Weak Interactions

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- Observation of $W^{\pm}$ and Z^0 bosons in $p\bar{p}$ collisions
- Z^0 production at e^+e^- colliders
  - (continued next page)

Weak Interactions

- (continued)
- Weak decays of quarks. The GIM model and the CKM matrix
- Neutral K mesons
- CP violation in the neutral kaon system
- Cosmological CP violation
- D^0-$\overline{D}^0$ and B^0-$\overline{B}^0$ mixing

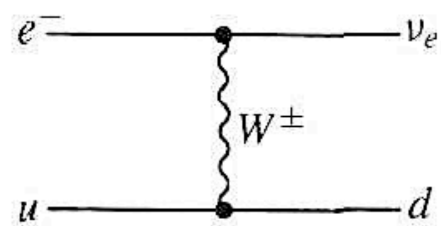
- Reference: Donald H. Perkins, Introduction to High Energy Physics, Fourth Edition

Classification

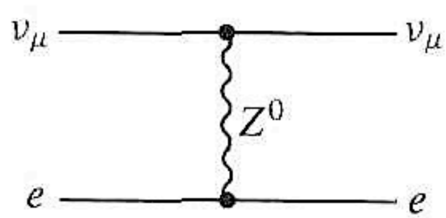
- Weak interactions are mediated by the "intermediate bosons" $W^{\pm}$ and Z^0
- Just as the EM force between two current carrying wires depends on the EM current, the weak interaction is between two weak currents, describing the flow of conserved weak charge, g

$$j \propto \psi^* \psi$$

- Two types of interactions:
  - CC (charged current)



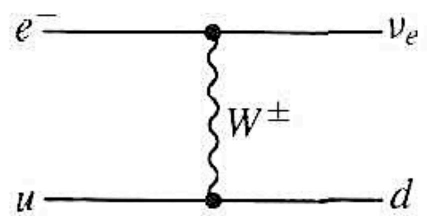
  - NC (neutral current)



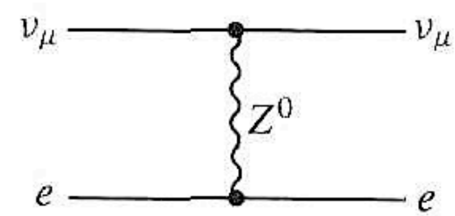
Classification

- Weak interactions occur between all types of leptons and quarks, but are often hidden by the stronger EM and strong interactions.

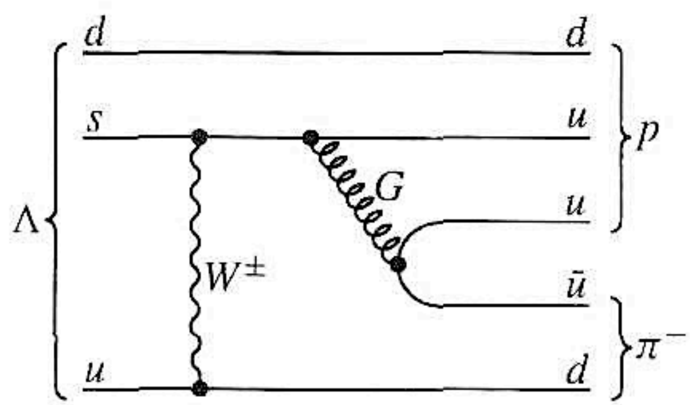
- Semi-leptonic



Leptonic



- Non-leptonic



Lepton universality

- Unit of weak charge
  - all the leptons carry the same weak charge and therefore couple to the $W^{\pm}$ with the same strength
  - The quarks DO NOT carry the same unit of weak charge

- Muon decay

$$\Gamma(\mu \to e \nu_e \bar{\nu}_\mu) = \frac{1}{\tau} \propto G^2 m_\mu^5$$
$$= \frac{G^2 m_\mu^5}{192\pi^3}$$

- experimental: $\tau_\mu = 2.197 \times 10^{-6}$ sec

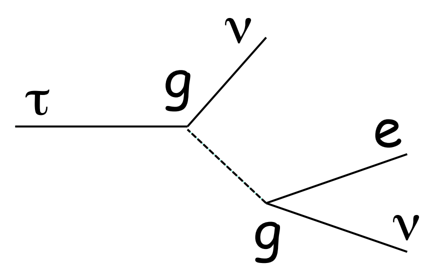
Lepton universality

- Tau decay

$$\Gamma(\tau \to e\nu_e\bar{\nu}_\tau) = B(\tau \to e\,\nu\,\nu)\,\frac{1}{\tau_\tau} \propto G^2 m_\tau^5$$

$$= \frac{G^2 m_\tau^5}{192\pi^3}$$

- $B(\tau \to e\,\nu\,\nu) = 17.80 \pm 0.06\%$
- Test universality: since $\Gamma \sim G^2 \sim g^4$



$$g_\tau^4 \propto B(\tau \to e\,\nu\,\nu) / (m_\tau^5\,\tau_\tau)$$

$$\left(\frac{g_\tau}{g_\mu}\right)^4 = B(\tau \to e\nu_e\bar{\nu}_\tau)\left(\frac{m_\mu}{m_\tau}\right)^5\left(\frac{\tau_\mu}{\tau_\tau}\right)$$

Lepton universality

- Test universality:

$$\left(\frac{g_\tau}{g_\mu}\right)^4 = B(\tau \to e\nu_e\bar{\nu}_\tau)\left(\frac{m_\mu}{m_\tau}\right)^5\left(\frac{\tau_\mu}{\tau_\tau}\right)$$

With $\tau_\mu = 2.197 \times 10^{-6}$ s, $\tau_\tau = (291.0 \pm 1.5) \times 10^{-15}$ s, $m_\mu = 105.658$ MeV
$m_\tau = 1777.0$ MeV and $B(\tau \to e\,\nu\,\nu) = 17.80 \pm 0.06\%$

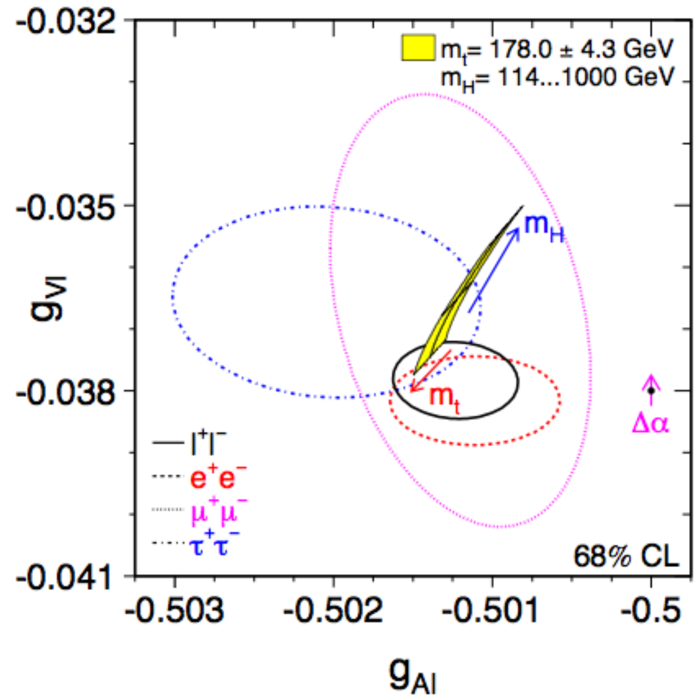
$$\frac{g_\tau}{g_\mu} = 0.999 \pm 0.003$$

$$\frac{g_\mu}{g_e} = 1.001 \pm 0.004$$

Lepton universality

- Lepton universality also holds for the Z couplings:

$$Z^0 \to e^+e^- : \mu^+\mu^- : \tau^+\tau^- = 1 : 1.000 \pm 0.004 : 0.999 \pm 0.005$$



- From the muon lifetime we can compute the Fermi constant, G:

$$G/(\hbar c)^3 = 1.1664 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

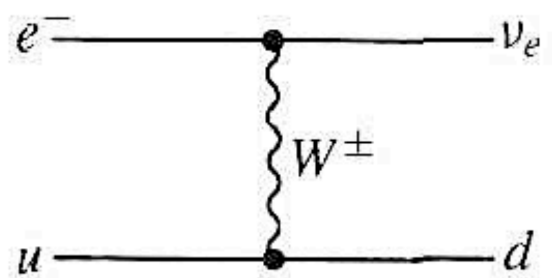
Nuclear β-decay: Fermi theory

- The decay of the neutron provides the prototype weak interaction

$$n \rightarrow p + e^- + \bar{\nu}_e$$

- The quark level interaction is

$$d \rightarrow u + e^- + \bar{\nu}_e$$

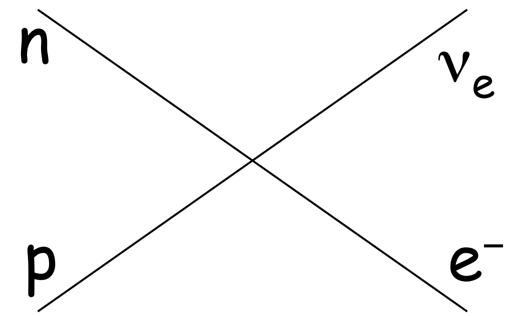


Nuclear β-decay: Fermi theory

- Since $q^2 \ll M_W$, interaction is effectively pointlike

$$W = \frac{2\pi}{\hbar} G^2 |M|^2 \frac{dN}{dE_0}$$

(Fermi's Golden Rule)

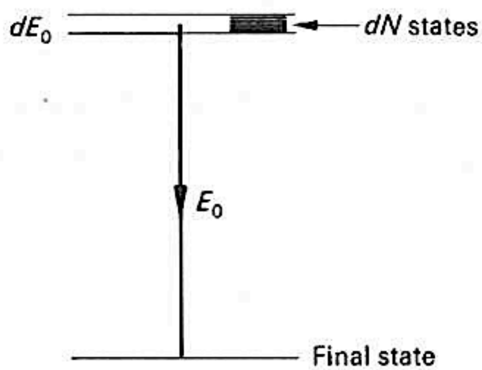


- $|M|^2 \approx 1$ if J(leptons) = 0 (Fermi transitions)
- $|M|^2 \approx 3$ if J(leptons) = 1 (Gamow-Teller transitions)

- dN/dE is the density of states

Nuclear β-decay: Fermi theory

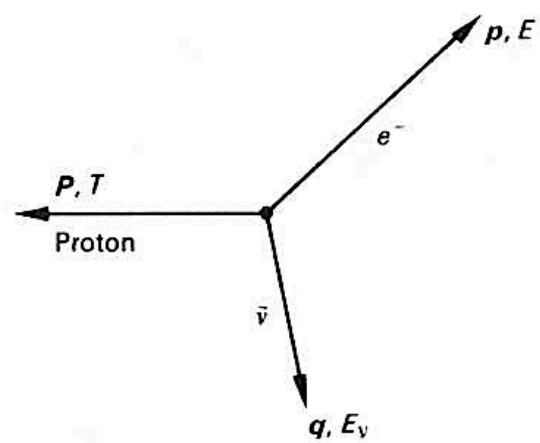
- <u>Density of states</u>
  - number of ways of sharing the available energy in the range $E_0 \rightarrow E_0 + dE_0$



  - Consider the neutron decay

$$\mathbf{P} + \mathbf{q} + \mathbf{p} = 0$$

$$T + E_\nu + E = E_0$$



Nuclear β-decay: Fermi theory

- $m_\nu \approx 0$, so $E_\nu \approx qc$
- $E_0 \approx 1$ MeV (very approx.), so $Pc \approx 1$ MeV
- $T = P^2c^2/(2Mc^2) = P^2/(2M) \approx 10^{-3}$ MeV, which is negligible
- energy, E_0, is shared entirely between electron and neutrino:

