Discrete Symmetries: C, P, CP, CPT

- Outline
  - P Violation, C Violation and CP Conservation
    - Muons, neutrinos and pions
  - CP Violation
    - K^0_L and B decays
  - Flavor Oscillations and the CPT Theorem
  - CP Violation in the Standard Model

Conservation Rules

	Interaction		
Conserved quantity	strong	EM	weak
energy-momentum, charge, baryon number, lepton number	yes	yes	yes
CPT	yes	yes	yes
P (parity)	yes	yes	no
C (charge conjugation parity)	yes	yes	no
CP (or T)	yes	yes	10^{-3} violation
I (isospin)	yes	no	no

Parity nonconservation in β-decay

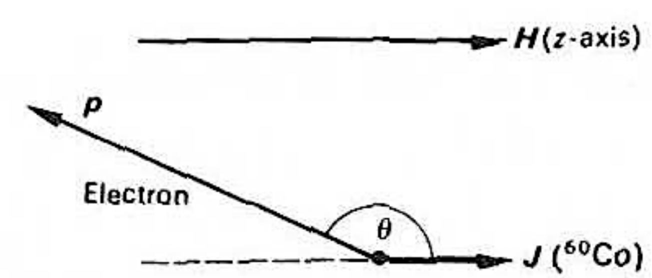
- 1956 - Lee and Yang conclude weak interactions violate parity conservation (invariance under spatial inversion)

  $K^+ \to 2\pi$ (final state has <u>even</u> parity)

  $K^+ \to 3\pi$ (final state has <u>odd</u> parity)

- 1957 - Wu et al test this
  - ^{60}Co at 0.01K inside solenoid
  - high proportion of spin 5 cobalt nuclei are aligned with field

$$^{60}Co \to {}^{60}Ni + e^- + \bar{\nu}_e$$

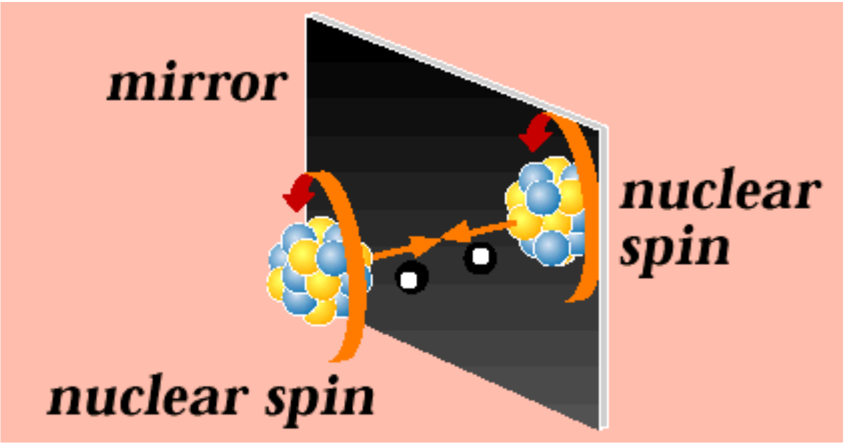


$$I(\theta) = 1 + \alpha \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E} = 1 + \alpha \frac{v}{c} \cos\theta$$

experiment showed $\alpha = -1$

Parity nonconservation in β-decay

$$I(\theta) = 1 + \alpha \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E}$$



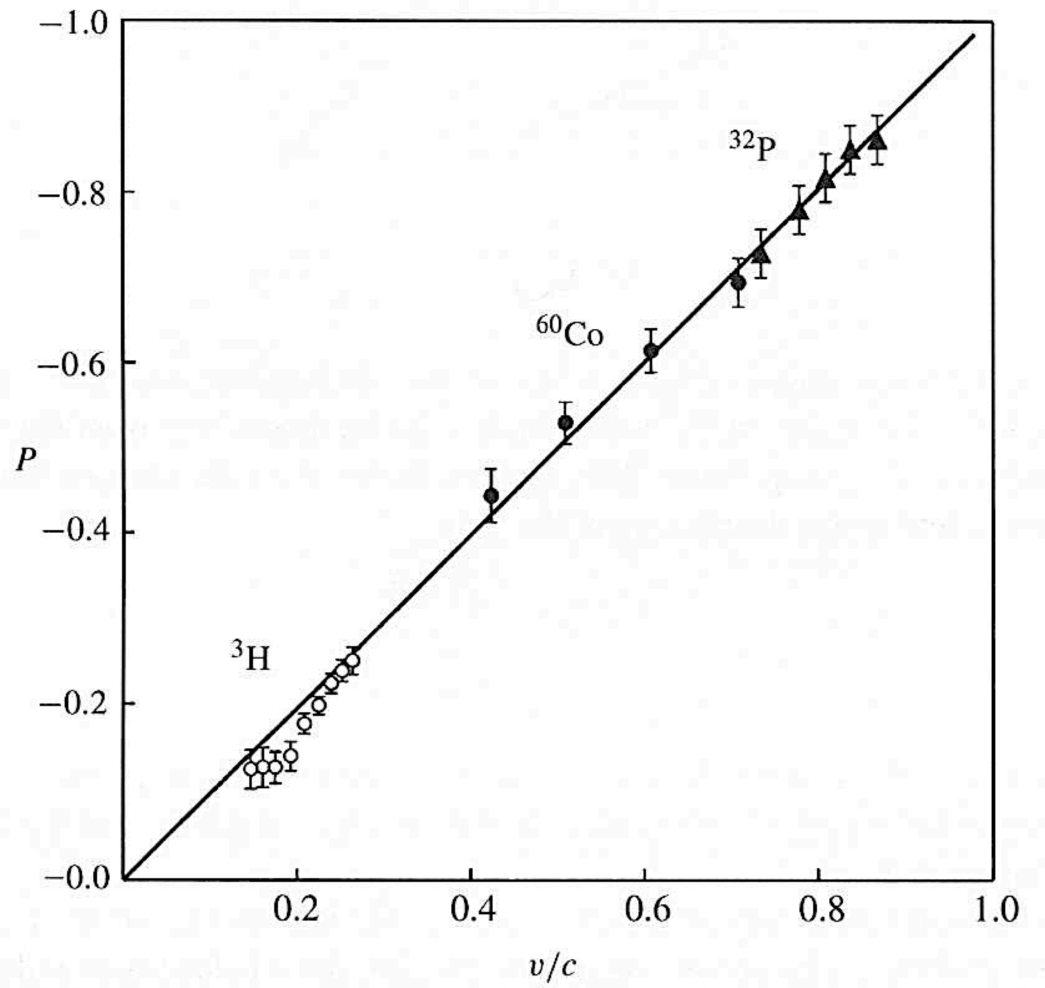
- The fore-aft asymmetry implies parity is violated
  - spin **(s)** does not change sign under reflection
  - momentum (p) changes sign under reflection
  - therefore $\boldsymbol{\sigma} \cdot \mathbf{p}$ changes sign under reflection
  - If parity were conserved, the experiment would find **a** = 0

$$\alpha = \begin{cases} +1 & \text{for } e^+\text{, thus } P = +v/c \\ -1 & \text{for } e^-\text{, thus } P = -v/c \end{cases}$$

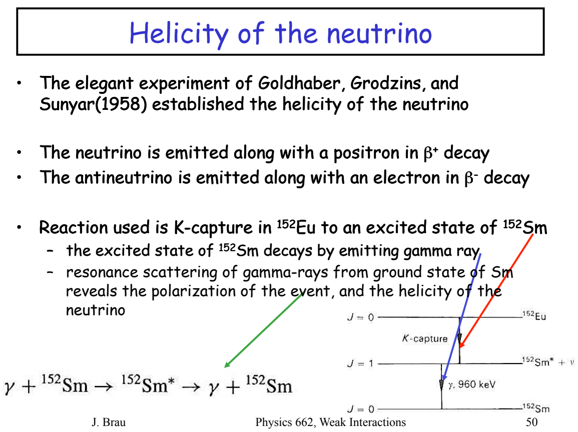
where $P = \alpha\, v/c$

Parity nonconservation in β-decay

$P = \alpha\, v/c$

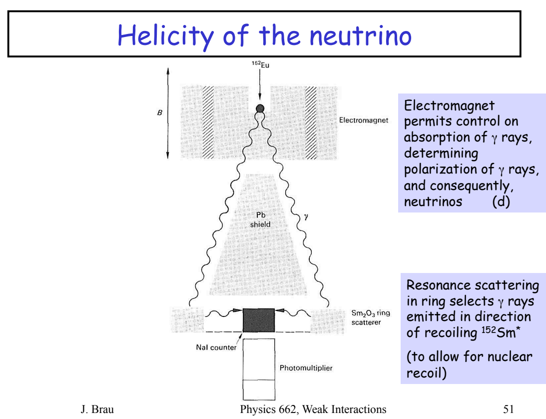


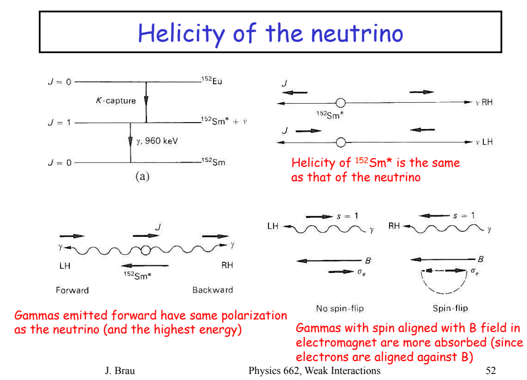
Neutrino Helicity



Experiment described last term established negative helicity of the neutrino.

Neutrinos are left-handed and anti-neutrinos are right-handed



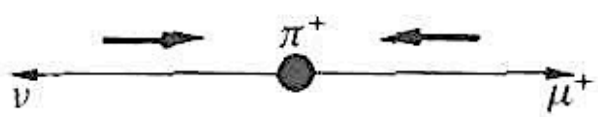


Pion and Muon decay

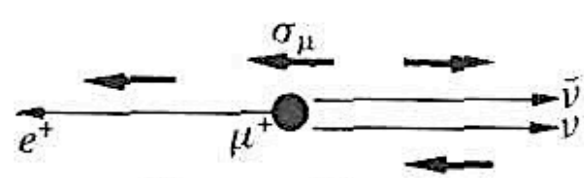
As Wu et al were observing lepton helicities in Co β-decay, others observed effects in decay of pions and muons

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$



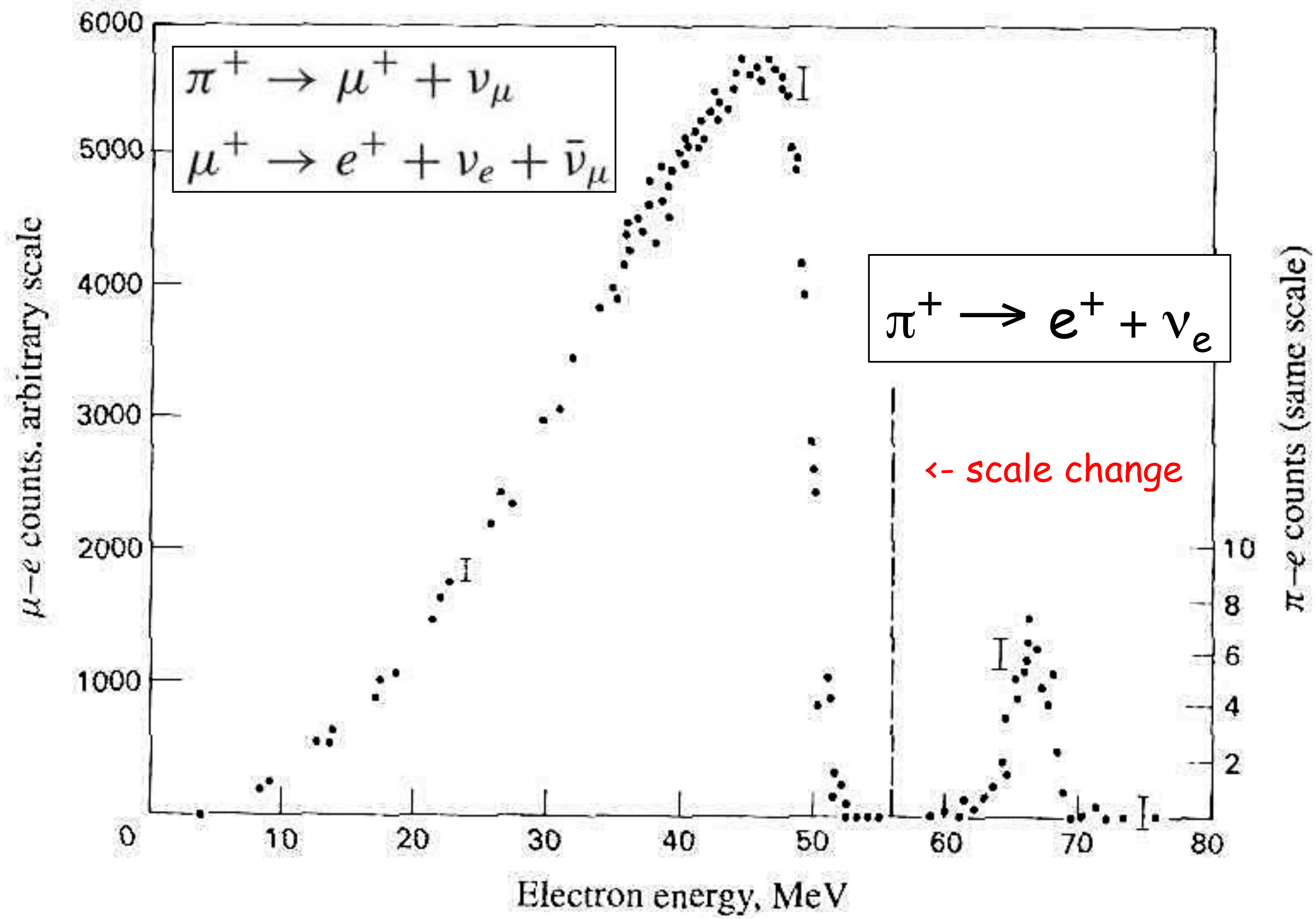
Pion rest frame



Muon rest frame

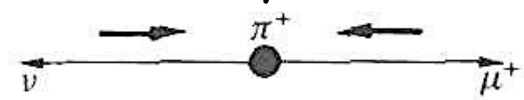
favored configuration

Pion and Muon decay



Pion and Muon decay

- Pions decay in flight $\pi^+ \rightarrow \mu^+ + \nu_\mu$
- Forward going muons, those with highest momentum and negative helicity, were selected



