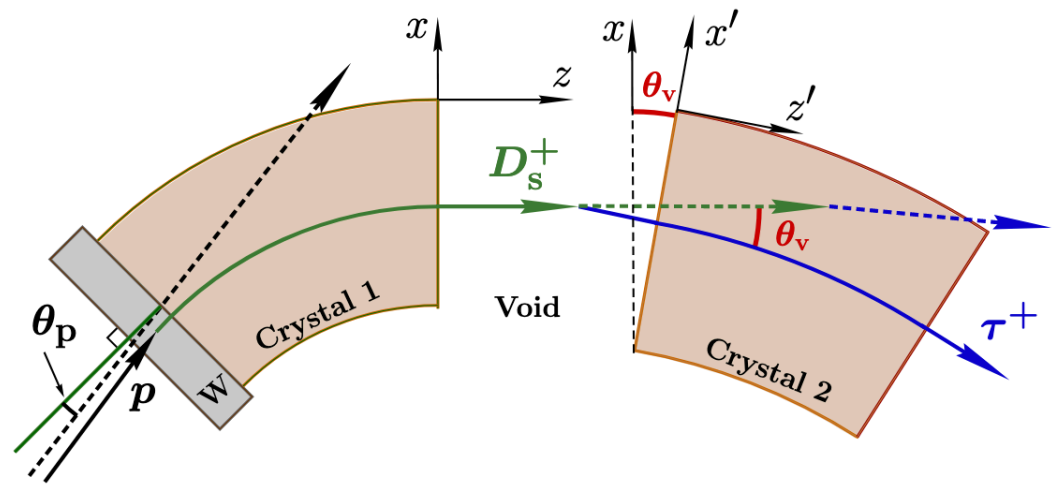
Fantasy? Measure tau lepton g-2

- Higher mass $\rightarrow$ larger BSM effects!
- But tau lifetime = 2.3×10^{-13} seconds



$$pp \rightarrow D_s^+ X$$

$$D_s^+ \rightarrow \tau^+ \nu_\tau$$



Conclusion

- Muon $g-2$ anomaly, a_μ , was recently measured at Fermilab with unprecedented precision, 460 ppb.
 - This results from numerous upgrades on Brookhaven experiment, after it was transported to Fermilab.
- Results are inconsistent with theory by 4.2σ (perturbative expansion approach)
 - However, lattice QCD estimate agrees with experiment.
- Future - 5 (and increasing) times more data from Fermilab.
 - Also, Japanese experiment at J-PARC (farther future: tau $g-2$??)

Acknowledgements

- This talk uses materials from:
 - <http://www.ep.ph.bham.ac.uk/general/seminars/slides/Steve-Maxfield-2013.pdf>
 - http://www.hep.fsu.edu/~wahl/Quarknet/summer2015/lectures/muon_g-2.pdf
 - <http://g2pc1.bu.edu/%7Eroberts/BLRoberts-Schwinger-fest-g-2-final-v2.pptx>
 - <https://indico.uis.no/event/16/contributions/55/attachments/19/34/go>
 - <https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.126.141801>
 - <https://www.sciencedirect.com/science/article/pii/S0370157320302556?via%3Dihub>
 - <https://www.nature.com/articles/s41586-021-03418-1>