  $qc = E_0 - E$
- Number of states available to an electron, in volume V, in momentum range (p,p+dp) inside solid angle $d\Omega$:

$$\frac{V\,d\Omega}{(2\pi)^3\hbar^3}p^2dp$$

- Electron phase space factor is $\frac{4\pi p^2dp}{(2\pi)^3\hbar^3}$ and neutrino $\frac{4\pi q^2dq}{(2\pi)^3\hbar^3}$

Nuclear β-decay: Fermi theory

- The proton momentum P is fixed, once the electron and neutrino is specified (there is no more freedom):

$$\mathbf{P} = -(\mathbf{p} + \mathbf{q})$$

- so the number of final states is:

$$dN = \frac{(4\pi)^2}{(2\pi)^6 \hbar^6} p^2 q^2 dp dq$$

- for a given p and E of the electron, the neutrino momentum is fixed $q = (E_0 - E)/c$ in the range dq = dE_0/c

- Hence:

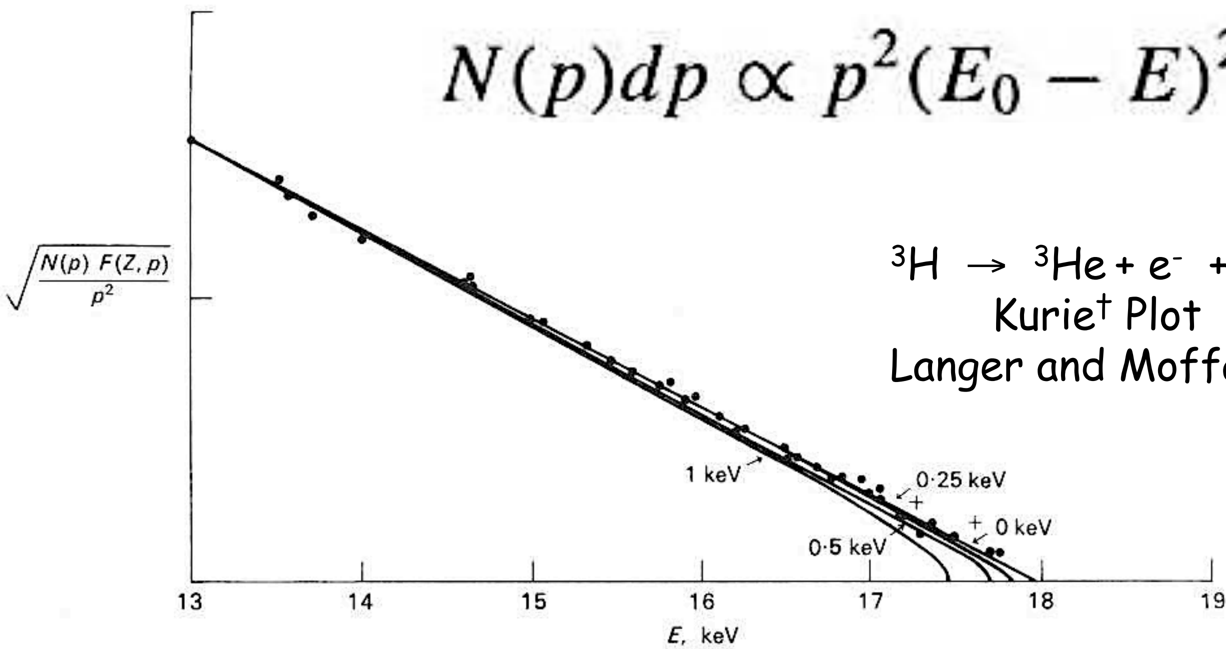
$$\frac{dN}{dE_0} = \frac{1}{4\pi^4 \hbar^6 c^3} p^2 (E_0 - E)^2 dp$$

Nuclear β-decay: Fermi theory

$$\frac{dN}{dE_0} = \frac{1}{4\pi^4\hbar^6 c^3} p^2 (E_0 - E)^2 dp$$

- This expression gives the electron spectrum, since the matrix element $|M|^2$ is a constant when integrated over angle

$$N(p)dp \propto p^2 (E_0 - E)^2 dp$$



$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$
Kurie† Plot
Langer and Moffatt (1952)

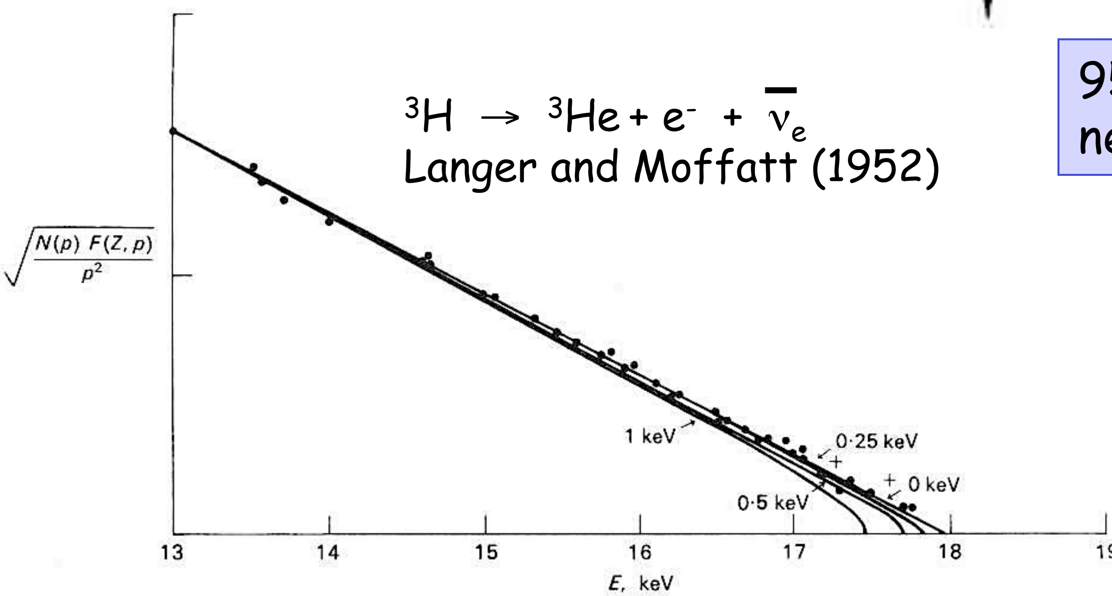
† Franz N. D. Kurie

Nuclear β-decay: Fermi theory

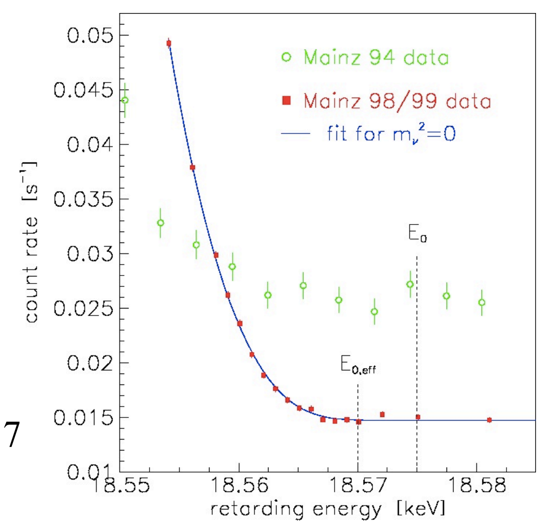
- For a non-zero neutrino mass, the expression has a simple change

$$N(p)dp \propto p^2(E_0 - E)^2 dp$$

$$N(p)dp \propto p^2(E_0 - E)^2 \sqrt{1 - \left(\frac{m_\nu c^2}{E_0 - E}\right)^2}\, dp$$



95% CL Limit on electron neutrino mass (< 2 eV)



Inverse β-decay: neutrino interactions

- The physical existence of the neutrino required a demonstration of its interaction

$$\bar{\nu}_e + p \to n + e^+$$

$$\sigma(\bar{\nu}_e + p \to n + e^+) = \frac{G^2}{\pi} |M|^2 \frac{p^2}{v_i v_f}$$

$$\sigma(\bar{\nu}_e + p \to e^+ + n) \simeq 10^{-43} E^2 \text{ cm}^2$$ (E in MeV)

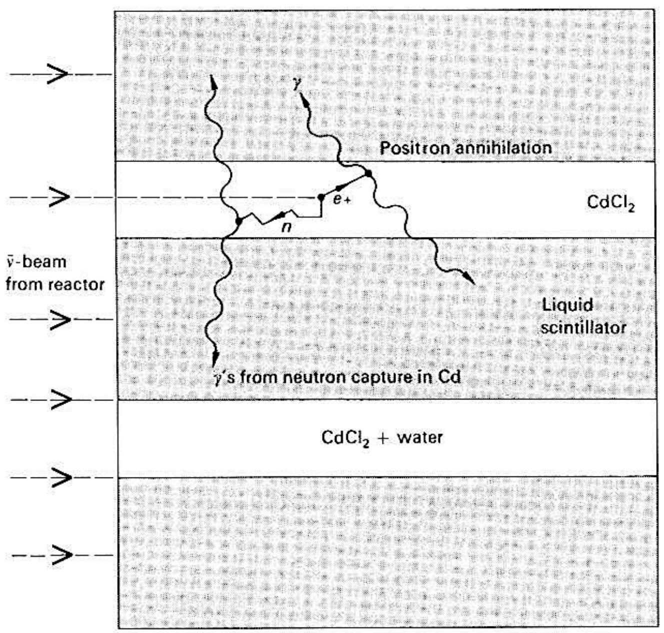
- mean free path of a 1 MeV antineutrino is 50 light years of water !

Inverse β-decay: neutrino interactions

- Neutrinos from 1000 MW reactor core
- Flux ~ 10^{13} $cm^{-2}s^{-1}$
- Target:
  - $CdCl_2$ + water
- Few events observed per hour

Reines and Cowan (1956)

$$\bar{\nu}_e + p \to n + e^+$$



- Events identified by sequence of pulses in scintillator separated by μseconds
- Prompt pulse (10^{-9}s)
  - positron annhilates to gamma rays
- Delayed pulse (10^{-6}s)
  - neutrons moderate and are radiatively captured by cadmium

Parity nonconservation in β-decay

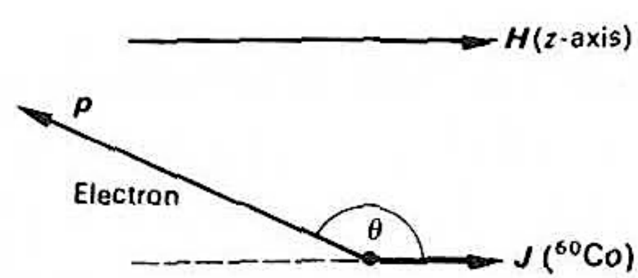
- 1956 - Lee and Yang conclude weak interactions violate parity conservation (invariance under spatial inversion)

  $K^+ \to 2\pi$ (final state has <u>even</u> parity)

  $K^+ \to 3\pi$ (final state has <u>odd</u> parity)