In V-A theory, π decay proceeds through A coupling

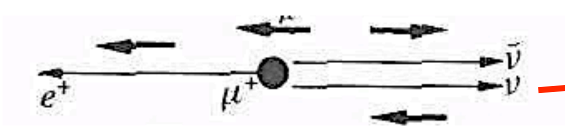
- Muons were stopped and angle of positron was measured

$$\frac{dN}{d\Omega} = 1 - \frac{\alpha}{3}\cos\theta$$

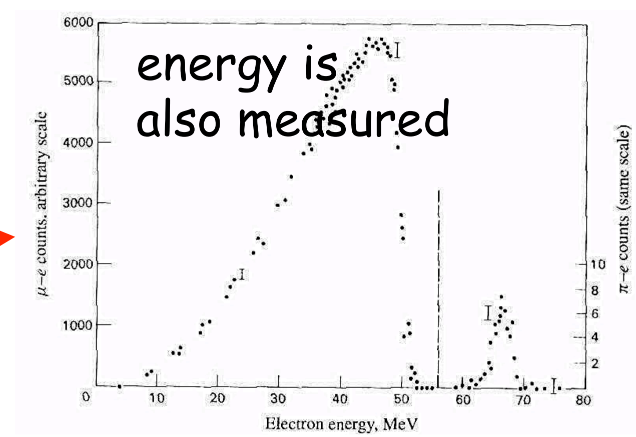
$\alpha = 1$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

This result is predicted by V-A



Energy spectrum suggests this configuration



energy is also measured

Pion and Muon decay

- Pion decay has two modes:

$$\pi^+ \to \mu^+ \nu_\mu$$

$$\pi^+ \to e^+ \nu_e$$

- V-A theory predicts the polarization of the charged antilepton

$$P = \frac{N_R - N_L}{N_R + N_L} = +\frac{v}{c}$$

- But charged antilepton must have same helicity as neutrino (-1), so probability is:

$$\frac{N_L}{N_R + N_L} = \frac{1}{2}\left(1 - \frac{v}{c}\right) \qquad 1 - \frac{v}{c} = \frac{2m^2}{m_\pi^2 + m^2}$$

Pion and Muon decay

- Phase space also affects the rate:

$$E_0 = m_\pi = p + \sqrt{p^2 + m^2}$$

- so the phase space factor is:

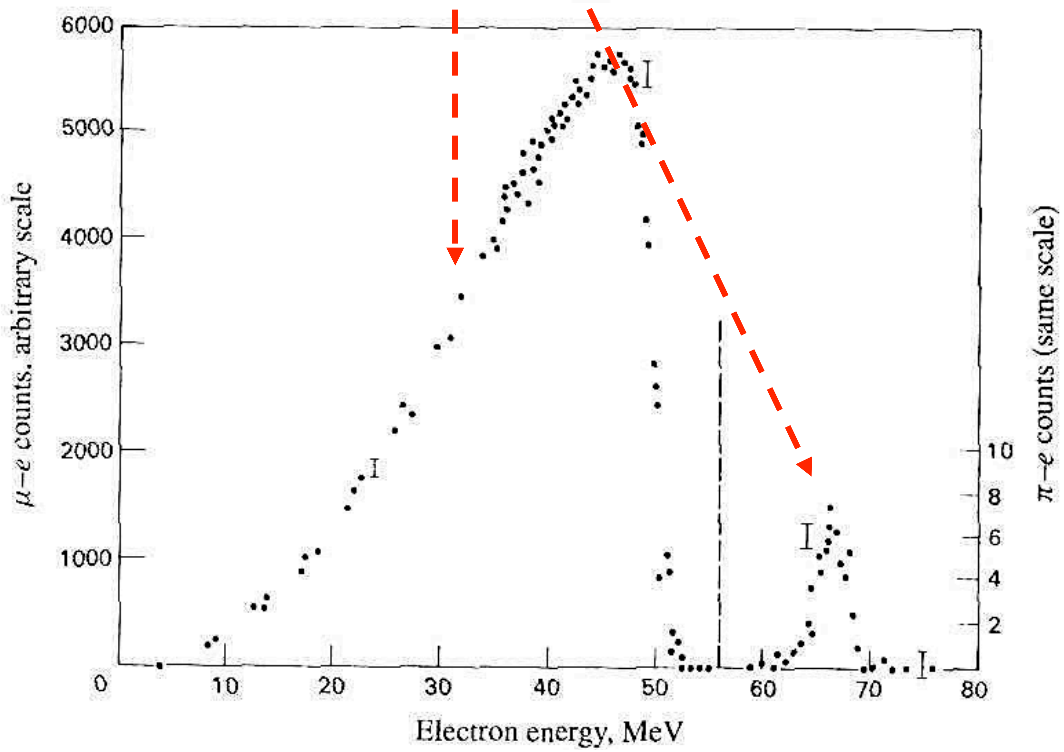
$$p^2 \frac{dp}{dE_0} = \frac{(m_\pi^2 + m^2)(m_\pi^2 - m^2)^2}{4m_\pi^4}$$

- combining this with the polarization:

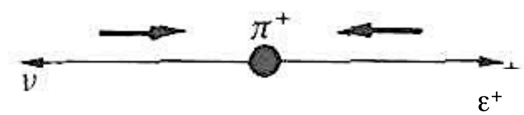
$$p^2 \frac{dp}{dE_0}\left(1 - \frac{v}{c}\right) = \frac{m^2}{2}\left(1 - \frac{m^2}{m_\pi^2}\right)^2$$

Pion and Muon decay

$$R = \frac{\pi \to e + \nu}{\pi \to \mu + \nu} = \frac{m_e^2}{m_\mu^2} \frac{1}{(1 - m_\mu^2/m_\pi^2)^2} = 1.28 \times 10^{-4}$$



$$R_{\text{exp}} = 1.23 \times 10^{-4}$$



Rate is suppressed by requirement that antilepton be left-handed

Pion and Muon decay

- Alternative interaction in pion decay to the A (axial vector) coupling is the P (pseudoscalar) coupling
- There are distinctively different predictions
  - The neutral lepton and charged antilepton would favor having the same helicity, resulting in the probability for the configuration:

$$\frac{N_L}{N_R + N_L} = \frac{1}{2}\left(1 + \frac{v}{c}\right)$$

  - The predicted branching ratio is then:

$$R = \frac{\pi \to e + \nu}{\pi \to \mu + \nu} = \frac{1}{(1 - m_\mu^2/m_\pi^2)^2} = 5.5$$

  - while experiment yields:

$$R_{\exp} = 1.23 \times 10^{-4}$$

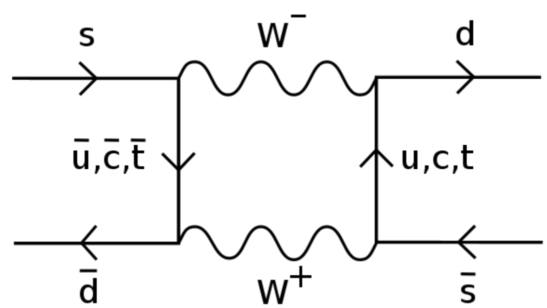
CP Violation

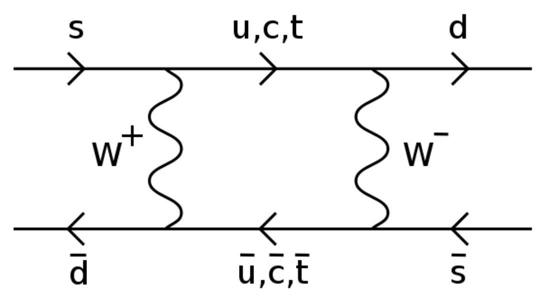
- Until 1964 it was believed that CP symmetry was respected in the weak interaction
  - P and C were known to be separately violated
- A small team led by Cronin and Fitch studying the long-lived neutral kaon discovered that it violated CP with a rate of 2×10^{-3}

$$K_L^0 = \sqrt{\tfrac{1}{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

- $K_L^0 \rightarrow \pi\ \pi\ \pi$ (> 99 %)
- $CP = -1 \rightarrow CP = -1$
- $K_L^0 \rightarrow \pi\ \pi$ (0. 2 %)

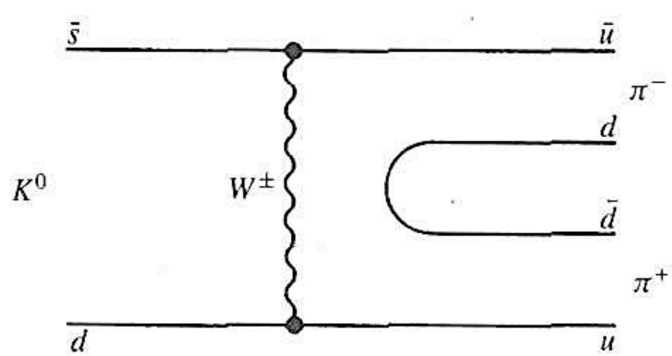
  $CP = -1 \rightarrow CP = +1$

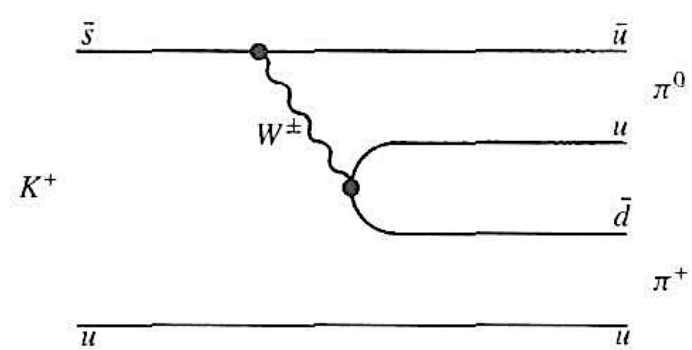




Neutral K mesons

I	I_3	S	Meson	Quark combination	Decay	Mass, MeV
$\frac{1}{2}$	$\frac{1}{2}$	$+1$	K^+	$u\bar{s}$	$K^+ \to \mu\nu$	494
$\frac{1}{2}$	$-\frac{1}{2}$	$+1$	K^0	$d\bar{s}$	$K^0 \to \pi^+\pi^-$	498
$\frac{1}{2}$	$-\frac{1}{2}$	-1	K^-	$\bar{u}s$	$K^- \to \mu\nu$	494
$\frac{1}{2}$	$\frac{1}{2}$	-1	$\bar{K}^0$	$\bar{d}s$	$\bar{K}^0 \to \pi^+\pi^-$	498





Neutral K mesons

K^0 and $\overline{K^0}$ are particle and antiparticle
related by charge conjugation
reversal of I_3 and change of strangeness $\Delta S = 2$

These particles are clearly distinctive in strong interaction,
since SI conserves isospin and strangeness

However, in propagating through free space, mixing occurs:

$$\begin{tikzcd}
 & 2\pi & \\
K^0 \arrow[ur] \arrow[dr] & & \bar{K}^0 \arrow[ul] \arrow[dl] \\
 & 3\pi &
\end{tikzcd}$$

a pure K^0 state at t=0 will become a mixed state:

$$|K(t)\rangle = \alpha(t)|K^0\rangle + \beta(t)|\bar{K}^0\rangle$$

Neutral K mesons

CP eigenstates can be defined for the K^0 mesons

K^0 and $\overline{K^0}$ are not CP eigenstates:

$$CP|K^0\rangle \to \eta|\bar{K}^0\rangle, \qquad CP|\bar{K}^0\rangle \to \eta'|K^0\rangle$$

we can form the linear combinations:

$$|K_S\rangle = \sqrt{\tfrac{1}{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right), \qquad CP = +1$$
$$|K_L\rangle = \sqrt{\tfrac{1}{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right), \qquad CP = -1$$

resulting in:

$$CP|K_S\rangle \to |K_S\rangle, \qquad CP|K_L\rangle \to -|K_L\rangle$$

Two-pion final states

- $K^0 \rightarrow \pi\,\pi$ spin (K) = 0 so L=0
- $P = P_\pi P_\pi (-1)^L = P_\pi^{\;2} = +1$
- $C = C_\pi^{\;2} = +1$
- so CP = +1 for 2 pions

Three-pion final states

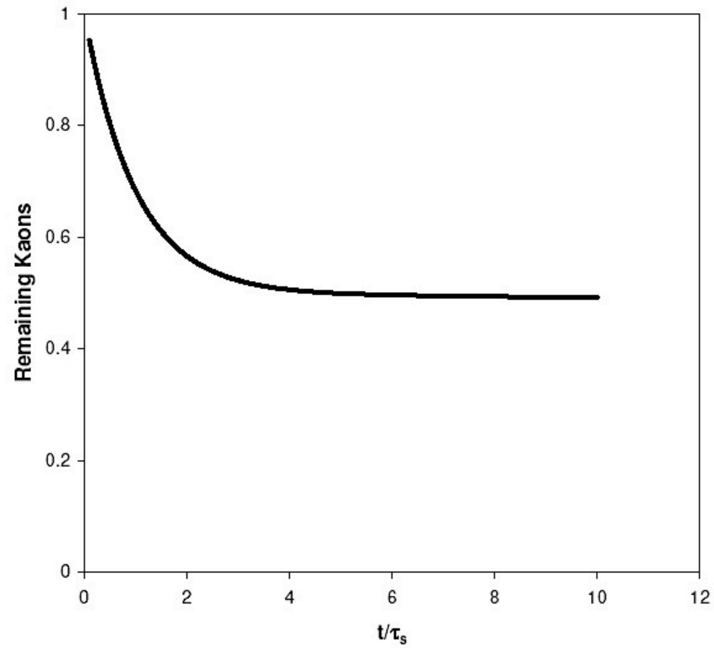
- $K^0 \rightarrow \pi\,\pi\,\pi$ spin (K) = 0 so $L=L_{12}+L_3 = 0$
  only true if $L_{12}=L_3$
- $P = P_\pi P_\pi P_\pi (-1)^{L12} (-1)^{L3} = P_\pi^{\,3} = -1$
- $C = C_\pi^{\,3} = +1$ since $C(\pi^0) = 1$
- so CP = -1 for 3 pions

Neutral K mesons

Consider K^0 production and subsequent decay

Why don't the K^0's decay away with a single exponential?

K^0's are mixtures of K_S and K_L



$$K_S \rightarrow 2\pi(CP=+1), \qquad \tau_S = 0.893 \times 10^{-10}\ \mathrm{s}$$
$$K_L \rightarrow 3\pi(CP=-1), \qquad \tau_L = 0.517 \times 10^{-7}\ \mathrm{s}$$

$0.8953 \pm 0.0005 \times 10^{-10}$ s
$0.5292 \pm 0.0009 \times 10^{-7}$ s

2011

Neutral K mesons

It can be shown that the 2π state has CP = +1,
and the 3π state has CP = -1

Thus, if CP is conserved,

$$K_S = \sqrt{\tfrac{1}{2}}(K^0 + \bar{K}^0) \rightarrow 2\pi(CP = +1),$$
$$K_L = \sqrt{\tfrac{1}{2}}(K^0 - \bar{K}^0) \rightarrow 3\pi(CP = -1),$$

Since the 2 and 3-pion decays have different Q values,
the decay rates are different:

$$K_S \rightarrow 2\pi(CP = +1), \qquad \tau_S = 0.893 \times 10^{-10}\ \mathrm{s}$$
$$K_L \rightarrow 3\pi(CP = -1), \qquad \tau_L = 0.517 \times 10^{-7}\ \mathrm{s}$$

which is why the CP=+1 state is called K-short(K_S)
and the CP=-1 state is called K-long (K_L)

Neutral K mesons

Strangeness oscillations

$$A_S(t) = A_S(0)e^{-(\Gamma_S/2+im_S)t}$$

$$A_L(t) = A_L(0)e^{-(\Gamma_L/2+im_L)t}$$

$$I(K^0) = \tfrac{1}{2}[A_S(t) + A_L(t)][A_S^*(t) + A_L^*(t)]$$
$$= \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

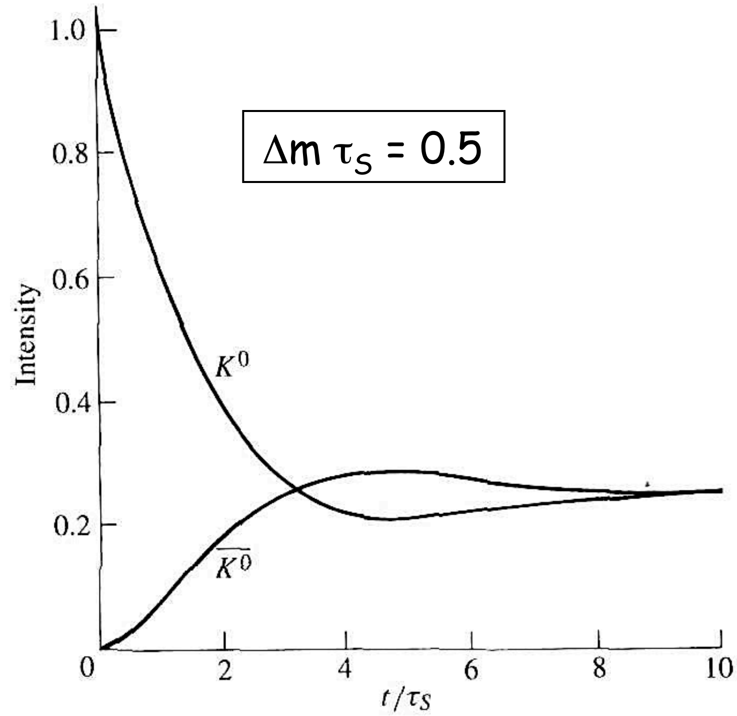
Assuming an initially pure K^0 beam, so $A_S(0) = A_L(0) = 1/\sqrt{2}$

$$\Delta m = m_L - m_S$$

$$I(\bar{K}^0) = \tfrac{1}{4}[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t]$$

$$\boxed{\Delta m = (3.491 \pm 0.009) \times 10^{-12}\ \text{MeV}}$$

Old value in Perkins



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$\Delta m = 0.5292 \pm 0.0009 \times 10^{10}\,\hbar\, s^{-1}$
$= 3.475 \pm 0.006 \times 10^{-12}$ MeV

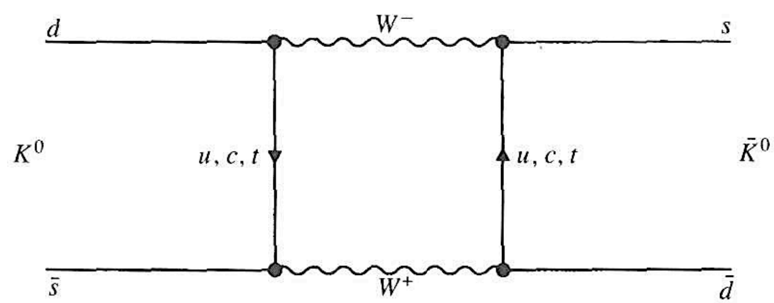
Neutral K mesons

$$\Delta m = 0.5292 \pm 0.0009 \times 10^{10} \hbar\, s^{-1}$$
$$= 3.475 \pm 0.006 \times 10^{-12} \text{ MeV}$$

This mass difference corresponds to the rate at which K^0 oscillates into $\bar{K}^0$.

This oscillation proceeds through the second-order weak interaction

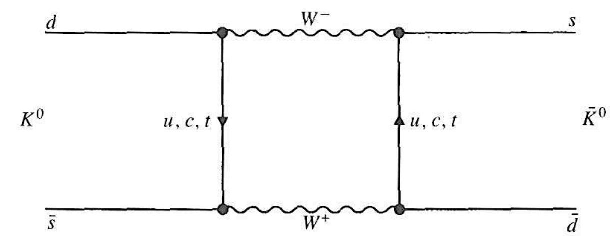
The c quark dominates (CKM matrix)



$$\Delta m = \frac{G^2}{4\pi} m_K f_K^2 m_c^2 \cos^2\theta_c \sin^2\theta_c$$

Neutral K mesons

$$\Delta m = \frac{G^2}{4\pi} m_K f_K^2 m_c^2 \cos^2\theta_c \sin^2\theta_c$$



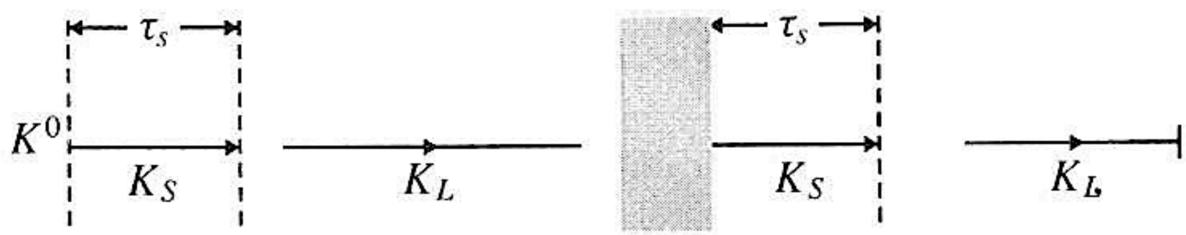
f_K is the called the "kaon decay constant"
$f_K \approx 1.2\ m_\pi$

$m_K f_K^2$ relates to $|\psi(0)|^2$, the wavefunction at the origin, or the point at which the quarks interact

measurement of Δm allowed the mass of the charmed quark to be estimated ($m_c \approx 1.5$ GeV) before it was discovered.

Neutral K mesons

K^0 Regeneration



$$K_L = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)$$

K^0 and $\overline{K}^0$ are absorbed differently

$$\frac{1}{\sqrt{2}}(fK^0 - \bar{f}\overline{K}^0) = \frac{1}{2}(f+\bar{f})K_L + \frac{1}{2}(f-\bar{f})K_S$$

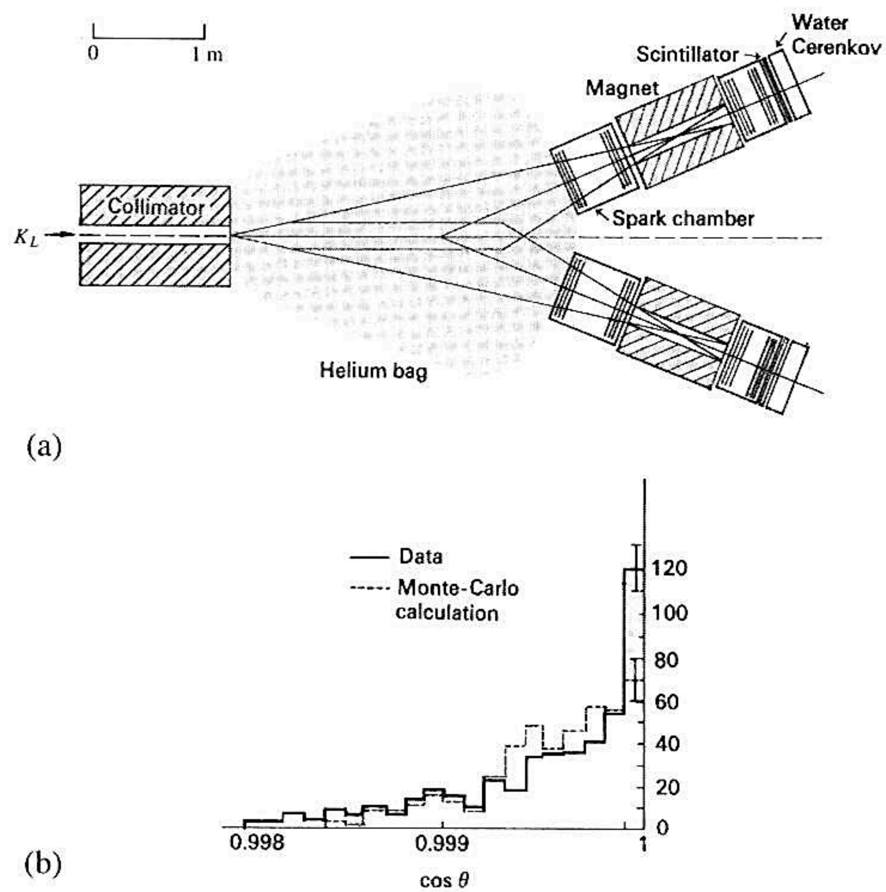
CP violation in the neutral kaon system

K_L is the CP=-1 eigenstate, and is not supposed to decay to two pions, just to three pions

1964 - experiment finds that K_L does decay to two pions with BR = 0.003

$$K_L \rightarrow \pi^+\pi^- \quad (0.002)$$
$$K_L \rightarrow \pi^0\pi^0 \quad (0.001)$$

This violates CP since K_L was is the CP=-1 eigenstate, but the CP of two pions is +1



CP violation in the neutral kaon system

Let K_1 represent the CP = +1 state and K_2 the CP = -1 state

$$K_L = \frac{1}{\sqrt{1+|\varepsilon|^2}}(K_2 + \varepsilon K_1)$$

$$K_S = \frac{1}{\sqrt{1+|\varepsilon|^2}}(K_1 - \varepsilon K_2)$$

ε is a small parameter quantifying the CP violation, which must be determined experimentally

$$|\eta_{+-}| = \frac{\mathrm{ampl}(K_L \to \pi^+\pi^-)}{\mathrm{ampl}(K_S \to \pi^+\pi^-)} = (2.29 \pm 0.02) \times 10^{-3}$$

$$|\eta_{00}| = \frac{\mathrm{ampl}(K_L \to \pi^0\pi^0)}{\mathrm{ampl}(K_S \to \pi^0\pi^0)} = (2.28 \pm 0.02) \times 10^{-3}$$

$|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$
$|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3}$
$|\eta_{00}| = (2.221 \pm 0.011) \times 10^{-3}$