- 1957 - Wu et al test this
  - ^{60}Co at 0.01K inside solenoid
  - high proportion of spin 5 cobalt nuclei are aligned with field

$$^{60}Co \to {}^{60}Ni + e^- + \bar{\nu}_e$$



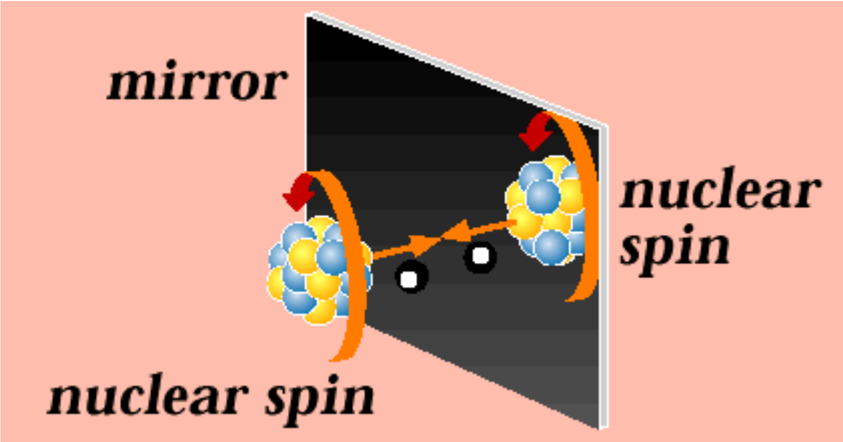
$$I(\theta) = 1 + \alpha \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E}$$
$$= 1 + \alpha \frac{v}{c} \cos\theta$$

experiment showed α = -1

Parity nonconservation in β-decay

$$I(\theta) = 1 + \alpha \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E}$$

- The fore-aft asymmetry implies parity is violated
  - spin (σ) does not change sign under reflection
  - momentum (p) changes sign under reflection
  - therefore $\boldsymbol{\sigma} \cdot \mathbf{p}$ changes sign under reflection
  - If parity were conserved, the experiment would find $\alpha = 0$

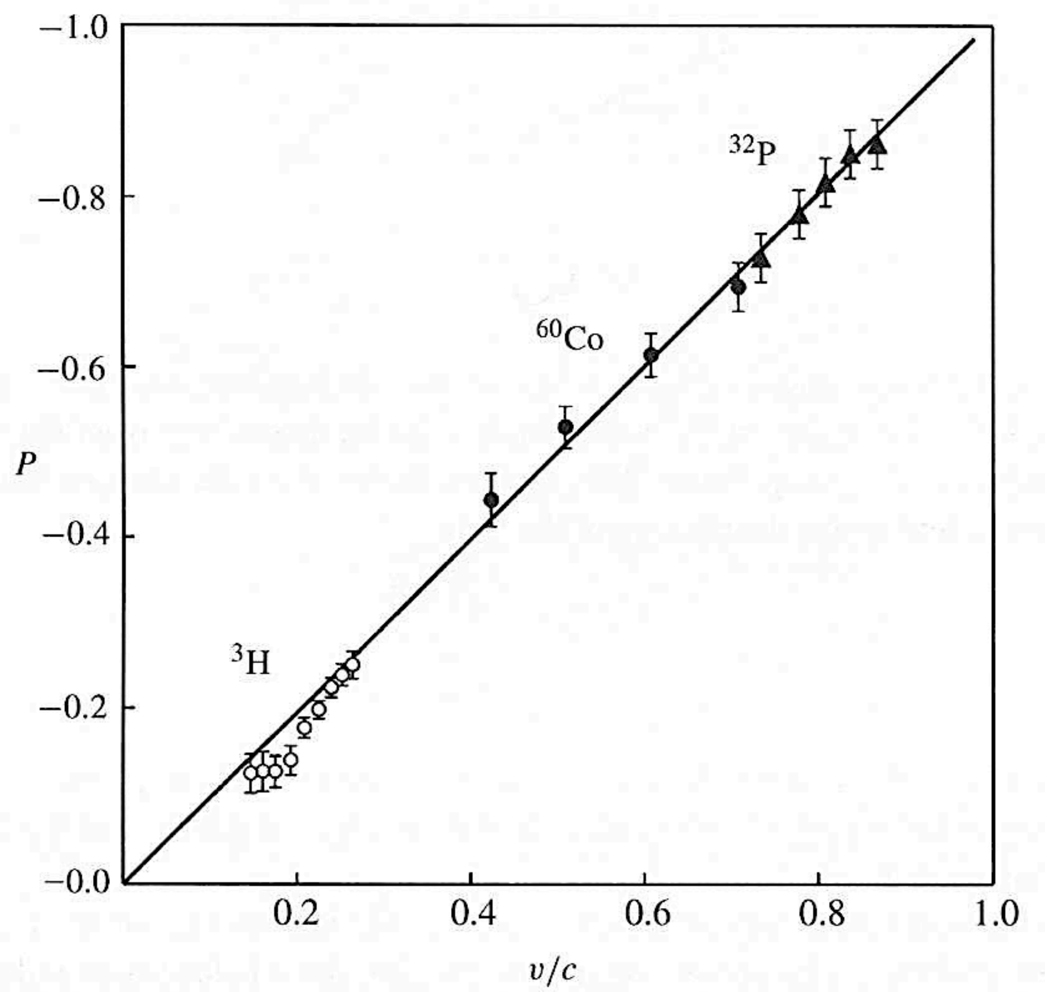


$$\alpha = \begin{cases} +1 & \text{for } e^+\text{, thus } P = +v/c \\ -1 & \text{for } e^-\text{, thus } P = -v/c \end{cases}$$

where $P = \alpha\, v/c$

Parity nonconservation in β-decay

$P = \alpha\, v/c$

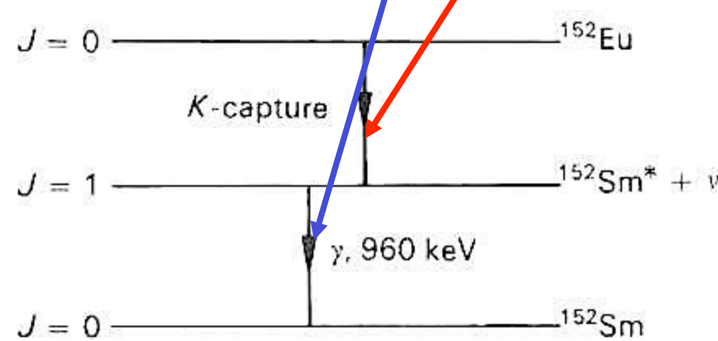


Helicity of the neutrino

- The elegant experiment of Goldhaber, Grodzins, and Sunyar(1958) established the helicity of the neutrino

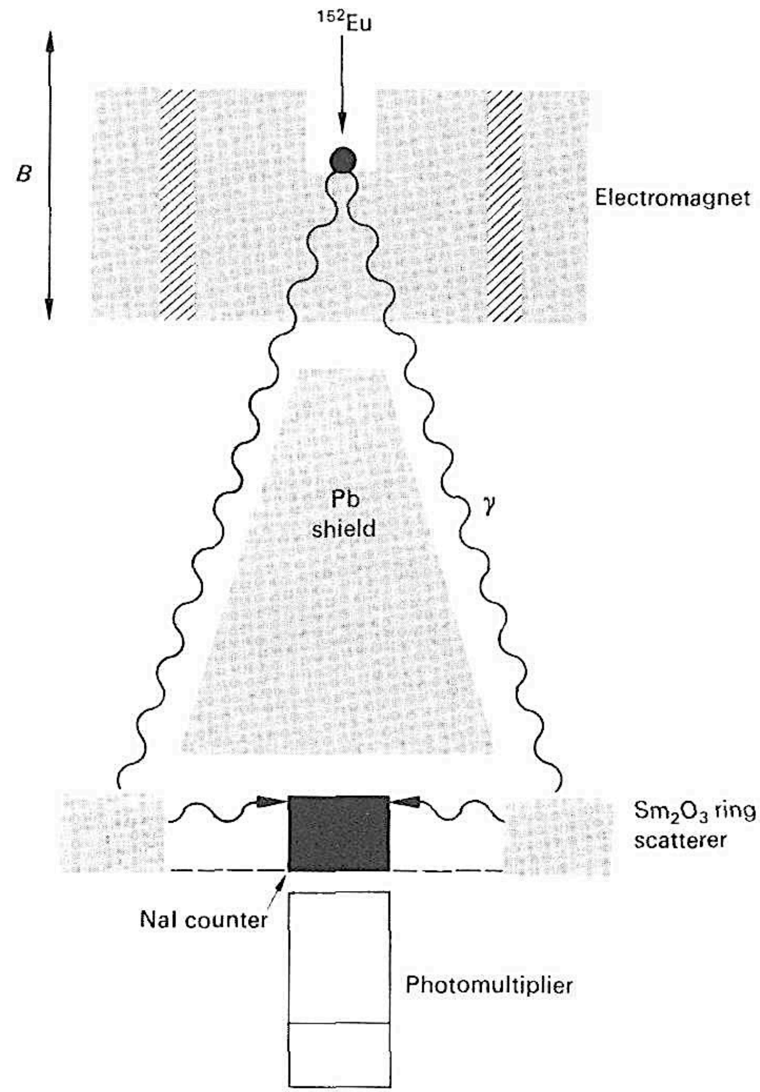
- The neutrino is emitted along with a positron in β^+ decay
- The antineutrino is emitted along with an electron in β^- decay

- Reaction used is K-capture in ^{152}Eu to an excited state of ^{152}Sm
  - the excited state of ^{152}Sm decays by emitting gamma ray
  - resonance scattering of gamma-rays from ground state of Sm reveals the polarization of the event, and the helicity of the neutrino



$$\gamma + {}^{152}\mathrm{Sm} \rightarrow {}^{152}\mathrm{Sm}^* \rightarrow \gamma + {}^{152}\mathrm{Sm}$$

Helicity of the neutrino

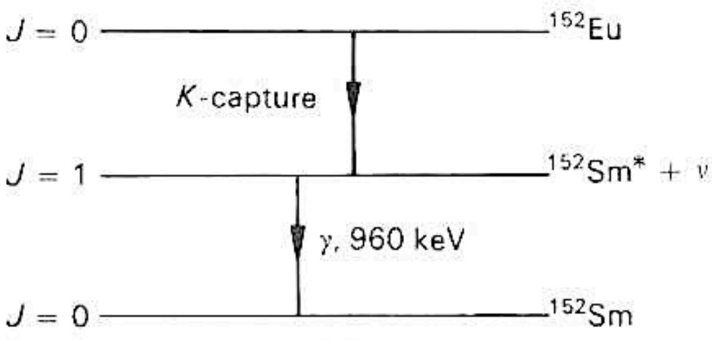


Electromagnet permits control on absorption of γ rays, determining polarization of γ rays, and consequently, neutrinos (d)

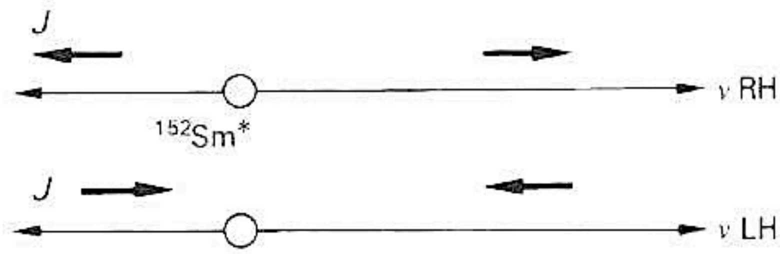
Resonance scattering in ring selects γ rays emitted in direction of recoiling $^{152}Sm^*$

(to allow for nuclear recoil)

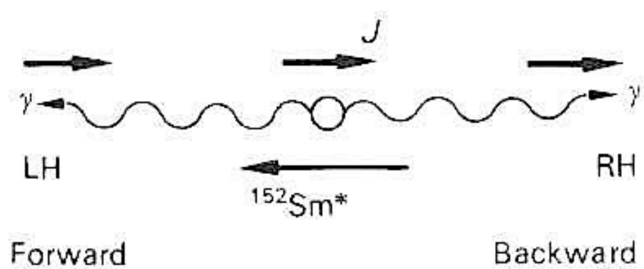
Helicity of the neutrino



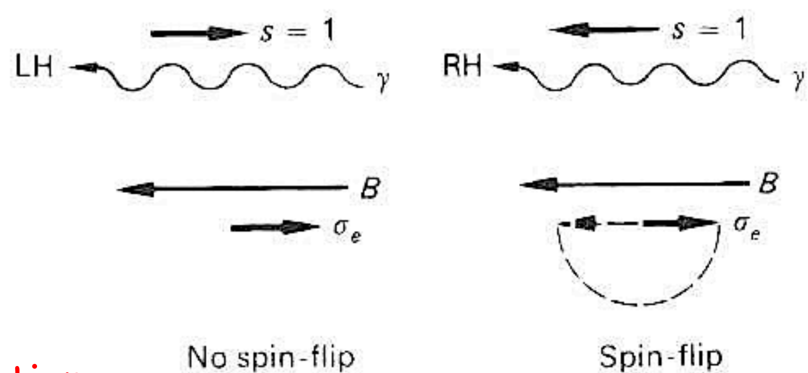
(a)