2011

CP violation in the neutral kaon system

Interference between K_S and K_L Amplitudes

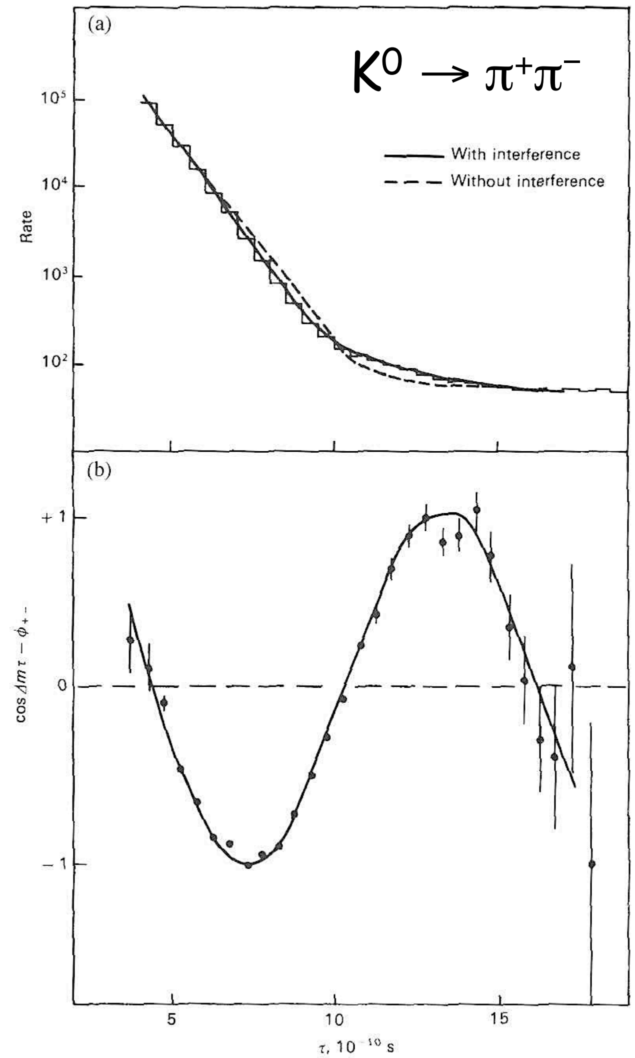
We've seen, starting from a pure K^0 beam:

$$I(K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

Now, K_L will contribute 2π with amplitude η_{+-}

$$\frac{I_{2\pi}(t)}{I_{2\pi}(0)} = e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2|\eta_{+-}|\, e^{-[(\Gamma_L+\Gamma_S)/2]t}\cos(\Delta m\, t + \phi_{+-})$$

where ϕ is the phase angle between the K_s and K_L amplitudes

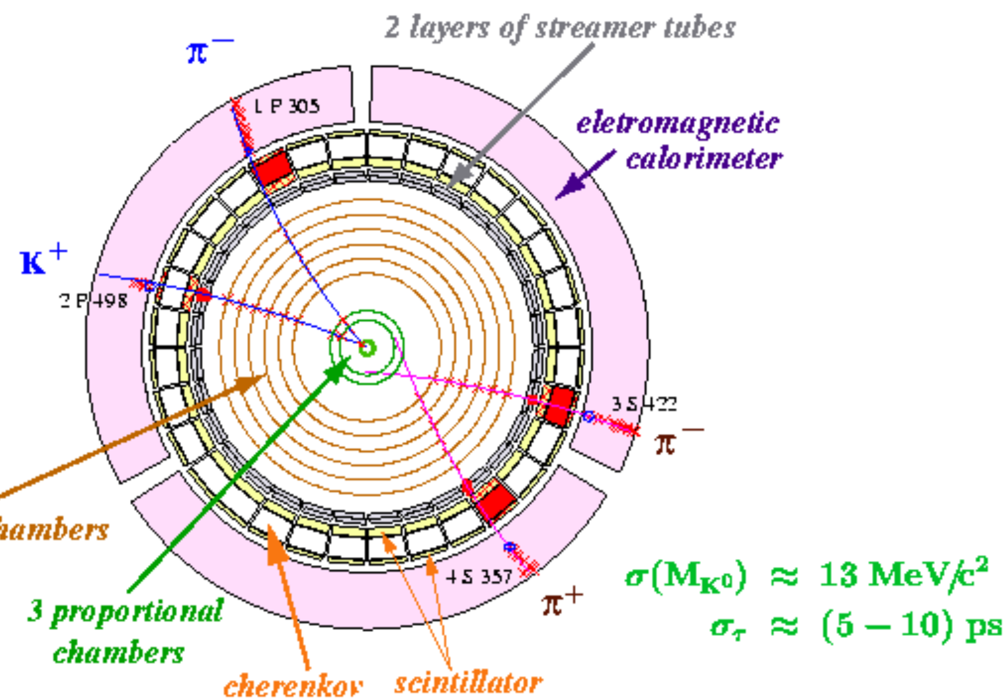


CP violation in the neutral kaon system

CPLEAR

$$p\bar{p} \rightarrow K^-\pi^+K^0$$
$$p\bar{p} \rightarrow K^+\pi^-\bar{K}^0$$

Measured asymmetry between K^0 and K^0



CP violation in the neutral kaon system

Interference between K_S and K_L Amplitudes

$$\frac{I_{2\pi}(t)}{I_{2\pi}(0)} = e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2|\eta_{+-}|\, e^{-[(\Gamma_L+\Gamma_S)/2]t}\cos(\Delta m\, t + \phi_{+-})$$

where ϕ is the phase angle between the K_s and K_L amplitudes

CPLEAR

$$p\bar{p} \to K^- \pi^+ K^0$$
$$p\bar{p} \to K^+ \pi^- \bar{K}^0$$

Measured asymmetry between K^0 and K^0

$$A_{+-}(t) = \frac{2|\eta_{+-}|e^{[(\Gamma_S-\Gamma_L)/2]t}\cos(\Delta m\, t + \phi_{+-})}{1 + |\eta_{+-}|^2 e^{(\Gamma_S-\Gamma_L)t}}$$

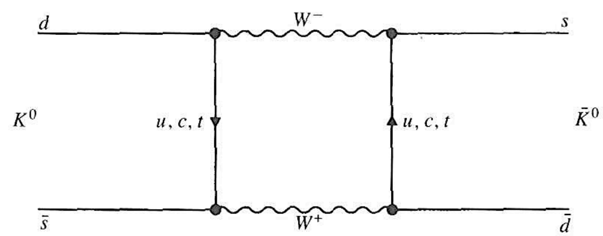
$\phi_{+-} = 43.7^\circ \pm 0.6^\circ \qquad \phi_{00} = 43.5^\circ \pm 1.0^\circ$

$\phi_{+-} = (43.51 \pm 0.05)^\circ \qquad \phi_{00} = (43.52 \pm 0.06)^\circ \qquad$ (2010 PDG)

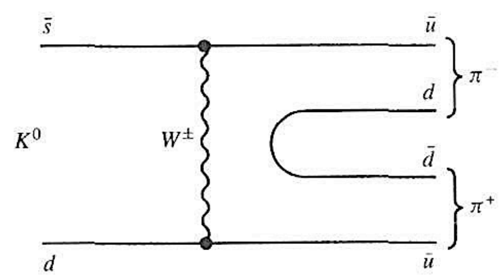
CP violation in the neutral kaon system

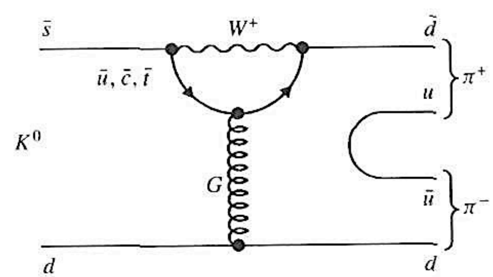
There are two possible sources of CP violation in the K^0 decay:

1. Indirect CP violation - admixture of ε



2. Direct CP violation $\Rightarrow$ ε', from interference





Penguin diagram

$$\eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}} \simeq \varepsilon + \varepsilon'$$
$$\eta_{00} = |\eta_{00}|e^{i\phi_{00}} \simeq \varepsilon - 2\varepsilon'$$

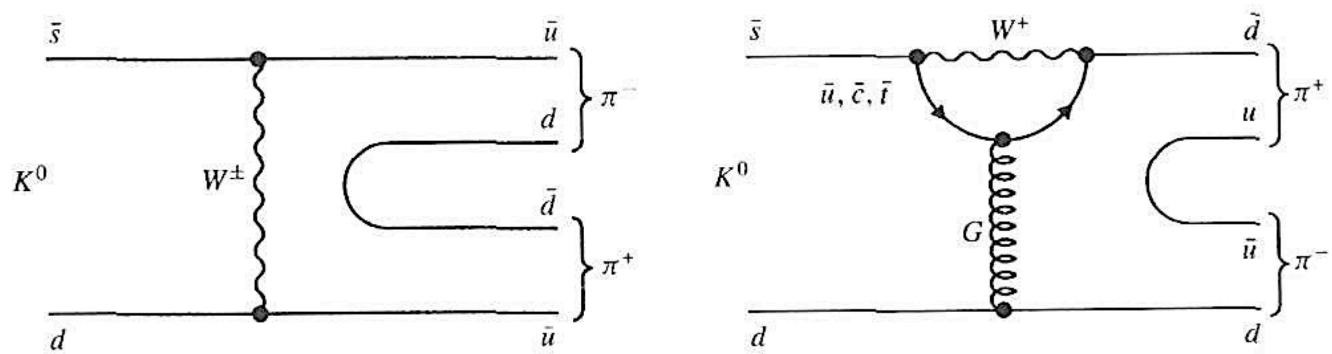
CP violation in the neutral kaon system

Measuring precisely ε'/ε has been a major effort at CERN and Fermilab for many years

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.65 \pm 0.26) \times 10^{-3} \qquad \text{(2010 PDG)}$$

Direct CP violation is established.

This is a puzzle for the Standard Model because the interference (and therefore ε') should be small with m_t = 175 GeV



CP violation in the neutral kaon system

CP Violation in leptonic decays

$$K_L \to e^+ + \nu_e + \pi^-$$
$$K_L \to e^- + \bar{\nu}_e + \pi^+$$

These decays transform into one another under the CP operation, so CP-invariance demands they be equal

$$\Delta = \frac{\text{rate}(K_L \to e^+ + \nu_e + \pi^-) - \text{rate}(K_L \to e^- + \bar{\nu}_e + \pi^+)}{\text{rate}(K_L \to e^+ + \nu_e + \pi^-) + \text{rate}(K_L \to e^- + \bar{\nu}_e + \pi^+)}$$
$$= (0.327 \pm 0.012) \times 10^{-2}$$

Since e^+ must come from K^0 and e^- from $\overline{K^0}$

$$\Delta = \frac{|1+\varepsilon|^2 - |1-\varepsilon|^2}{|1+\varepsilon|^2 + |1-\varepsilon|^2} \simeq 2\,\mathrm{Re}\,\varepsilon$$

$$\Rightarrow \quad \Delta(\varepsilon' = 0) = 2\eta\cos\phi = (0.332 \pm 0.004) \times 10^{-2}$$

CP violation in the neutral kaon system

Superweak Interaction?

Wolfenstein postulated (1964) that CP violation arises from a new superweak interaction ($\Delta S = 2$)

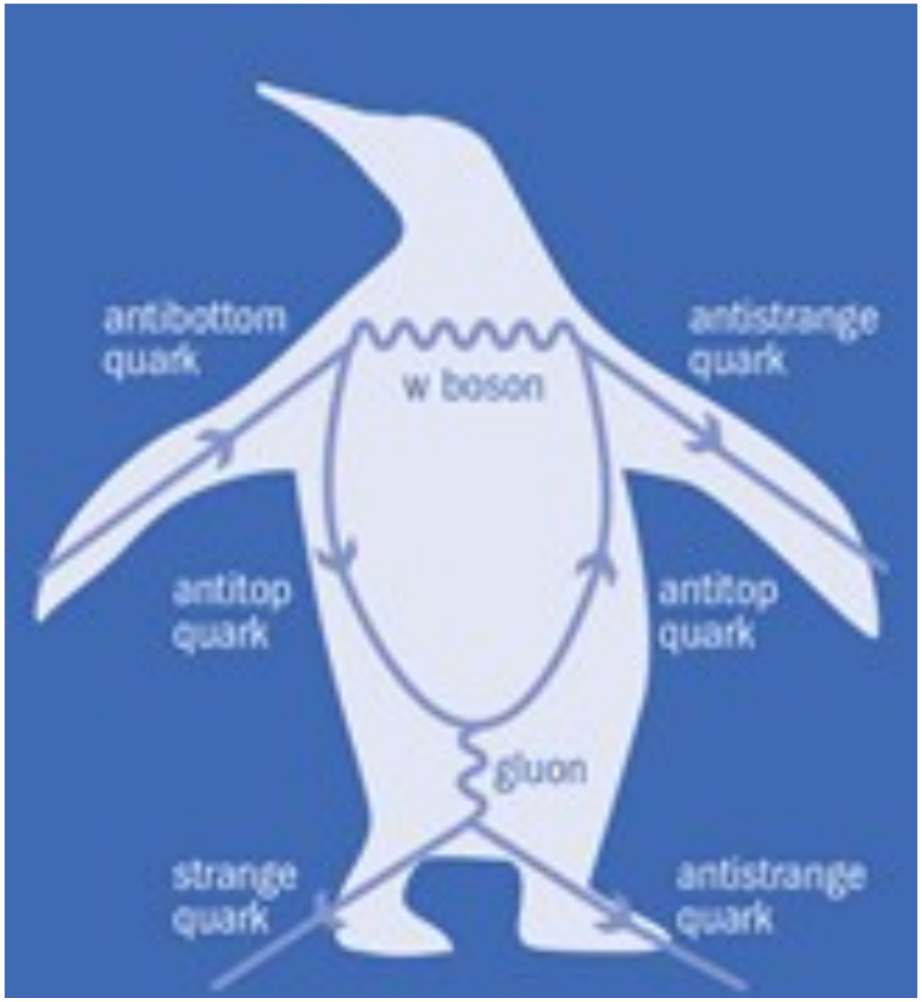
an alternative to the six quark model explanation