Helicity of ^{152}Sm* is the same as that of the neutrino



Gammas emitted forward have same polarization as the neutrino (and the highest energy)



Gammas with spin aligned with B field in electromagnet are more absorbed (since electrons are aligned against B)

Lepton Polarization in β-decay

With determination that the neutrino is left-handed, and given the velocity dependence of the electron helicity, the polarization of the leptons in β-decay are:

	Positron	Electron	Neutrino	Anti-neutrino
Particle polarization	+ v/c	- v/c	-1	+1

Projection Operators

For massless particles, the operators

$$P_{R,L} = \frac{1}{2}\left(1 \pm \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E}\right)$$

project out states of particular helicity from two-component spinors:

$\psi = \psi_R + \psi_L.$

So

$$P_R\psi = \frac{1}{2}\left(1 + \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E}\right)(\psi_L + \psi_R) = \psi_R$$

$$P_L\psi = \frac{1}{2}\left(1 - \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E}\right)(\psi_L + \psi_R) = \psi_L$$

Projection Operators

For massive particles, the operators shown here, project out $P = \pm v/c$

$$P_{R,L} = {}^1/_2\,(1 \pm \gamma_5)\,, \qquad \gamma_5 = \iota\gamma_1\gamma_2\gamma_3\gamma_4$$

where $\gamma_k = \beta\,\alpha_k = \begin{bmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{bmatrix}$, k = 1,2,3

and

$$\gamma_4 = \beta \qquad \gamma_5 = \iota\,\gamma_1\gamma_2\gamma_3\gamma_4$$

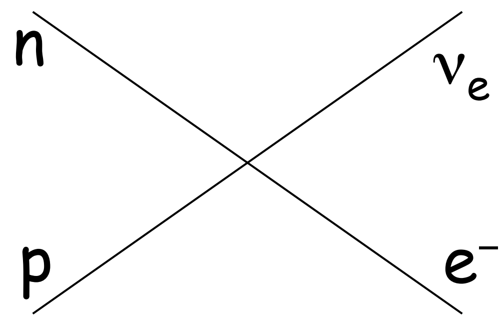
$$\beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -\iota \\ \iota & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Weak interaction theory

Fermi's original theory of the weak interaction

uses EM interaction as model

$$M \propto \frac{e^2}{q^2} J_{\text{baryon}} J_{\text{lepton}}$$



$$J_{\text{lepton}} = \psi_e^* \gamma_4 \gamma_\mu \psi_e \equiv \bar{\psi}_e \gamma_\mu \psi_e$$

$$J_{\text{baryon}} = \bar{\psi}_p \gamma_\mu \psi_p$$

$$\boxed{\bar{\psi}_e = \psi_e^* \gamma_4}$$

$$M = G J_{\text{baryon}}^{\text{weak}} J_{\text{lepton}}^{\text{weak}} = G(\bar{\psi}_p O \psi_n)(\bar{\psi}_e O \psi_\nu)$$

Fermi assumed O is a vector operator, as it is for the electromagnetic interaction

Weak interaction theory

The discovery of parity violation in the weak interaction indicated O must be a combination of two types of interactions, of opposite parity

Relativistic invariance allows five types of operators

V	vector	γ_μ
A	axial vector	$\gamma_5\gamma_\mu$
S	scalar	1
P	pseudoscalar	γ_5
T	tensor	$\gamma_\mu\gamma_\nu$

The V-A interaction

V or A are favored by the observation of opposite helicities for the fermions and antifermions

General combination of V and A:

$$\tfrac{1}{2}(C_V + \gamma_5 C_A)$$

$$\gamma_5 = \iota\gamma_1\gamma_2\gamma_3\gamma_4$$

Left-handedness of the neutrino requires $C_A = -C_V$

A and V have opposite parity $\Rightarrow$ "maximal parity violation"

Conservation of weak currents

The weak charge is not conserved by the strong interaction (unlike the electromagnetic charge, which is unchanged by the strong interaction)

$C_A = -C_V$ for the leptonic weak interactions, but not for those involving hadrons at low energy

neutron β-decay	$C_A / C_V = -1.26$
Λ β-decay	$C_A / C_V = -0.72$
$\Sigma^- \to n\, e\, \nu$ β-decay	$C_A / C_V = +0.34$

Conserved vector current (CVC)
Partially conserved axial-vector current (PCAC)

Non-conservation of weak currents

The weak charge is not conserved by the strong interaction (unlike the electromagnetic charge, which is unchanged by the strong interaction)

The violation of weak charge conservation by low energy hadrons is understood to originate from the strong quark-quark interactions.

For example, the difference $|C_A / C_V| - 1$ in neutron decay was accounted for by virtual emission and re-absorption of pions by the nucleon (Goldberg-Treiman).

The weak boson and Fermi couplings

In modern theory, g is the coupling of a lepton to the W. Relating this to the Fermi constant:

$$\frac{g^2}{8M_W^2} \equiv \frac{G}{\sqrt{2}}$$

Then, having measured G, and making assumptions of g, a prediction of M_W can be obtained.

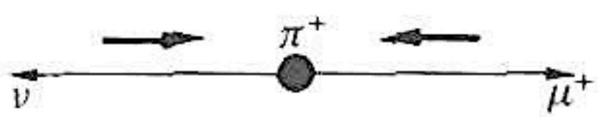
G has been slightly redefined (replaced by $G/\sqrt{2}$) from original Fermi theory, since original theory only had vector interaction, and modern theory has V-A

Pion and Muon decay

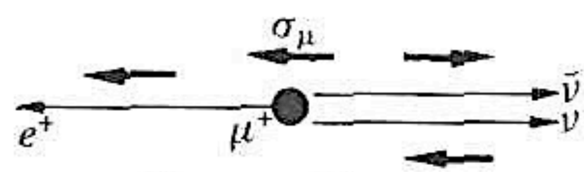
As Wu et al were observing lepton helicities in Co β-decay, others observed effects in decay of pions and muons

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$



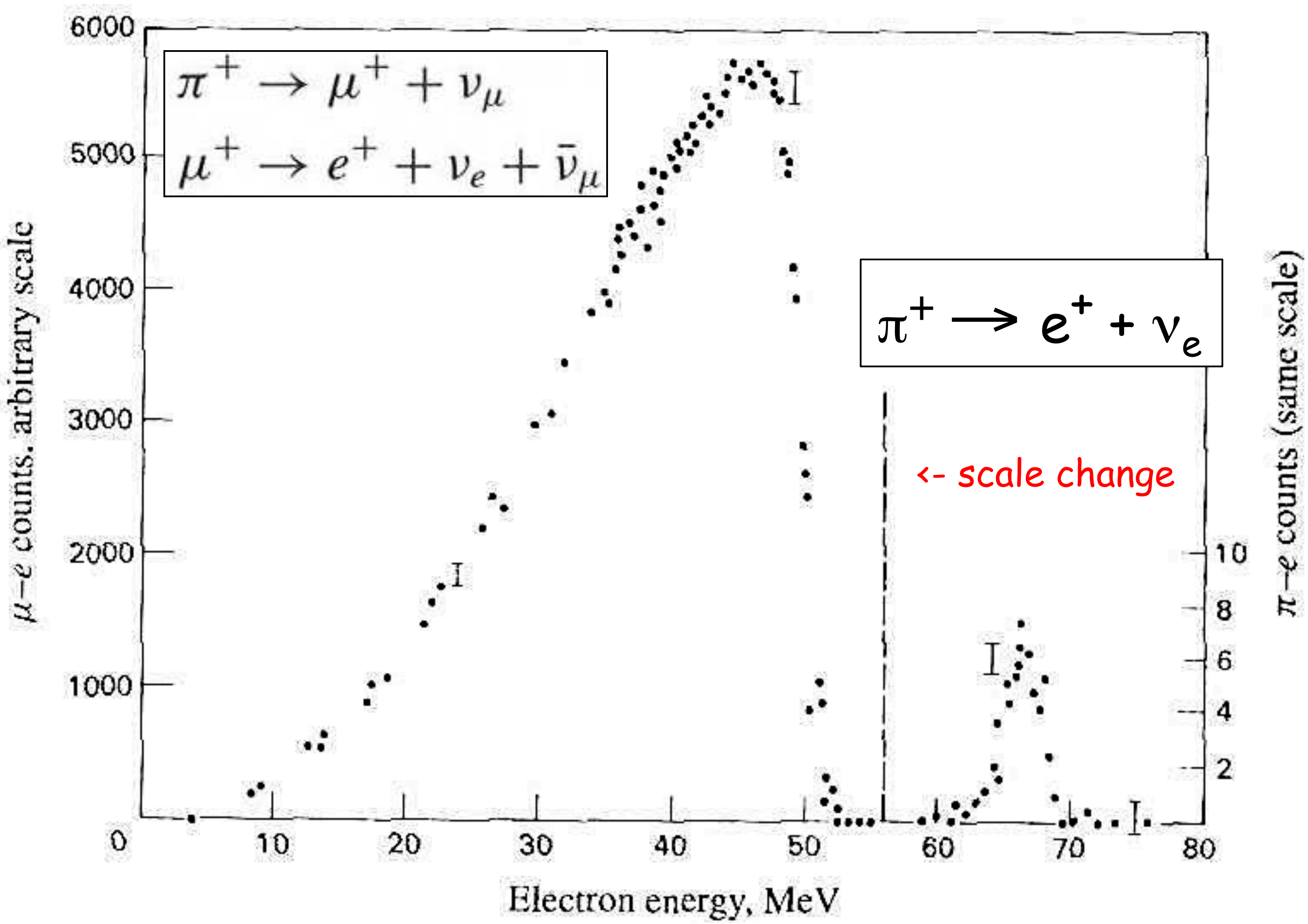
Pion rest frame



Muon rest frame

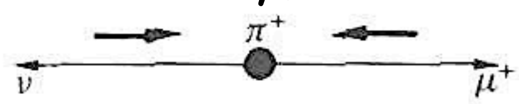
favored configuration

Pion and Muon decay



Pion and Muon decay

- Pions decay in flight $\pi^+ \rightarrow \mu^+ + \nu_\mu$
- Forward going muons, those with highest momentum and negative helicity, were selected



In V-A theory, π decay proceeds through A coupling

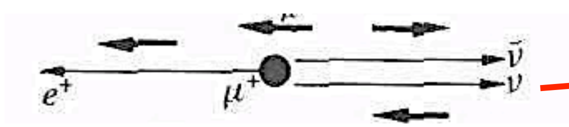
- Muons were stopped and angle of positron was measured

$$\frac{dN}{d\Omega} = 1 - \frac{\alpha}{3}\cos\theta$$

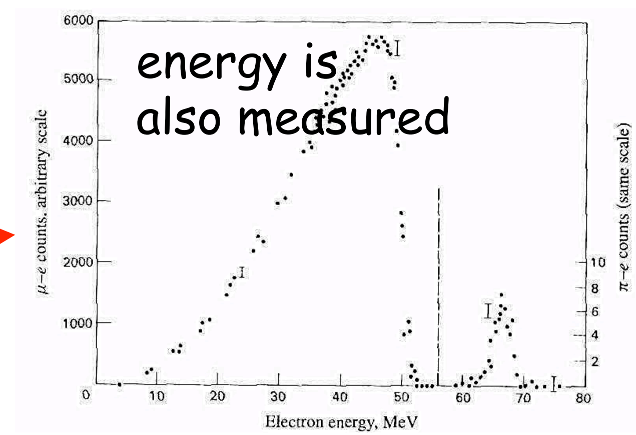
α = 1

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

This result is predicted by V-A



Energy spectrum suggests this configuration



Pion and Muon decay

- Pion decay has two modes:

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$$\pi^+ \rightarrow e^+ \nu_e$$

- V-A theory predicts the polarization of the charged antilepton

$$P = \frac{N_R - N_L}{N_R + N_L} = +\frac{v}{c}$$

- But charged antilepton must have same helicity as neutrino (-1), so probability is:

$$\frac{N_L}{N_R + N_L} = \frac{1}{2}\left(1 - \frac{v}{c}\right) \qquad 1 - \frac{v}{c} = \frac{2m^2}{m_\pi^2 + m^2}$$

Pion and Muon decay

- Phase space also affects the rate:

$$E_0 = m_\pi = p + \sqrt{p^2 + m^2}$$

- so the phase space factor is:

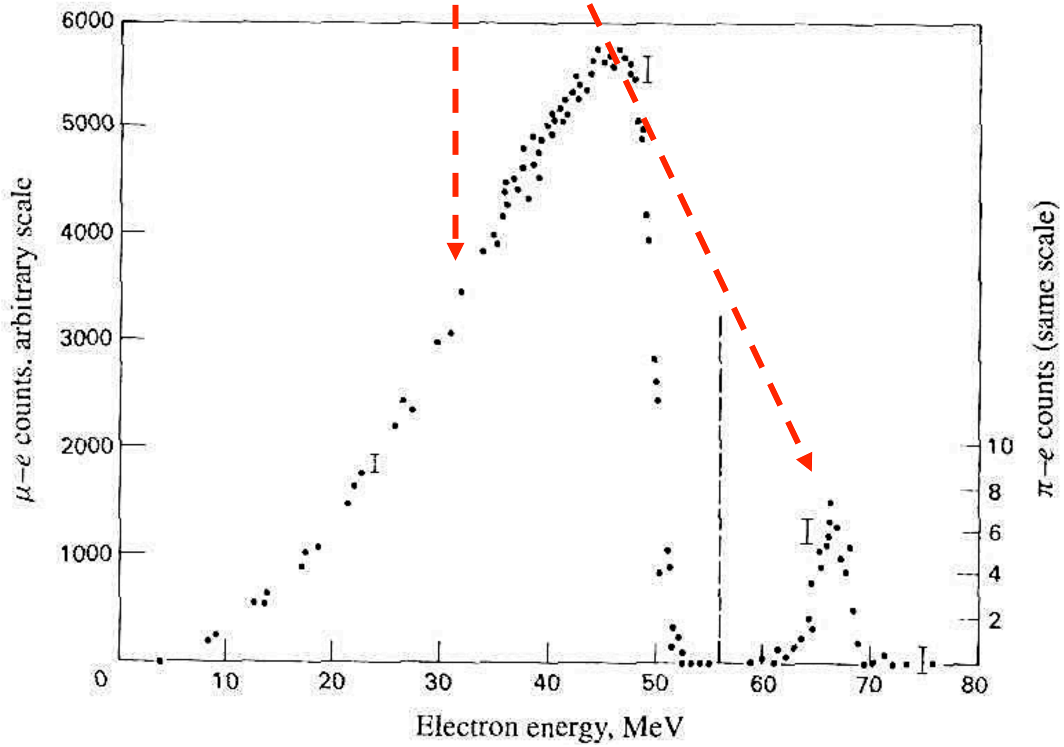
$$p^2 \frac{dp}{dE_0} = \frac{(m_\pi^2 + m^2)(m_\pi^2 - m^2)^2}{4m_\pi^4}$$

- combining this with the polarization:

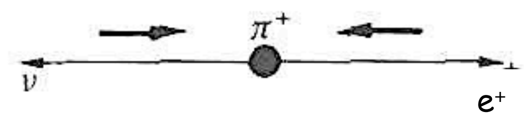
$$p^2 \frac{dp}{dE_0}\left(1 - \frac{v}{c}\right) = \frac{m^2}{2}\left(1 - \frac{m^2}{m_\pi^2}\right)^2$$

Pion and Muon decay

$$R = \frac{\pi \to e + \nu}{\pi \to \mu + \nu} = \frac{m_e^2}{m_\mu^2} \frac{1}{(1 - m_\mu^2/m_\pi^2)^2} = 1.28 \times 10^{-4}$$



$$\boxed{R_{\text{exp}} = 1.23 \times 10^{-4}}$$



Rate is suppressed by requirement that antilepton be left-handed

Pion and Muon decay

- Alternative interaction in pion decay to the A (axial vector) coupling is the P (pseudoscalar) coupling
- There are distinctively different predictions
  - The neutral lepton and charged antilepton would <u>favor</u> having the same helicity, resulting in the probability for the configuration:

$$\frac{N_L}{N_R + N_L} = \frac{1}{2}\left(1 + \frac{v}{c}\right)$$

  - The predicted branching ratio is then:

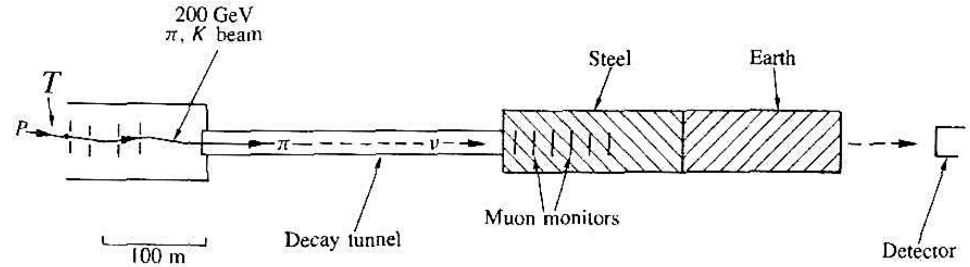
$$R = \frac{\pi \to e + \nu}{\pi \to \mu + \nu} = \frac{1}{(1 - m_\mu^2/m_\pi^2)^2} = 5.5$$

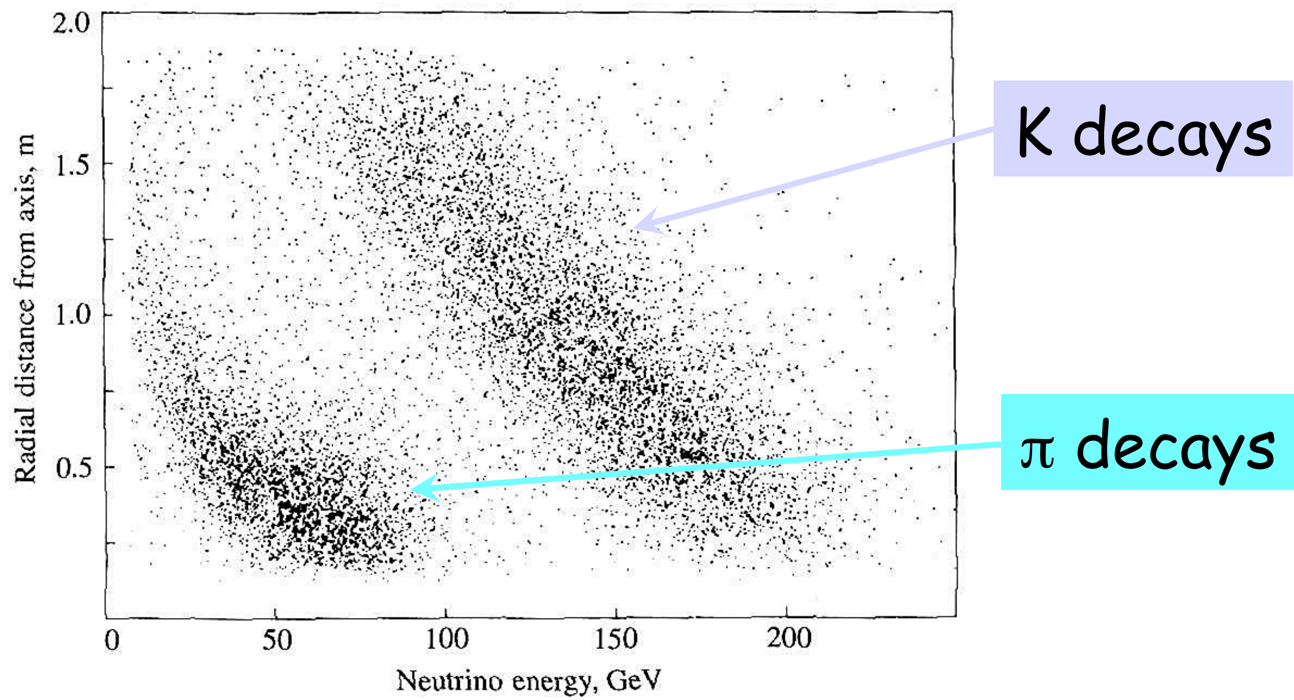
  - while experiment yields:

$$R_{\text{exp}} = 1.23 \times 10^{-4}$$

Neutral weak currents

- Neutrino beams and experiments





Neutral weak currents

- Accelerator neutrino beams gave evidence in 1962 for lepton flavors

$$\nu_\mu + N \to \mu^- + X$$

$$\nu_e + N \to e^- + X$$

- In 1973, neutrino interactions without a charge lepton were discovered: Neutral-current events

$$\nu_\mu + N \to \nu_\mu + X$$

$$\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X$$

$$\nu_\mu + e^- \to e^- + \nu_\mu$$

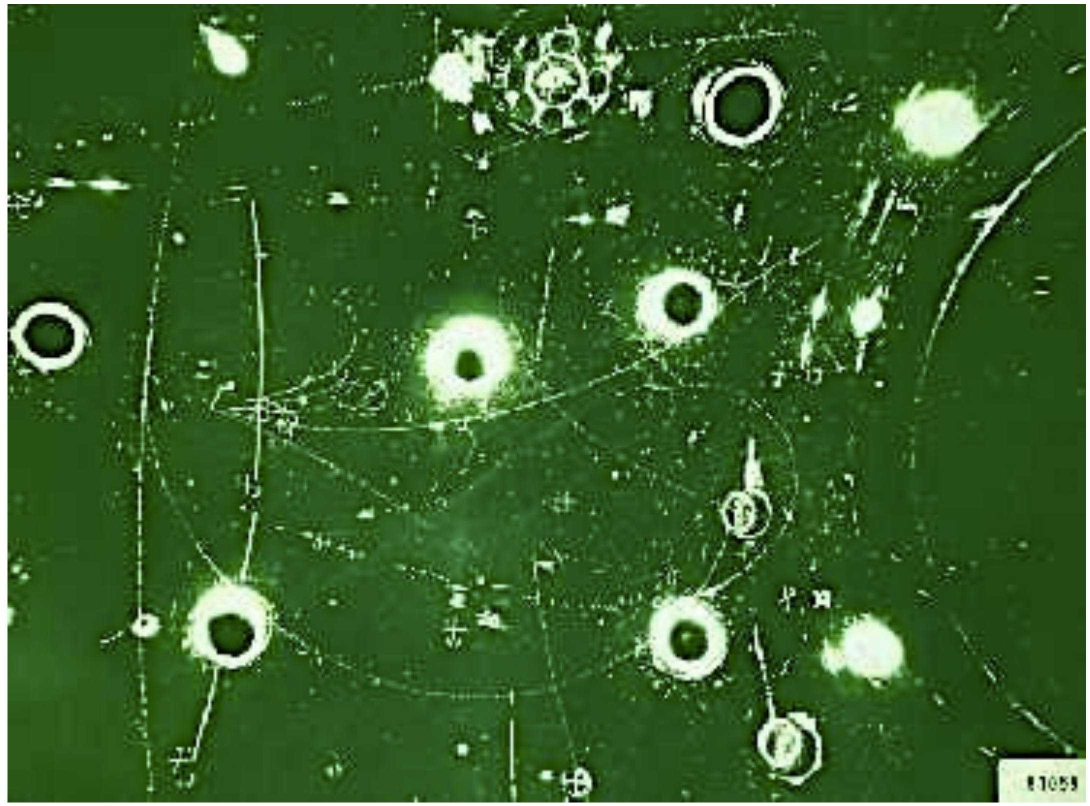
$$\bar{\nu}_\mu + e^- \to e^- + \bar{\nu}_\mu$$