Prediction:

$\varepsilon' = 0$ (no Direct CP violation)

which now has been ruled out

Penguin


antibottom
quark
w boson
antistrange
quark
antitop
quark
antitop
quark
gluon
strange
quark
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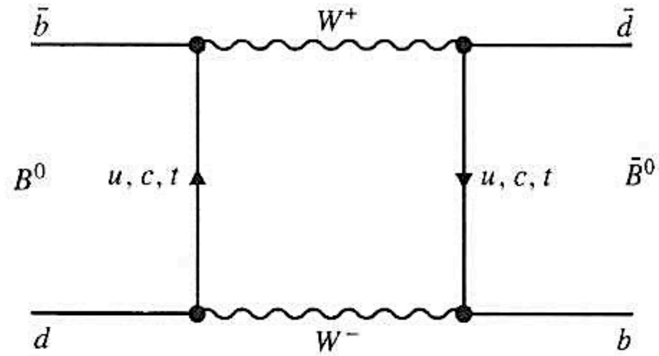
quark

B^0-$\overline{B^0}$ mixing

Mixing in the bottom meson is dominated by the t quark exchange, and is substantial

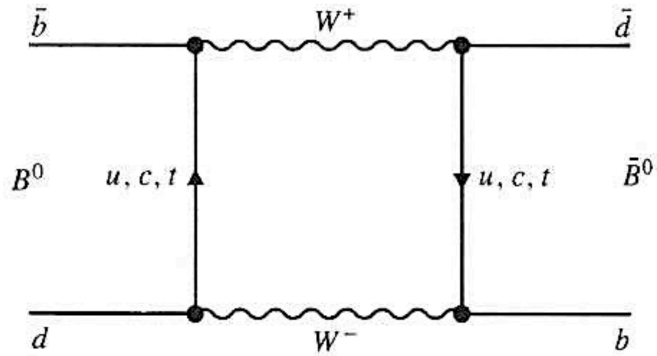


for kaons we had $P(t)dt = \frac{\Gamma e^{-\Gamma t}}{2}\left[\frac{e^{-y\Gamma t}}{2} + \frac{e^{+y\Gamma t}}{2} \pm \cos x\Gamma t\right]dt$

$\Gamma = (\Gamma_S + \Gamma_L)/2 \approx \Gamma_S/2 = \frac{1}{2}/0.89 \times 10^{-10}\,s = 0.56 \times 10^{10}\,s^{-1}$
$y = (\Gamma_S - \Gamma_L) / 2\Gamma \approx 1$
$x = \Delta m / \Gamma = 0.53 \times 10^{10}\,s^{-1}/0.56 \times 10^{10}\,s^{-1} \approx 0.95$

B^0-$\overline{B^0}$ mixing

Mixing in the bottom meson is dominated by the t quark exchange, and is substantial



$$|B_{H,L}\rangle = q\,|B_q^0\rangle \pm p\,|\overline{B}_q^0\rangle\,, \quad |q/p| \approx 1$$

$$\Delta m_q = m_H - m_L$$

$$\Delta\Gamma_q = \Gamma_L - \Gamma_H$$

$$\Gamma(B_d) = 1/(1.519 \pm 0.007 \times 10^{-12}\ s)$$
$$\Delta\Gamma_d/\Gamma_d = 0.015 \pm 0.018$$
$$\Delta m_d = (0.507 \pm 0.004) \times 10^{12}\ s^{-1}$$

B^0-$\overline{B^0}$ mixing

$$P(t)dt = \frac{\Gamma e^{-\Gamma t}}{2}\left[\frac{e^{-y\Gamma t}}{2} + \frac{e^{+y\Gamma t}}{2} \pm \cos x\Gamma t\right] dt$$

For B's, $\Gamma = (\Gamma_S + \Gamma_L)/2$

$y = (\Gamma_S - \Gamma_L) / 2\Gamma \approx 0$ (B^0 and $\overline{B^0}$ decay to common channels)

$x = \Delta m / \Gamma = 0.507 \times 1.519 = 0.77$

$$P(t)dt = \frac{\Gamma e^{-\Gamma t}}{2}\left[1 \pm \cos x\Gamma t\right] dt$$

$\Rightarrow \quad x = 0.77$

B^0-$\overline{B^0}$ mixing

CKM matrix should be unitary

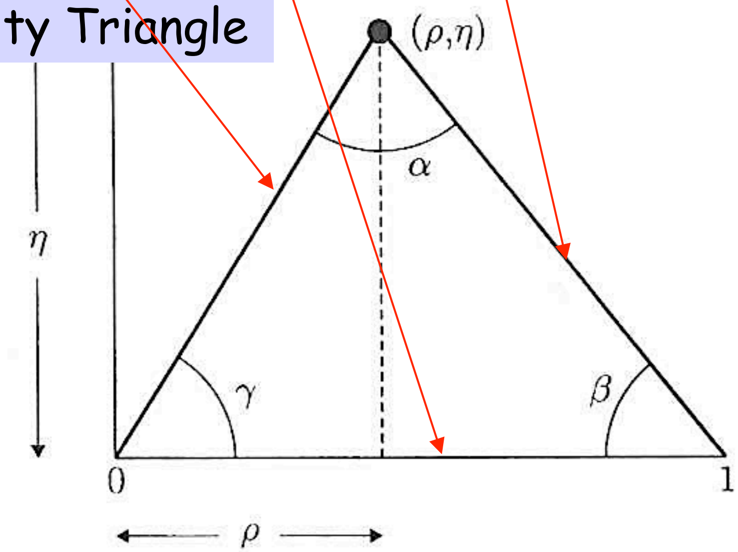
$$V_{CKM} = \begin{vmatrix} V_{ud} = 0.975 & V_{us} = 0.221 & V_{ub} = 0.005 \\ V_{cd} = 0.221 & V_{cs} = 0.974 & V_{cb} = 0.04 \\ V_{td} = 0.01 & V_{ts} = 0.041 & V_{tb} = 0.999 \end{vmatrix}$$

Therefore off-diagonal terms will vanish, such as:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Unitarity Triangle

$$V_{CKM} = \begin{vmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{vmatrix}$$



B_S mixing

$$|B_{H,L}\rangle = q\,|B_S^0\rangle \pm p\,|\bar{B}_S^0\rangle\,, \quad |q/p| \approx 1$$

$$\Delta m_q = m_H - m_L$$

$$\Delta\Gamma_q = \Gamma_L - \Gamma_H$$

$$\Gamma(B_s) = 1/(1.497 \pm 0.015 \times 10^{-12}\ s)$$
$$\Delta\Gamma(B_s) = 0.100 \pm 0.013 \times 10^{12}\ s^{-1}$$

$$\Gamma_L^{-1} = 1.393 \pm 0.019 \times 10^{-12} s \qquad \Gamma_H^{-1} = 1.618 \pm 0.024 \times 10^{-12} s$$

$$\Delta m_s = (17.69 \pm 0.08) \times 10^{12}\ s^{-1}$$

B_s mixing

$$P(t)dt = \frac{\Gamma e^{-\Gamma t}}{2}\left[\frac{e^{-y\Gamma t}}{2} + \frac{e^{+y\Gamma t}}{2} \pm \cos x\Gamma t\right]dt$$

$\Gamma = (\Gamma_S + \Gamma_L)/2 = 1/(1.497 \times 10^{-12}\ s)$

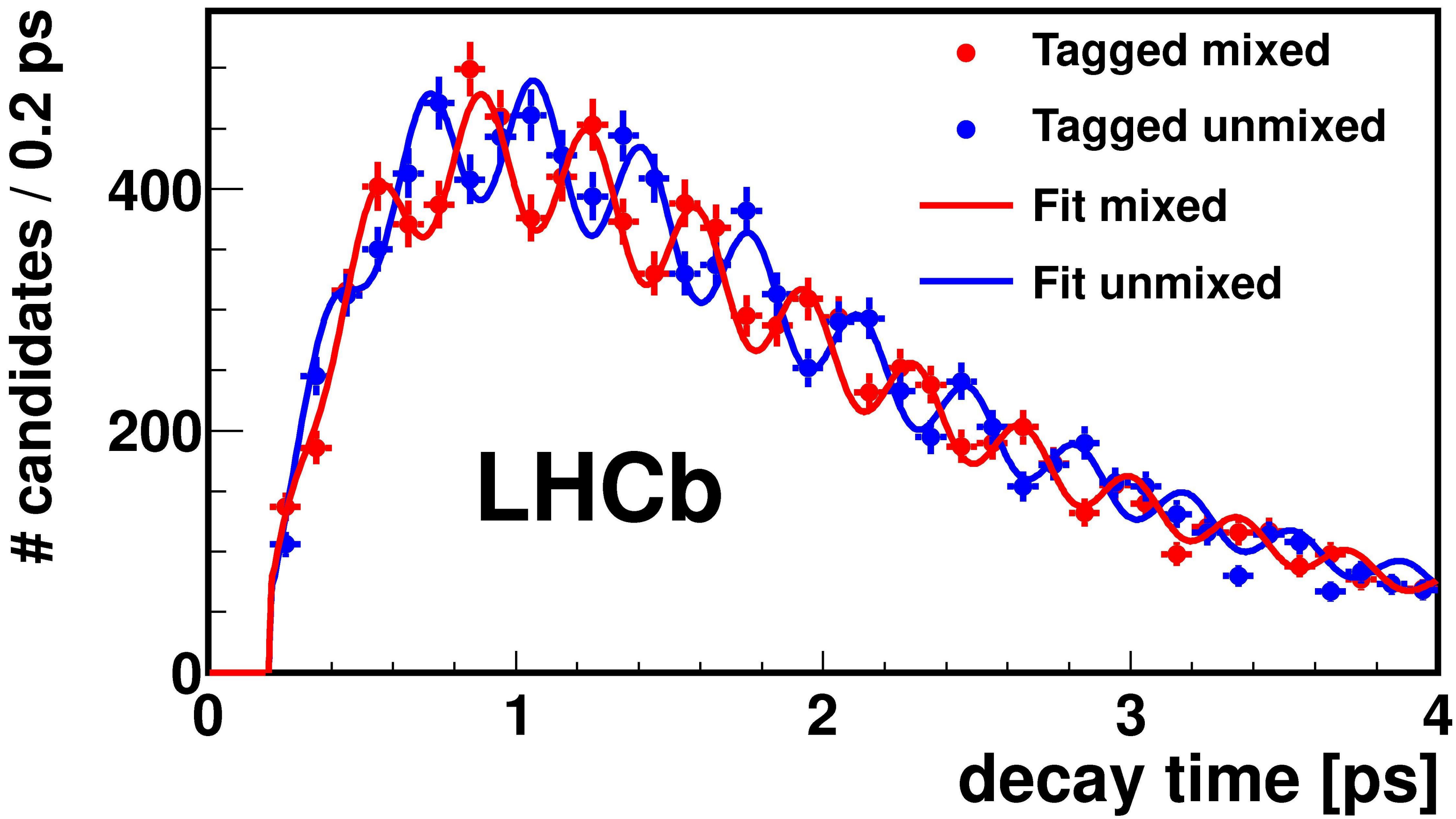
$y = (\Gamma_S - \Gamma_L) / 2\Gamma = 0.100 \times 1.497 / 2 = 0.075$

$x = \Delta m / \Gamma = 17.69 \times 1.497 = 26$

$$P(t)dt = \frac{\Gamma e^{-\Gamma t}}{2}\left[1 \pm \cos x\Gamma t\right]dt$$

$\Rightarrow \quad x = 26$

B_s mixing



Tagged mixed
Tagged unmixed
Fit mixed
Fit unmixed
LHCb
candidates / 0.2 ps
400
200
0
0
1
2
3
4
decay time [ps]

B_d & B_s mixing

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2$$

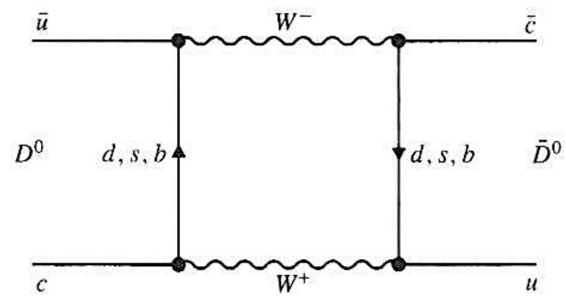
$\xi = 1.237 \pm 0.032$ from lattice QCD

then

$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.2111 \pm 0.0010(\mathrm{exp}) \pm 0.0055(\mathrm{lattice})$$