Neutral weak currents

- First neutral current interaction seen

$$\bar{\nu}_\mu + e^- \rightarrow e^- + \bar{\nu}_\mu$$

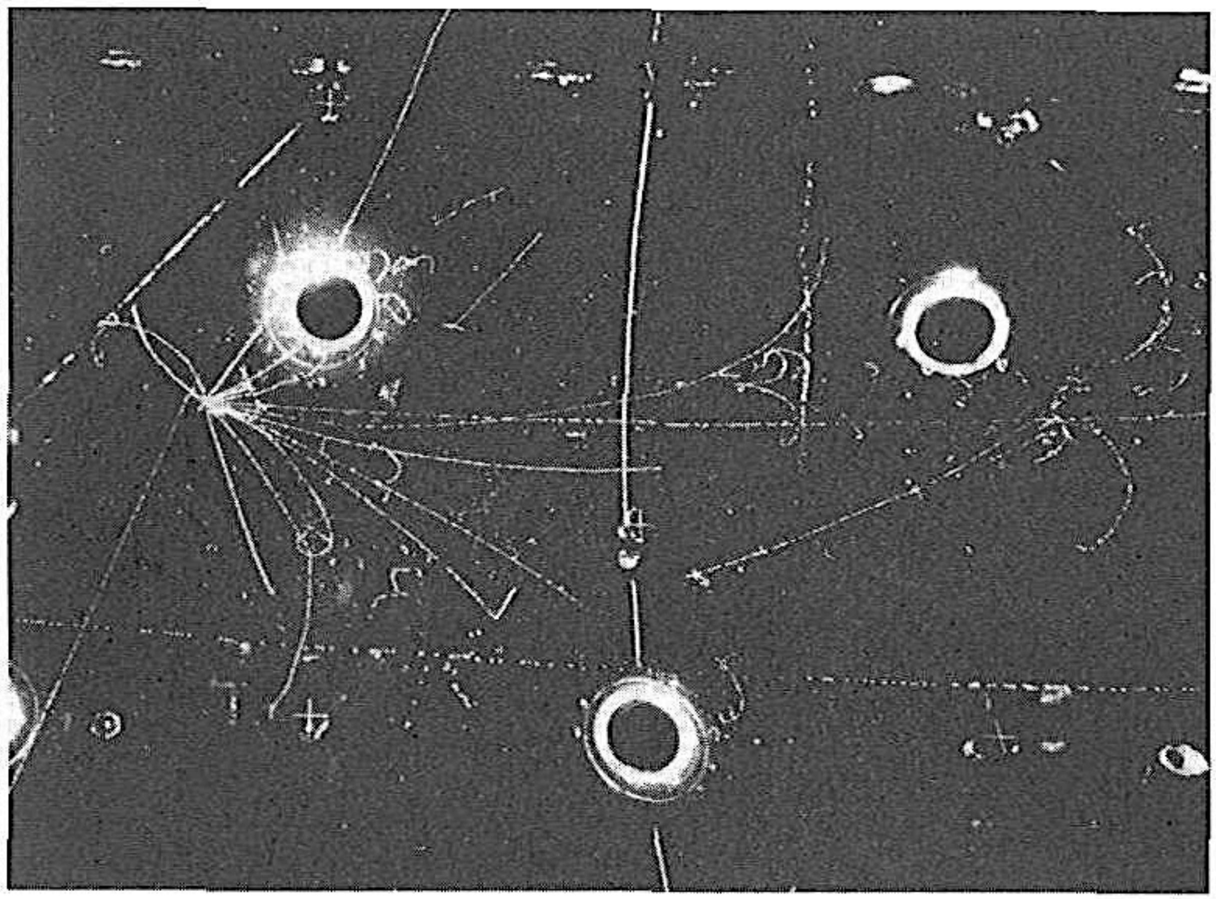
antineutrinos



Neutral weak currents

- Charge current interaction (Gargamelle bubble chamber - CERN)
- 15 tons Freon (CF_3Br)

neutrinos

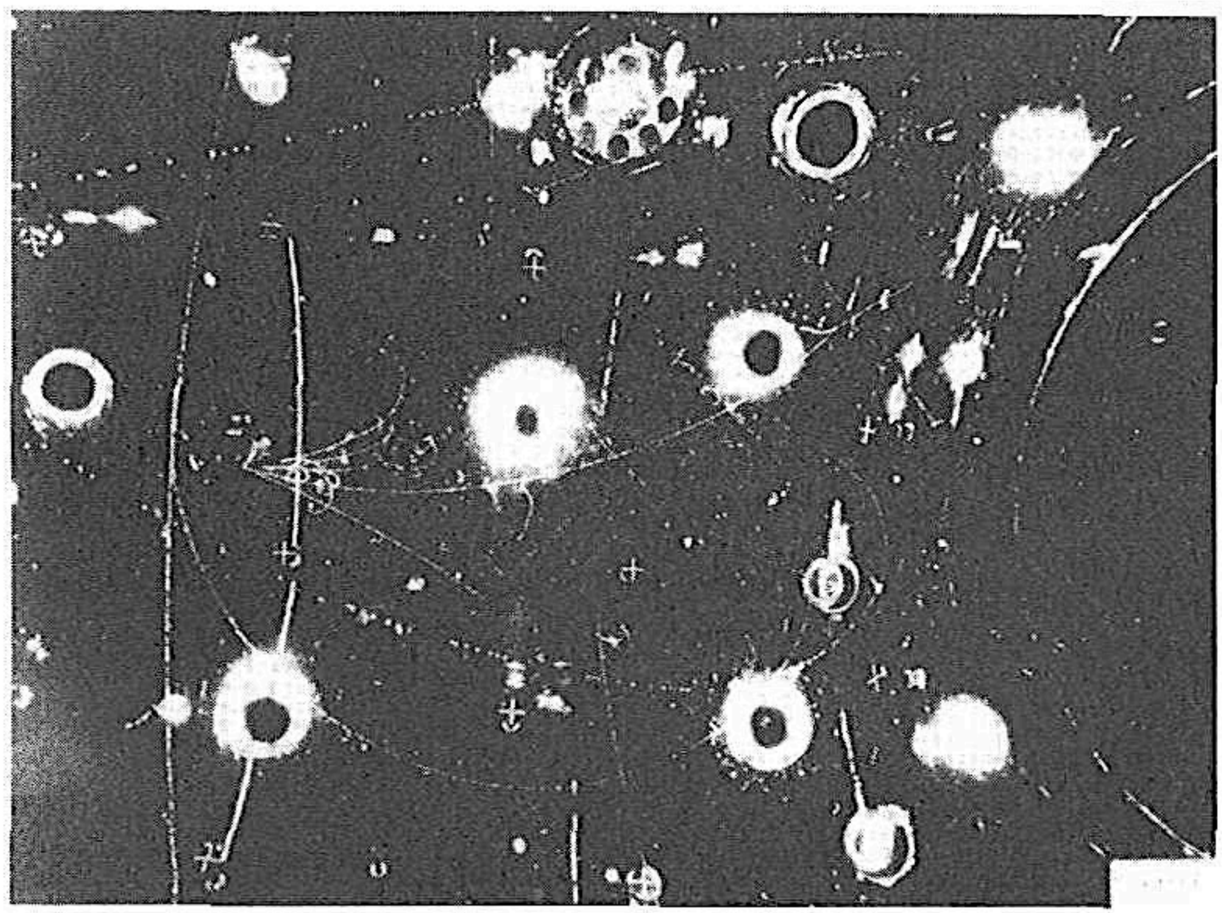


muon

Neutral weak currents

- Neutral current interaction

neutrinos



Neutral weak currents

- The discovery of neutral current interactions established the unified theory of weak and electromagnetic interactions
- Estimates of the masses of the $W^{\pm}$ and Z^0 were possible
- Once the masses were known, experiments could be planned to search for the intermediate bosons

Prediction of $W^{\pm}$ Boson

- From the standard Glashow-Salam-Weinberg model, bouyed by the observation of neutral currents, the mass of the W boson is predicted from the coupling constant, g, which can be assumed to be the same in electromagnetic and weak interactions (g=e):

$$e^2/M_W^2 \approx G \qquad \Longrightarrow \qquad M_W \approx \frac{e}{\sqrt{G}} \approx 90\,\mathrm{GeV} - 100\,\mathrm{GeV}$$

where $G \sim 1\times10^{-5}\,\mathrm{GeV}^{-2}$, $e^2 = 4\pi/137$.

- Later, the improved model based on the Higgs Mechanism:

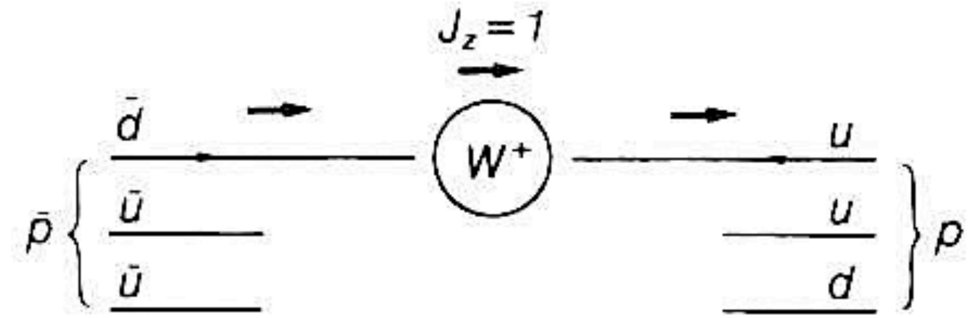
where θ_W =weak mixing angle.

$$M_W^2 = \frac{g^2\sqrt{2}}{8G} = \frac{e^2\sqrt{2}}{8G\sin^2\theta_W}$$

- $\sin^2\theta_W$ value from Gargamelle was 0.3-0.4, predicts a value of $M_W \approx 80\,\mathrm{GeV}$.

Observation of $W^{\pm}$ and Z^0 bosons in $p\bar{p}$ collisions

- W and Z bosons first observed at CERN in 1983



- In 1978, A. Astbury et al proposed to transform the CERN SPS into a $p\bar{p}$ colliding beam machine, the SppS.

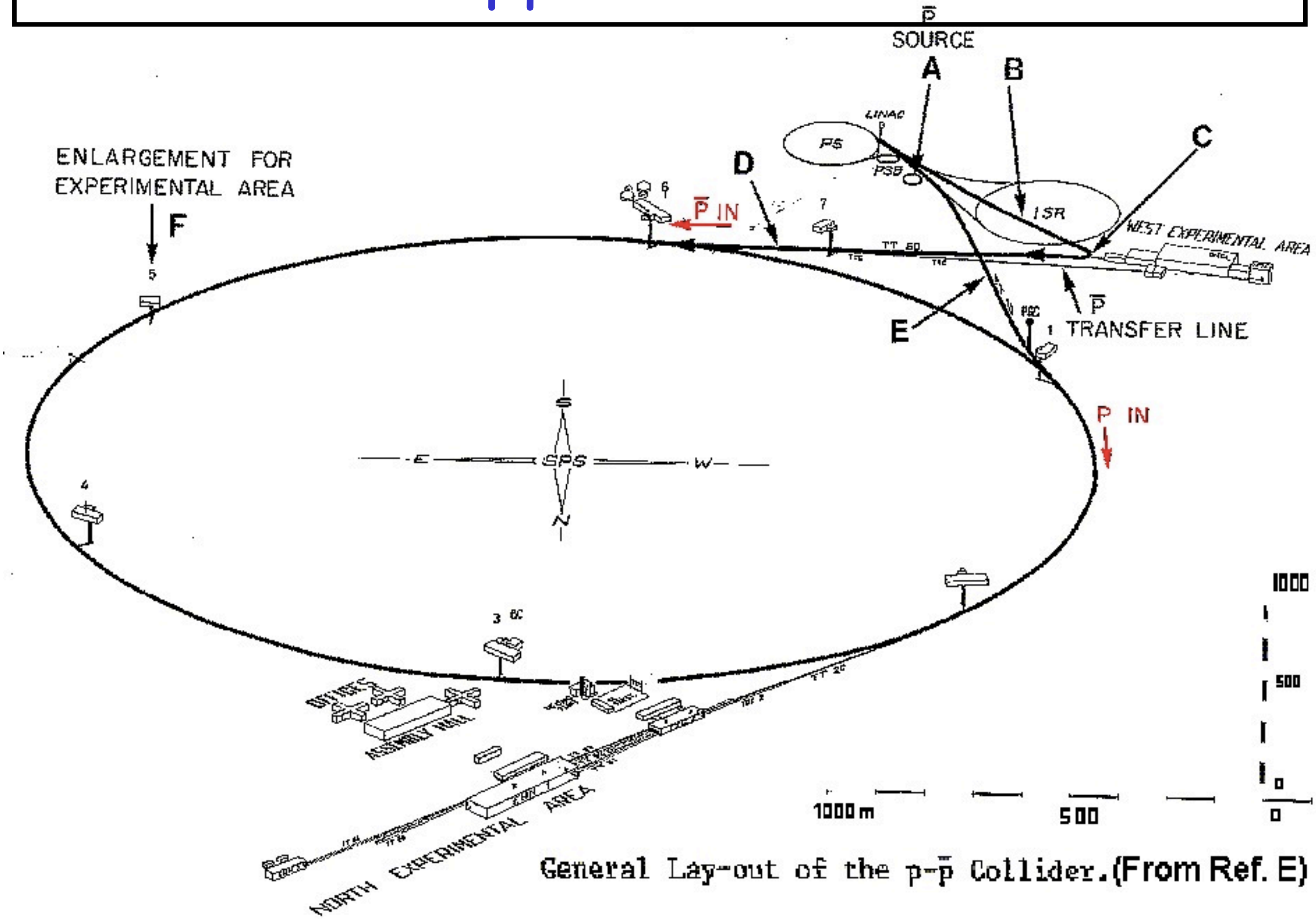
$$u + \bar{d} \to W^+ \to e^+ + \nu_e,\ \mu^+ + \nu_\mu$$

$$\bar{u} + d \to W^- \to e^- + \bar{\nu}_e,\ \mu^- + \bar{\nu}_\mu$$

$$\left.\begin{array}{l} u + \bar{u} \\ d + \bar{d} \end{array}\right\} \to Z^0 \to e^+e^-,\ \mu^+\mu^-$$

Required 80GeV in $q\bar{q}$ collision
 – corresponds to 400-600GeV in pp
CERN proposed center of mass energy $\sqrt{s}$=540GeV

SppS Collider



General Lay-out of the p-p̄ Collider.(From Ref. E)

CERN Proposal

- To calculate the number of interactions per second, Rate = σL, where σ is the cross-section and L is the luminosity.
- L is found from:

$$L = 1.43\text{x}10^{7} \frac{N_p N_{\bar{p}}}{n}$$

where N is total number of particles, and n is number of bunches of particles (5 in SppS).

- For all interactions (not just W producing ones) σ = 50mb. Produces value of $5\text{x}10^4$ int/sec.

Observation of $W^{\pm}$ and Z^0 bosons in $p\bar{p}$ collisions

$$\sigma(u\bar{d} \to W^+ \to e^+\nu_e) = \frac{1}{N_c} \frac{4\pi\lambda\!\!\!^{-2}\Gamma_{u\bar{d}}\Gamma_{e\nu}/4}{(2s_d+1)(2s_u+1)[(E-M_W)^2+\Gamma^2/4]} \frac{2J+1}{3}$$

$\lambda\!\!\!^{-} = 2/E$ (deBroglie wavelength) $= 2/M_W$

$(2J+1)/3$ accounts for V-A (LH quarks and RH antiquarks)

$1/N_c$ assures color match

$$\sigma_{\max}(u\bar{d} \to W^+ \to e^+\nu_e) = \frac{4\pi}{3M_W^2}\frac{\Gamma_{ud}\Gamma_{e\nu}}{\Gamma^2} = \frac{4\pi}{81M_W^2}$$
$$= 9.2 \text{ nb}$$

BR's:	BR(ud) = 1/3 BR(cs) = 1/3	BR($l\nu$) = 1/9

Z production 10 times smaller

Observation of $W^{\pm}$ and Z^0 bosons in $p\bar{p}$ collisions

Discovery experiment at CERN: 270 GeV x 270 GeV

$p\,\bar{p} \rightarrow W$ and $p\,\bar{p} \rightarrow Z$ cross sections from integration over quark distribution functions:

$$\sigma(p\,\bar{p} \rightarrow W \rightarrow e\nu) \approx 1 \text{ nb} = 10^{-33} \text{ cm}^2$$

$$\sigma(p\,\bar{p} \rightarrow Z \rightarrow ee) \approx 0.1 \text{ nb} = 10^{-34} \text{ cm}^2$$

while $\sigma(\text{total}) = 40 \text{ mb} = 4 \times 10^7 \text{ nb}$

very small signal relative to background

select high p_T events to suppress background

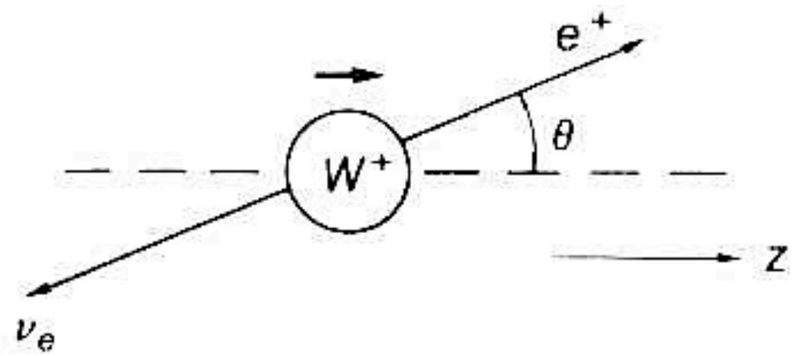
Observation of $W^{\pm}$ and Z^0 bosons in $p\bar{p}$ collisions

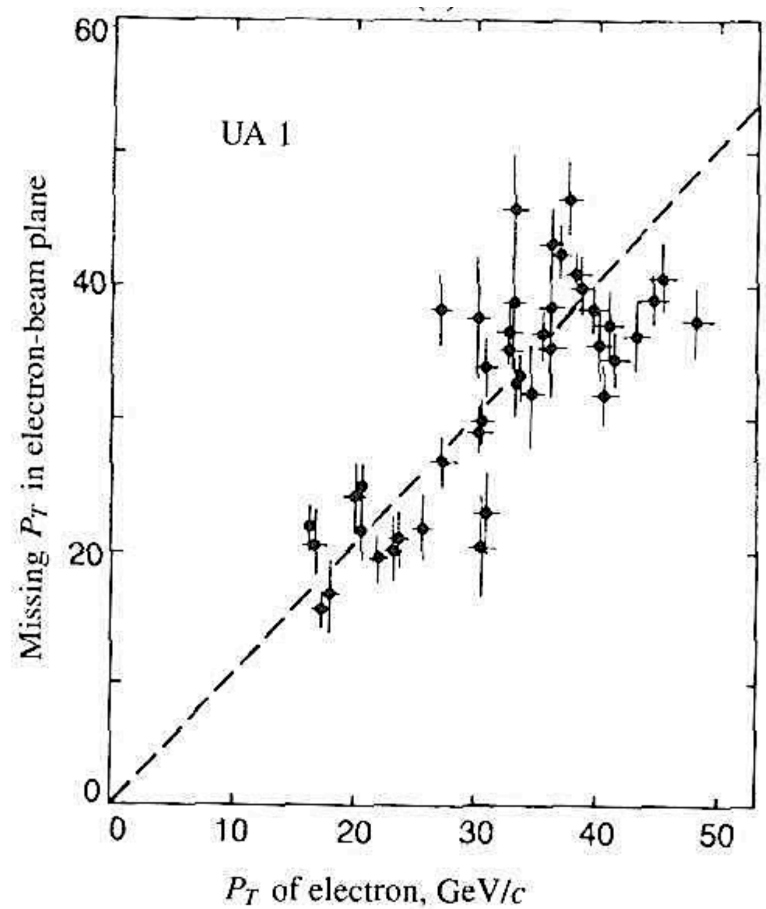
46 GeV electron in event (white), with p_T = 26 GeV/c

Missing p_T = 24 GeV/c (consistent with neutrino)

Observation of $W^{\pm}$ and Z^0 bosons in $p\bar{p}$ collisions

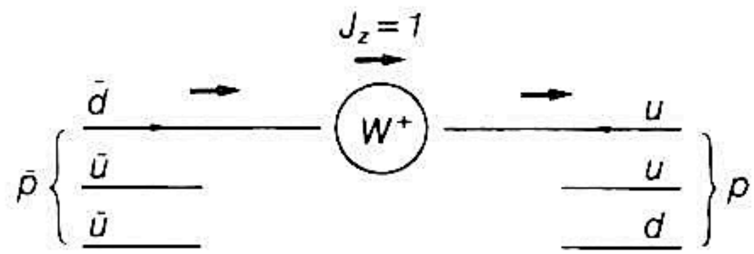
Many events observed with systematic balance of p_T of electron and missing p_T



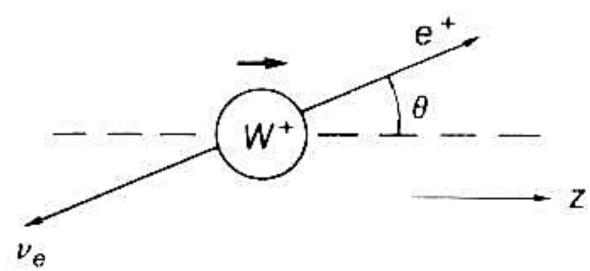


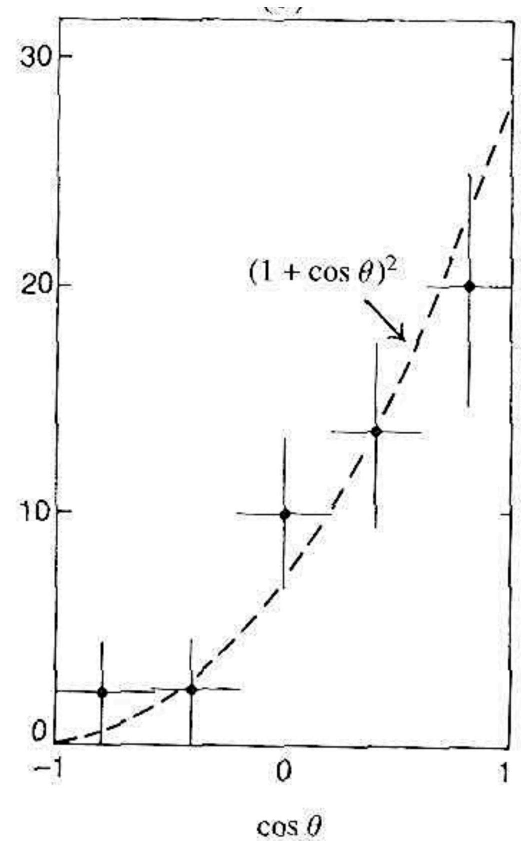
Observation of $W^{\pm}$ and Z^0 bosons in $p\bar{p}$ collisions