D^0-$\overline{D^0}$ mixing

Mixing which is seen for Kaons, can also occur in charm mesons as well



Mixing in the charm meson is expected to be very small:

the b quark diagram in D mixing depends on
$|V_{cb}V_{ub}|^2 \approx |0.004 \times 0.04|^2 \approx |0.00016|^2 \approx 3 \times 10^{-8}$

and the s quark diagram is suppressed by the small value of the s quark mass

D^0 mixing

CP eigenstates

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle \;, \quad |q/p| = 1$$

$$M_{1,2} = M \pm \Delta M$$

$$\Gamma_{1,2} = \Gamma \pm \Delta\Gamma$$

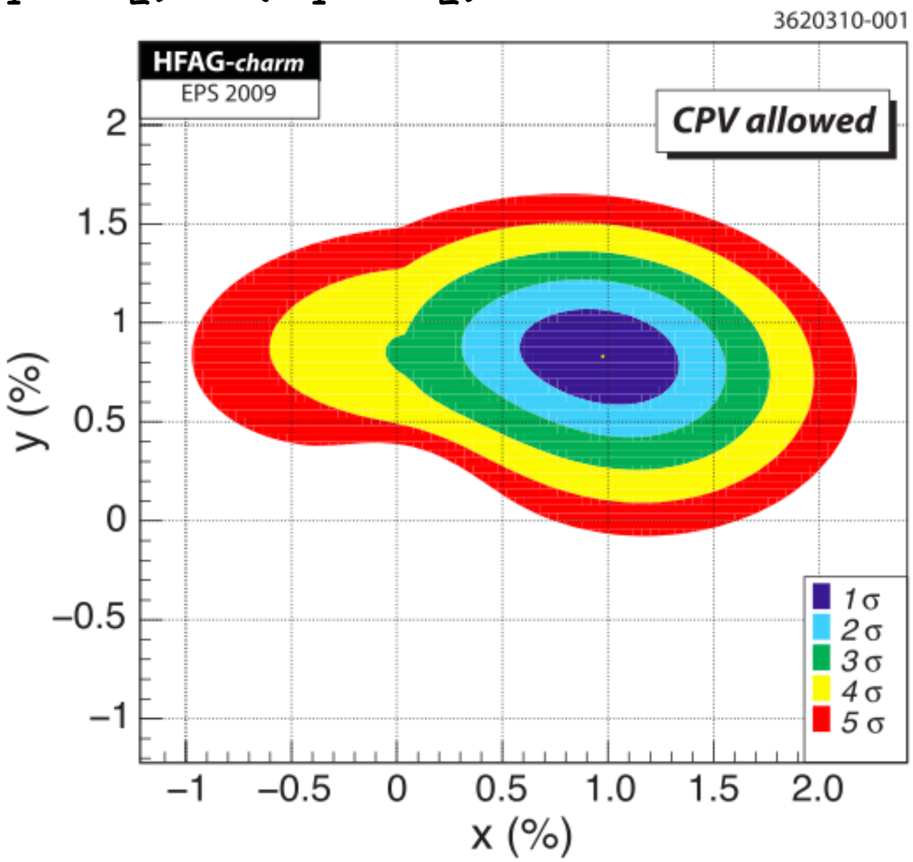
$$x = 2\,\Delta M \,/\, (\Gamma_1 + \Gamma_2)$$

$$y = (\Gamma_1 - \Gamma_2) \,/\, (\Gamma_1 + \Gamma_2)$$

D^0 mixing

$x = 2\,\Delta M / (\Gamma_1 + \Gamma_2)$

$y = (\Gamma_1 - \Gamma_2) / (\Gamma_1 + \Gamma_2)$



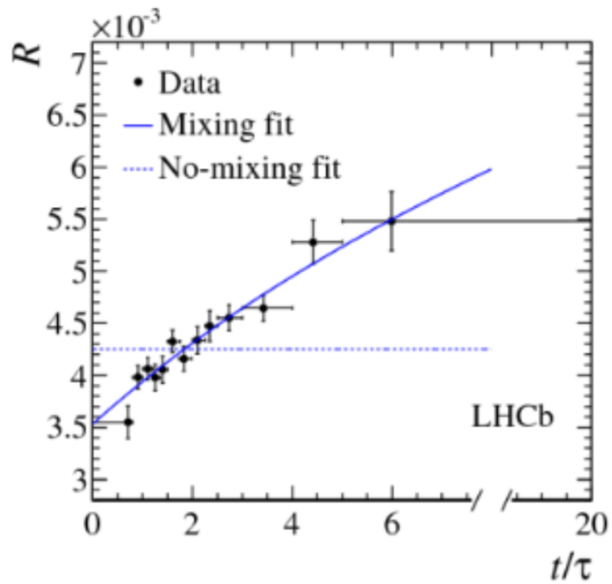
D^0 Mixing at LHCb

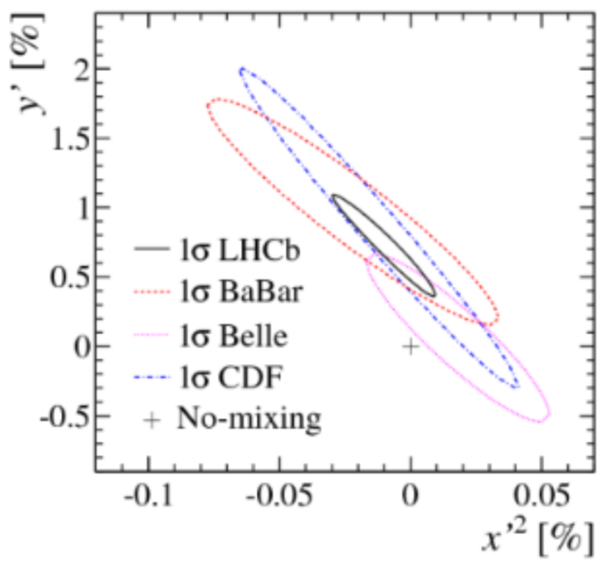
Initial state identified by charge of accompanying pion

$D^{*+} \rightarrow D^0\pi^+$ or $\overline{D}^{*-} \rightarrow D^0\pi^-$

Then look for ratio of "wrong-sign" ($D^0 \rightarrow K^+\pi^-$)

and "right-sign" ($D^0 \rightarrow K^-\pi^+$) (constant if no mixing)





LATEST: $x'^2 = (5.5 \pm 4.9) \times 10^{-5}$; $y' = (4.8 \pm 1.0) \times 10^{-3}$

CP Violation in the B System

CP violation shows up as large asymmetries in decay modes with very small BRs

eg. $B^0 \rightarrow K^+\pi^-, K^-\pi^+$

$$\frac{(\overline{B^0} \rightarrow K^-\pi^+) - (B^0 \rightarrow K^+\pi^-)}{(\overline{B^0} \rightarrow K^-\pi^+) + (B^0 \rightarrow K^+\pi^-)} = -0.082 \pm 0.006$$

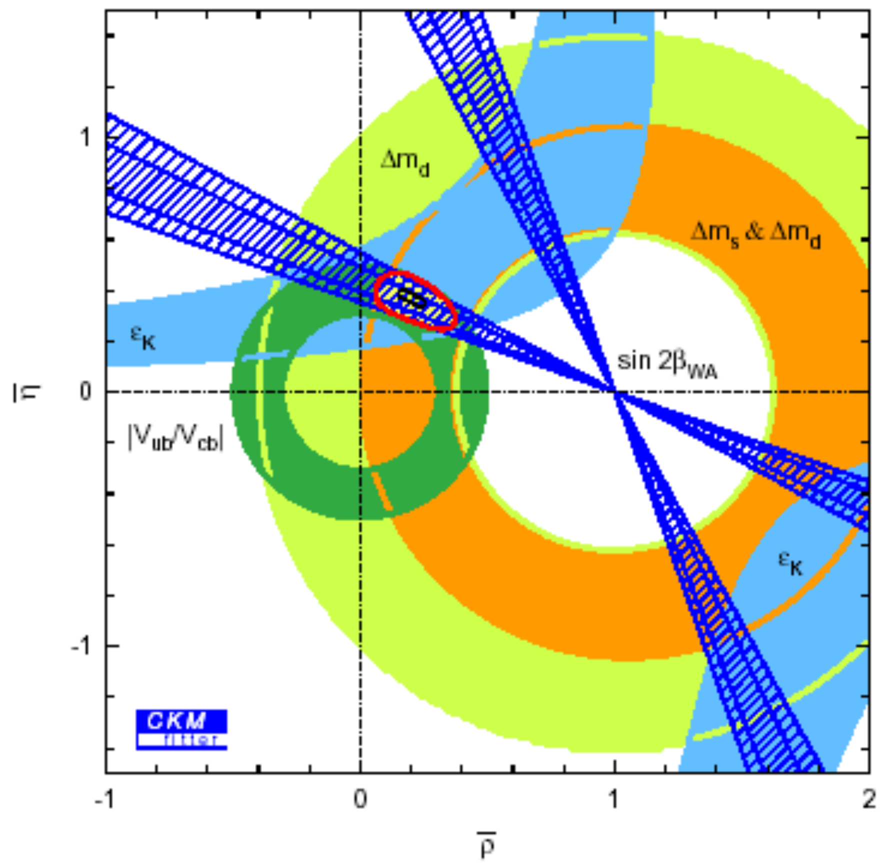
CP Violation in the B System

CP violation shows up as large asymmetries in decay modes with very small BRs

eg. $B^0 \rightarrow J/\psi\ K_S^0$

$$R \propto e^{-t/\tau}(1 \pm \sin 2\beta \sin \Delta m\, t)$$

sin 2β = 0.679 ± 0.020
http://www.slac.stanford.edu/xorg/hfag/



B mesons and CP violation

In searching for CP violation with Kaons one can let the K_1s decay away and then look for K_2 decays to 2π

B lifetimes for the two CP eigenstates are similar, so this approach does not work.

$$\frac{\Gamma_S^K - \Gamma_L^K}{\Gamma_S^K + \Gamma_L^K} \approx 1 \qquad \frac{\Gamma_S^B - \Gamma_L^B}{\Gamma_S^B + \Gamma_L^B} \approx 0$$

Instead, the time evolution of $B^0\overline{B^0}$ pairs is examined to measurable CP violation.

Look for rate differences to the same CP final state (f):

$$R(B^0 \rightarrow f) \neq R(\overline{B}^0 \rightarrow f)$$

B mesons, CP violation, and the CKM matrix

CKM in terms of W couplings to charge 2/3 quarks (best for illustrating physics!)

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

four real parameters, phase generates CP violation

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The “Wolfenstein” representaton:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

This representation uses $s_{12} >> s_{23} >> s_{13}$ and $c_{23}=c_{13}=1$
Here $\lambda=\sin\theta_{12} \approx \theta_{12}$, and A, ρ, η are all real and ≈ 1.

The Wolfenstein representation is good for relating CP violation to specific decay rates.

A non-zero η gives CP violation since it provides a phase in the decay amplitude.

Why do we need a phase to observe CP violation?

Who needs a phase ?

A difference between the particle and anti-particle decay rate to the same CP final state is evidence of CP violation:

$$R(B^0 \to f) \neq R(\overline{B}^0 \to f)$$

If the decay amplitude contains a phase that changes sign under CP then:

$$A = |A| e^{i\phi} \Rightarrow \overline{A} = |A| e^{-i\phi}$$

But this won't give CP violation since:

$$A^* A = |A| e^{-i\phi} |A| e^{i\phi} = \overline{A}^* \overline{A} = |A| e^{i\phi} |A| e^{-i\phi} = |A|^2$$

In order to have CP violation there must be:

a) two amplitudes

b) two phases (weak phase, strong phase)

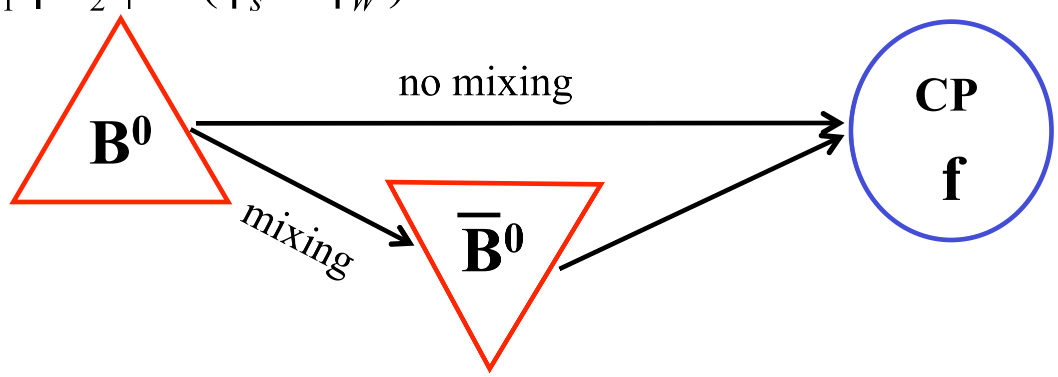
c) only one phase changes sign under CP (weak phase)

$$A = A_1 + A_2 = |A_1| e^{i\phi_W} e^{i\phi_s} + |A_2| \qquad \overline{A} = \overline{A}_1 + \overline{A}_2 = |A_1| e^{-i\phi_W} e^{i\phi_s} + |A_2|$$

$$A^* A = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\phi_s + \phi_W)$$

$$\overline{A}^* \overline{A} = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\phi_s - \phi_W)$$

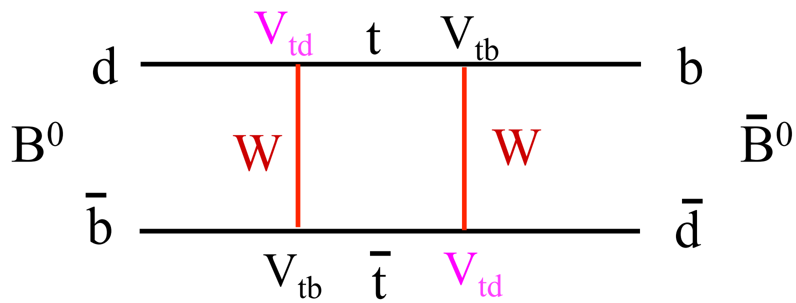
Use interference of B-meson decays to same final state (f) with/without mixing.

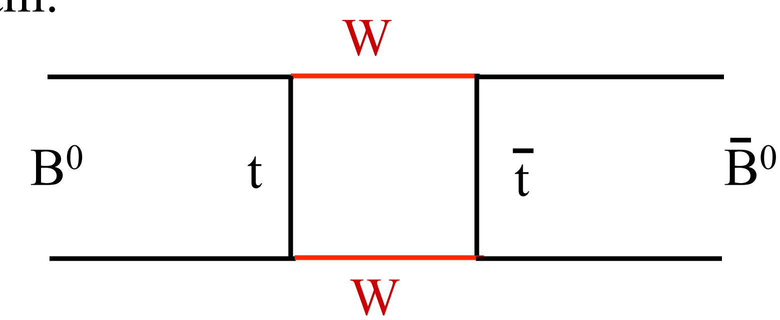


B Mixing, CP violation, and the CKM matrix

B mixing is exactly the same process that we discussed in "strangeness oscillations". Even simpler since the lifetimes are the same for both states (B_L, B_S).

A B^0 can oscillate into a $\bar{B}^0$ via a "box" diagram:

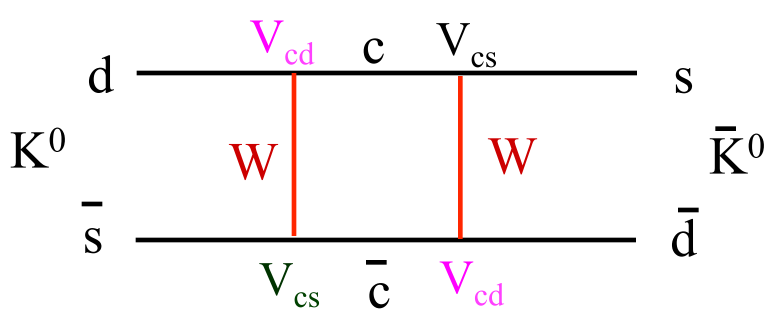


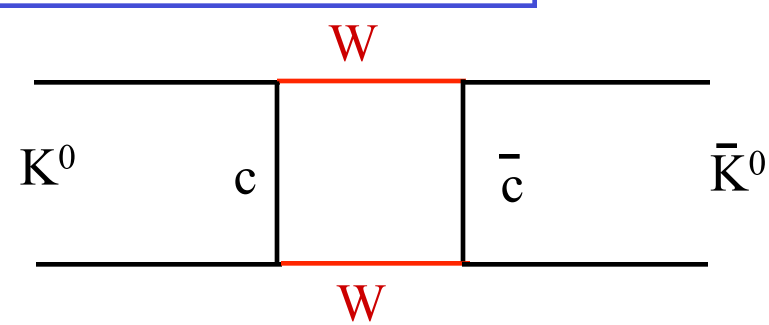


$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \underbrace{A\lambda^3(1-\rho-i\eta)}_{V_{td}} & -A\lambda^2 & \underbrace{1}_{V_{tb}} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

$\boxed{V_{td} \text{ provides the weak phase necessary for CP violation in B decay.}}$



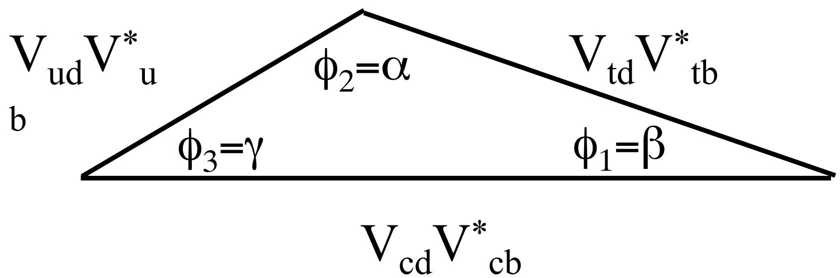


The Unitarity Triangle

Since the CKM matrix is unitary we must have:

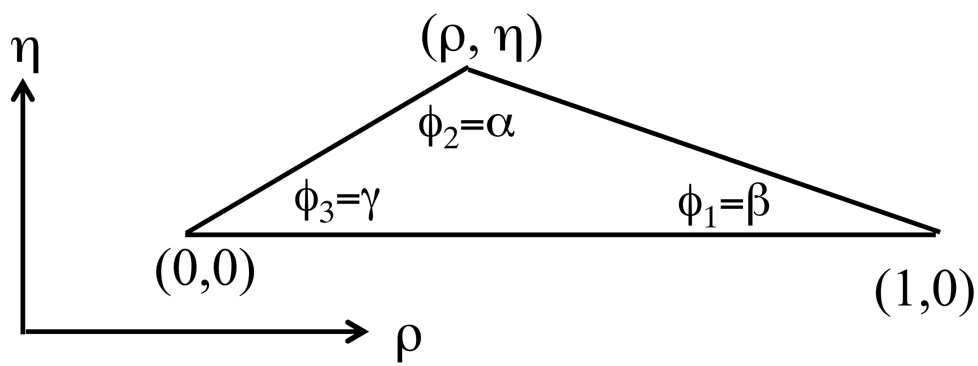
$$V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0$$

Since matrix elements can be complex numbers we can picture this relationship as a triangle.



$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

Convenient to normalize all sides to the base of the triangle ($V_{cd}V^{*}_{cb} = A\lambda^3$).
In the (ρ, η) plane the triangle now becomes:

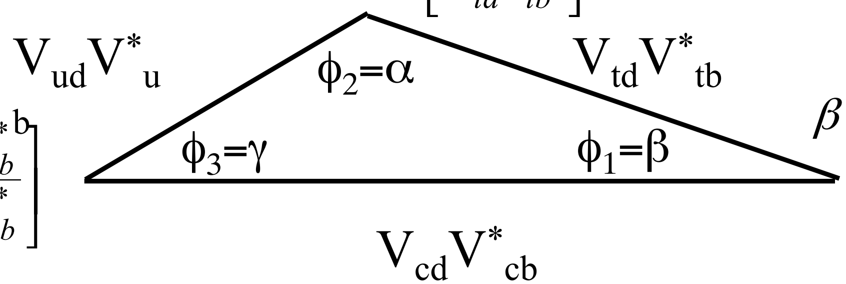


$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

One way to test the Standard Model is to measure the 3 sides & 3 angles and see if the triangles closes!

The Unitarity Triangle & B decays

$$\pi - \alpha = Arg\left[\frac{V_{ud}V_{ub}^*}{V_{td}V_{tb}^*}\right]$$



$$\gamma = Arg\left[\frac{V_{cd}V_{cb}^*}{V_{ud}V_{ub}^*}\right] \qquad \beta = Arg\left[\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

1) Sin2β:

$$b \to c\bar{c}s : \; B_d \to \psi K_s^0, B_d \to \psi K_L^0, \; B_d \to \psi K^{*0}$$

$B^0 \to \psi K_S$: get $V_{td}V^*_{tb}$ from B mixing, V_{cb} from $b \to c$, get V_{cd} from K^0 mixing.

$$b \to c\bar{c}d : \; B_d \to D\bar{D}, B_d \to D\bar{D}^*, \; B_d \to D^*\bar{D}, \; B_d \to D^*\bar{D}^*$$

$$b \to s\bar{s}s, d\bar{d}s : \; B_d \to \pi^0 K_s^0, B_d \to \phi K_s^0$$

2) Sin2α:

$$B_d \to \pi^0\pi^0, B_d \to \pi^+\pi^+, \; B_d \to \pi^+\rho^-, B_d \to \pi^0\rho^0$$

3) Sin2γ:

$$B_s \to K_s\rho^0, \; B_s \to D_S K$$

Steps to observing CP violation with B mesons

Produce B-mesons pairs using the reaction $e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0$

must build an asymmetric collider

Reconstruct the decay of one of the B-mesons's into a CP eigenstate

example CP: $B^0 \to \psi K_S$ and $\bar{B}^0 \to \psi K_S$

Reconstruct the decay of the other B-meson to determine its flavor ("tag")

use high momentum leptons: $B^0 \to (e^+ \text{ or } \mu^+)X$ and $\bar{B}^0 \to (e^- \text{ or } \mu^-)X$

flavor of CP eigenstate also determined at time of the "tag" decay.

Measure the distance (L) between the two B meson decays and convert to proper time

must reconstruct the position of both B decay vertices

$t = L/(\beta\gamma c)$

Fit the decay time difference (t) to the functional form:

$$dN/dt \propto e^{-\Gamma|t|}\left[1 \pm \eta_{cp}(\mathbf{\sin 2\phi})\sin(\Delta m t)\right]$$

$-B^0, +\bar{B}^0$

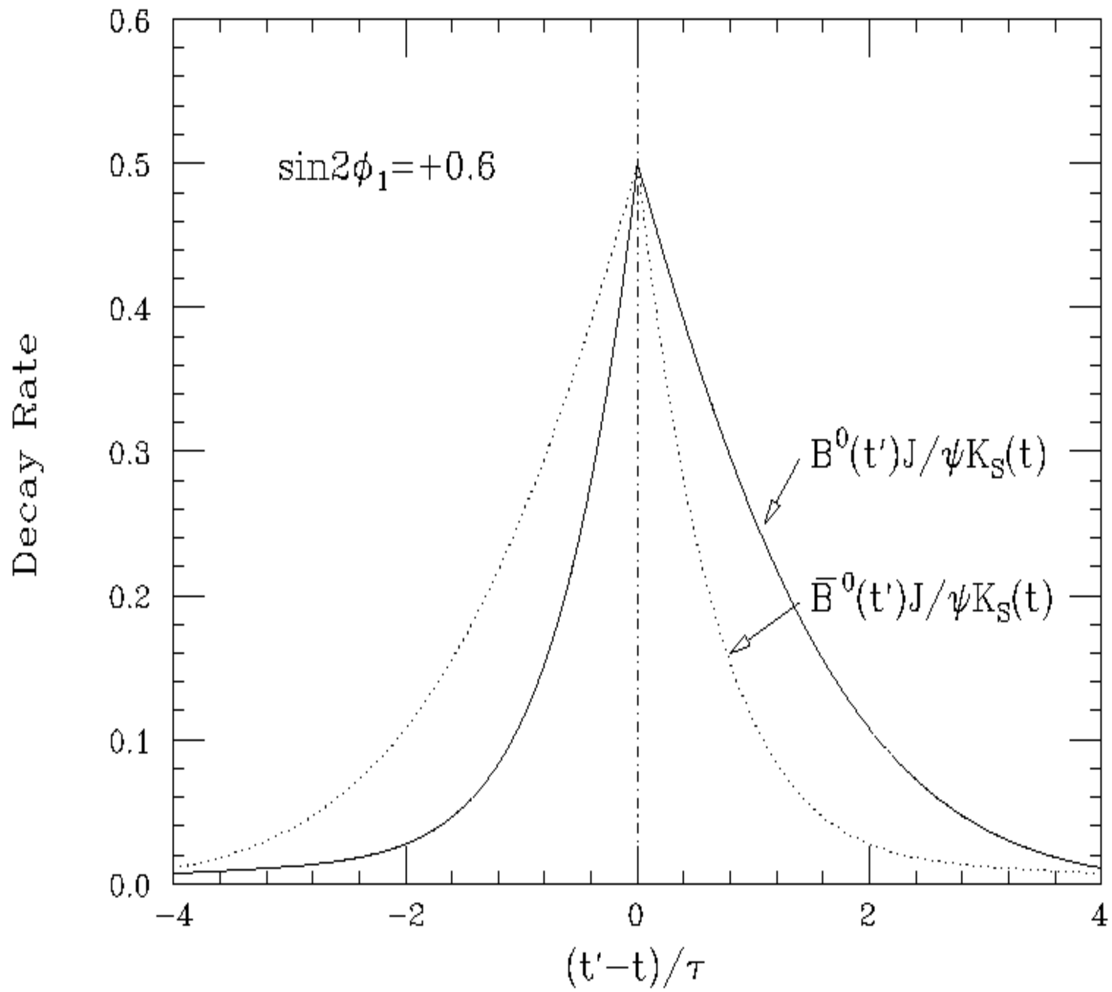
$\eta_{cp} = \pm 1$
CP of final state
$= -1$ for $B^0 \to \psi K_S$

CP violating phase

Δm=difference between B mass eigenstates
$\Delta m = 0.47 \times 10^{12}\hbar/s$

Determine **sin2ϕ**

Expected signature of CP violation with B mesons



Decay rate is not the same for B^0 and $\overline{B}^0$ tag.

Why do we need an asymmetric collider?

The source of B mesons is the Y(4S), which has $J^{PC}=1^{--}$.
The Y(4S) decays to two bosons with $J^{P}=0^{-}$.
Quantum Mechanics (application of the Einstein-Rosen-Podosky Effect) tells us that for a C=- initial state (Y(4S)) the rate asymmetry:

$$A = \frac{N_{(B_1 \to f_{CP})(\bar{B}_2 \to \bar{f}_{fl})} - N_{(B_1 \to f_{CP})(B_2 \to f_{fl})}}{N_{(B_1 \to f_{CP})(\bar{B}_2 \to \bar{f}_{fl})} + N_{(B_1 \to f_{CP})(B_2 \to f_{fl})}} = 0$$

N=number of events
f_{CP}= CP eigenstate (e.g. $B^0 \to \psi K_S$)
f_{fl}= flavor state (particle or anti-particle) (e.g. $B^0 \to e^+X$)

However, if we measure the time dependence of A we find:

$$A(t_1,t_2) = \frac{N(t_1,t_2)_{(B_1 \to f_{CP})(\bar{B}_2 \to \bar{f}_{fl})} - N(t_1,t_2)_{(B_1 \to f_{CP})(B_2 \to f_{fl})}}{N(t_1,t_2)_{(B_1 \to f_{CP})(\bar{B}_2 \to \bar{f}_{fl})} + N(t_1,t_2)_{(B_1 \to f_{CP})(B_2 \to f_{fl})}} \propto \sin 2\phi_{CP}$$

Need to measure the time dependence of decays to "see" CP violation using the B's produced at the Y(4S).

Why Do We Need an Asymmetric Collider?

In order to measure time we must measure distance: t=L/v.

How far do B mesons travel after being produced by the Υ(4S) (at rest) at a symmetric e^+e^- collider?

At a symmetric collider we have for the B mesons from Υ(4S) decay:

p_{lab} =0.3 GeV, m_B=5.28 GeV

$\tau_B = 1.6 \times 10^{-12}$ sec

Average flight distance <L>= $(\beta\gamma)c\tau_B$= (p/m)(468μm)=(0.3/5.28)(468μm)=(27μm)

This is too small to measure!!

If the beams have unequal energies then the entire system is Lorentz Boosted:

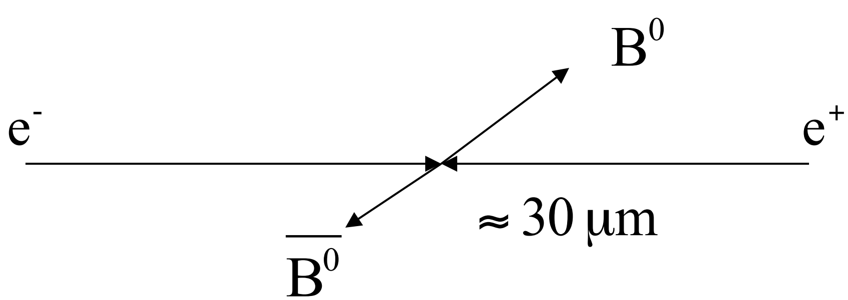
$\beta\gamma = p_{lab}/E_{cm} = (p_{high} - p_{low})/E_{cm}$

SLAC: 9 GeV+3.1 GeV $\beta\gamma$ = 0.55 <L>= 257μm

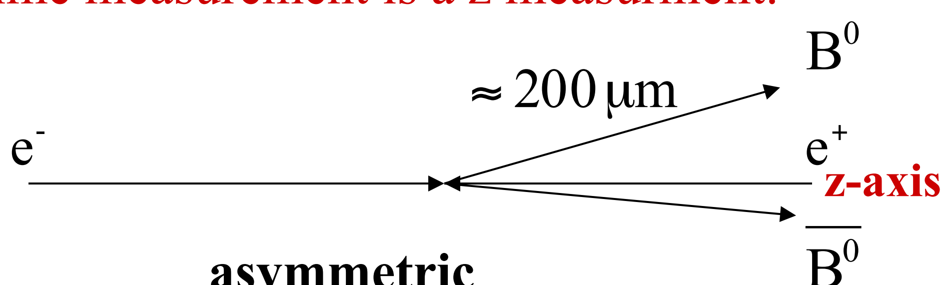
KEK: 8 GeV+3.5 GeV $\beta\gamma$ = 0.42 <L>= 197μm

We can measure these decay distances !

Because of the boost and the small p_{lab} the time measurement is a z measurment.



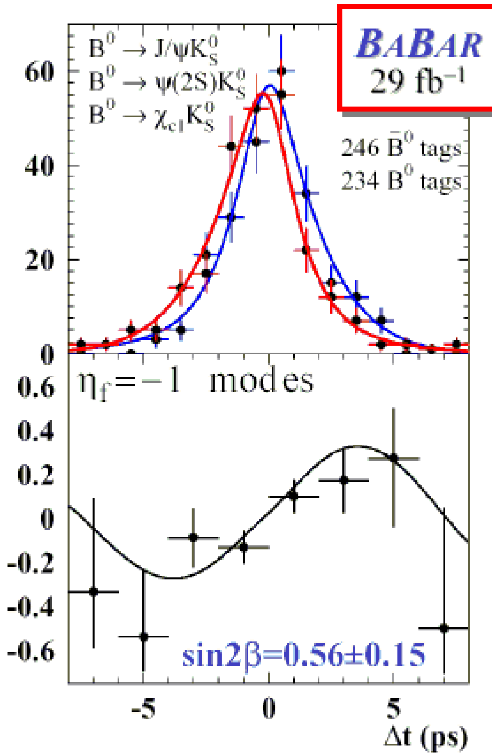
symmetric
CESR

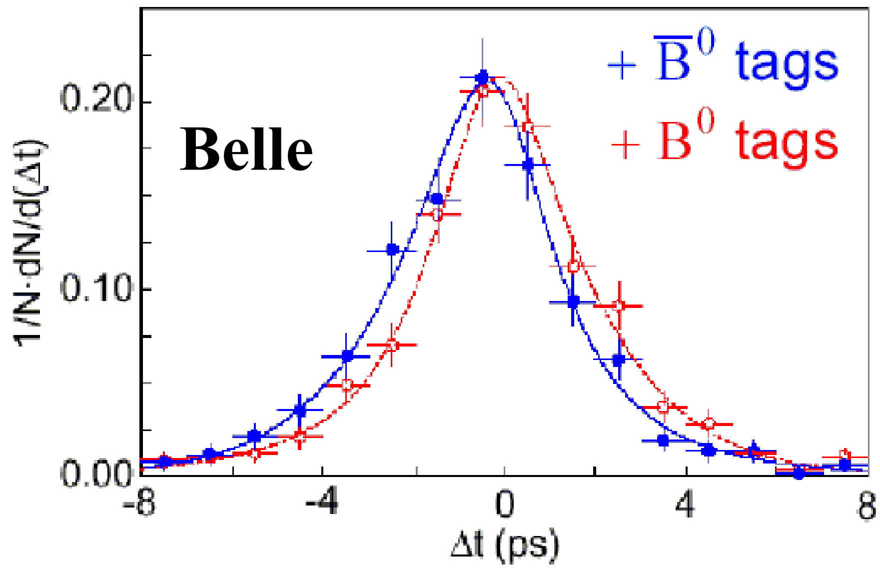


asymmetric
SLAC, KEK

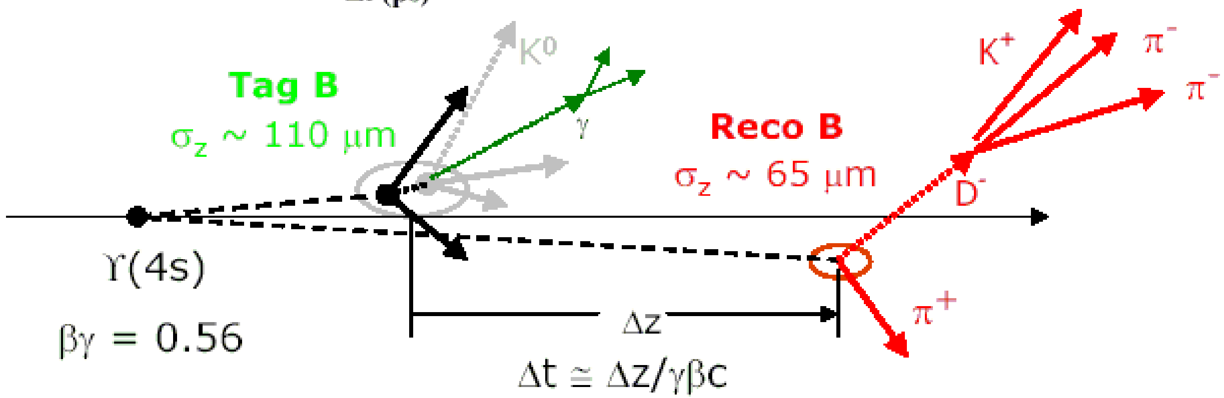
Recent Results on Sin2β

Fit distributions to: $e^{-\Gamma|t|}\left[\left\{1 \mp \sin 2\phi_1 \sin \Delta m \Delta t\right\}\right]$





$\sin 2\beta = 0.99 \pm 0.14_{(stat)} \pm 0.06_{(syst)}$



CP violation with K-mesons Vs. B-mesons

K_L mesons	B^0 mesons
observe CPV with "wrong" CP states (CP- $\rightarrow$CP+).	Look for differences in decay distributions (time)
$\approx$3/1000 decays violate CP use $K_L{}^0 \rightarrow 2\pi/3\pi$, $K_L{}^0 \rightarrow \pi l \nu$	CPV is large (10-15%) for certain final states. <u>But</u> the final states have small branching ratios: sin2β BRs $\approx 5 \times 10^{-4}$: $B^0 \rightarrow \psi K_s$, $B^0 \rightarrow \psi K_L$ sin2α BRs $\approx 5 \times 10^{-6}$: $B^0 \rightarrow \pi^+\pi^-$, $B^0 \rightarrow \pi^0\pi^0$
can make $\approx 10^7$ K_L/sec	can make $\approx 5B^0$/sec
explores limited region of the standard model.	CKM "triangles" provide a detailed check of standard model.
Very mature field (1964) maybe one more experiment	Field is just beginning. Experiments are planned for next 20 years (SLAC, KEK, CERN, Fermilab)

CPT Invariance

- Very general assumptions in quantum field theory:
  - spin-statistics relation: integer and half-integer spin particles obey Bose and Fermi statistics
  - Lorentz invariance

  lead to the CPT Theorem:
  - <u>all interactions</u> are invariant under the successive operation of C (=charge conjugation), P (=parity operation), and T (= time reversal)

- Masses, lifetimes, moments, etc. of particles and antiparticles must be identical

CPT Invariance

- Tests of the CPT Theorem

$(m_{W^+} - m_{W^-})\ /\ m_{\text{average}}$	-0.002 ± 0.007
$(m_{e^+} - m_{e^-})\ /\ m_{\text{average}}$	$<8 \times 10^{-9}$, CL = 90%
$\lvert q_{e^+} + q_{e^-} \rvert / e$	$<4 \times 10^{-8}$
$(g_{e^+} - g_{e^-})\ /\ g_{\text{average}}$	(-0.5+-2.1) $\times 10^{-12}$
$(\tau_{\mu^+} - \tau_{\mu^-})\ /\ \tau_{\text{average}}$	$(2 \pm 8) \times 10^{-5}$
$(g_{\mu^+} - g_{\mu^-})\ /\ g_{\text{average}}$	$(-0.11 \pm 0.12) \times 10^{-8}$
$(m_{\tau^+} - m_{\tau^-})/m_{\text{average}}$	$<2.8 \times 10^{-4}$, CL = 90%
$2\ (m_t - m_{\bar{t}})\ /\ (m_t + m_{\bar{t}})$	0.022 ± 0.022
$(m_{\pi^+} - m_{\pi^-})\ /\ m_{\text{average}}$	(2+-5) $\times 10^{-4}$
$(\tau_{\pi^+} - \tau_{\pi^-})\ /\ \tau_{\text{average}}$	(6+-7) $\times 10^{-4}$
$(m_{K^+} - m_{K^-})\ /\ m_{\text{average}}$	$(-0.6 \pm 1.8) \times 10^{-4}$
$(\tau_{K^+} - \tau_{K^-})\ /\ \tau_{\text{average}}$	$(0.10 \pm 0.09)\%$ (S = 1.2)
$K^{\pm} \rightarrow \mu^{\pm} \nu_{\mu}$ rate difference/average	$(-0.5 \pm 0.4)\%$
$K^{\pm} \rightarrow \pi^{\pm} \pi^{0}$ rate difference/average [1]	$(0.8 \pm 1.2)\%$

CPT Invariance (cont.)

phase difference $\phi_{00} - \phi_{+-}$		$(0.2 \pm 0.4)^{\circ}$
$\mathrm{Re}(\frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00}) - \frac{A_L}{2}$		$(-3 \pm 35) \times 10^{-6}$
$A_{CPT}(D^0 \to K^- \pi^+)$		0.008 ± 0.008
$\lvert m_p - m_{\overline{p}} \rvert / m_p$	[k]	$<2 \times 10^{-9}$, CL = 90%
$(\lvert \frac{q_{\overline{p}}}{m_{\overline{p}}} \rvert - \frac{q_p}{m_p}) / \frac{q_p}{m_p}$		$(-9 \pm 9) \times 10^{-11}$
$\lvert q_p + q_{\overline{p}} \rvert / e$	[k]	$<2 \times 10^{-9}$, CL = 90%
$(\mu_p + \mu_{\overline{p}}) / \mu_p$		$(-0.1 \pm 2.1) \times 10^{-3}$
$(m_n - m_{\overline{n}}) / m_n$		$(9 \pm 6) \times 10^{-5}$
$(m_\Lambda - m_{\overline{\Lambda}}) / m_\Lambda$		(-0.1+-1.1) $\times 10^{-5}$ (S = 1.6)
$(\tau_\Lambda - \tau_{\overline{\Lambda}}) / \tau_\Lambda$		-0.001 ± 0.009
$(\tau_{\Sigma^+} - \tau_{\overline{\Sigma}^-}) / \tau_{\Sigma^+}$		$(-0.6 \pm 1.2) \times 10^{-3}$
$(\mu_{\Sigma^+} + \mu_{\overline{\Sigma}^-}) / \mu_{\Sigma^+}$		0.014 ± 0.015
$(m_{\Xi^-} - m_{\overline{\Xi}^+}) / m_{\Xi^-}$		$(-3 \pm 9) \times 10^{-5}$
$(\tau_{\Xi^-} - \tau_{\overline{\Xi}^+}) / \tau_{\Xi^-}$		-0.01 ± 0.07
$(\mu_{\Xi^-} + \mu_{\overline{\Xi}^+}) / \lvert \mu_{\Xi^-} \rvert$		$+0.01 \pm 0.05$
$(m_{\Omega^-} - m_{\overline{\Omega}^+}) / m_{\Omega^-}$		$(-1 \pm 8) \times 10^{-5}$
$(\tau_{\Omega^-} - \tau_{\overline{\Omega}^+}) / \tau_{\Omega^-}$		0.00 ± 0.05