W's are polarized in production by V-A interaction



W decay asymmetric by V-A interaction

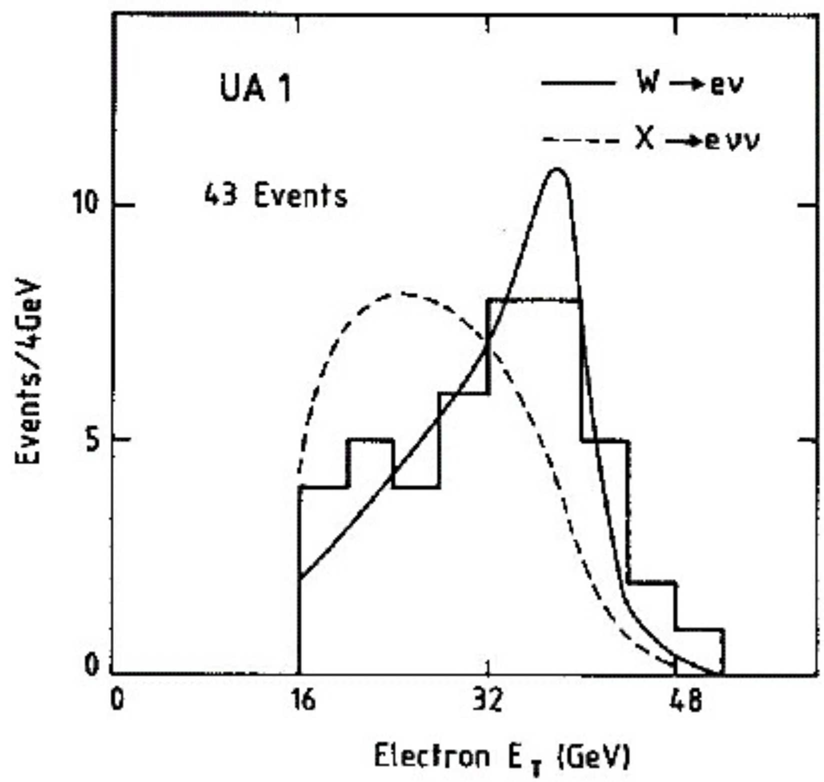




$$\frac{dN}{d(\cos\theta)} = |d^1_{1,1}|^2 = (1 + \cos\theta)^2$$

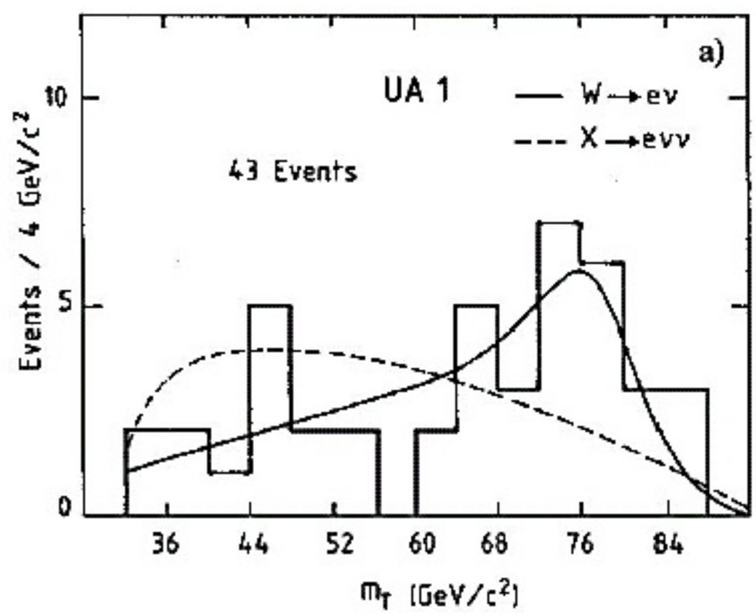
Mass measurement of $W^{\pm}$ bosons in $p\bar{p}$ collisions

First method:
- lepton E_T measurement
- compare with Monte Carlo
- could be affected by the
  - transverse energy in production



Mass measurement of $W^{\pm}$ bosons in $p\bar{p}$ collisions

Second method:
- transverse mass measurement
- compare with Monte Carlo
- independent of the transverse energy in production

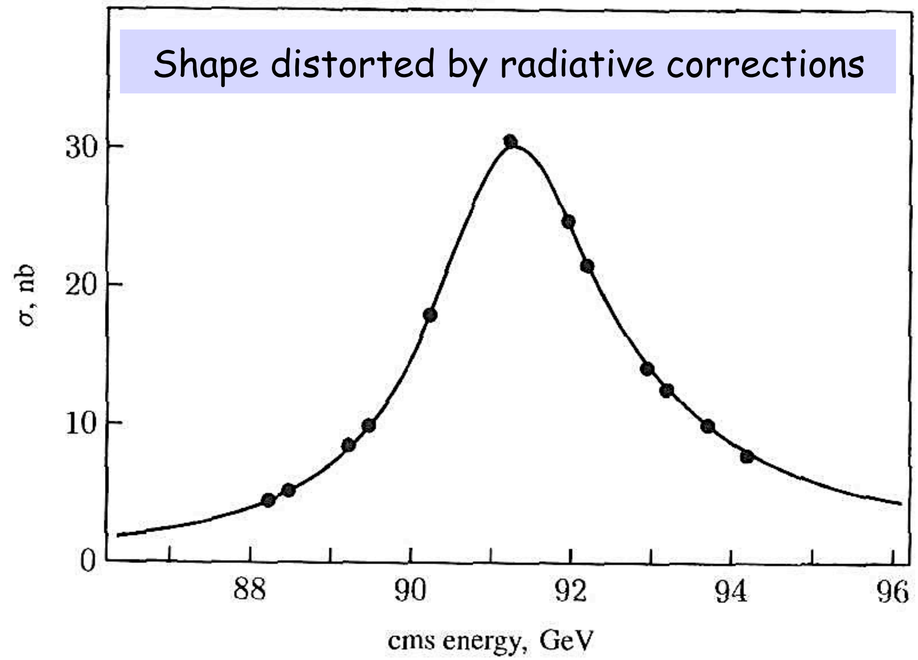


Z^0 production at e^+e^- colliders

Following the discovery of the W and Z bosons at CERN in 1983, LEP at CERN and SLC at SLAC were built and began in 1989 to produce and study the Z with high statistics

$e^+e^- \rightarrow Z \rightarrow$ anything

$$\sigma = \frac{4\pi\lambda\!\!\!^{-2}(2J+1)}{(2s+1)^2} \frac{\Gamma_e\Gamma/4}{[(E-E_0)^2+\Gamma^2/4]}$$



$$\sigma_{max}(e^+e^- \rightarrow Z^0 \rightarrow \text{anything}) = \frac{12\pi}{M_Z^2}\left(\frac{\Gamma_e}{\Gamma}\right)$$

where $\lambda\!\!\!^{-} = 2/E = 2/M_Z$

Z^0 production at e^+e^- colliders

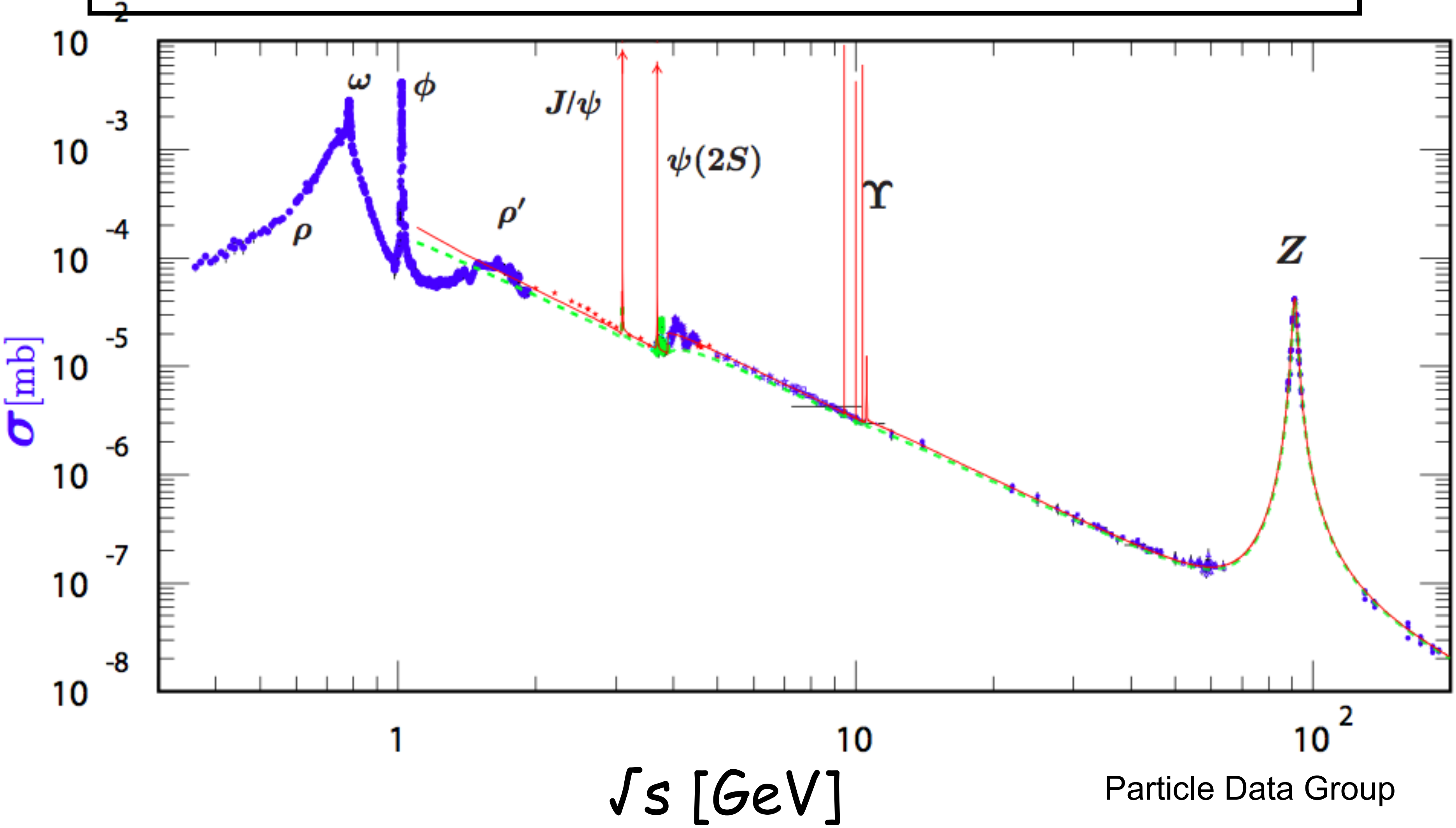
At the Z peak

$$R = \frac{\sigma_{\max}(e^+e^- \to Z^0)}{\sigma(\text{point})} \simeq \frac{9}{\alpha^2}\left(\frac{\Gamma_e}{\Gamma}\right) = 9/\alpha^2\ (0.034)$$

$\sigma(\text{point}) = \sigma(e^+ e^- \to \mu^+ \mu^-) = 4\pi\alpha^2/3s = 87\text{nb}/s(\text{GeV}^2)$

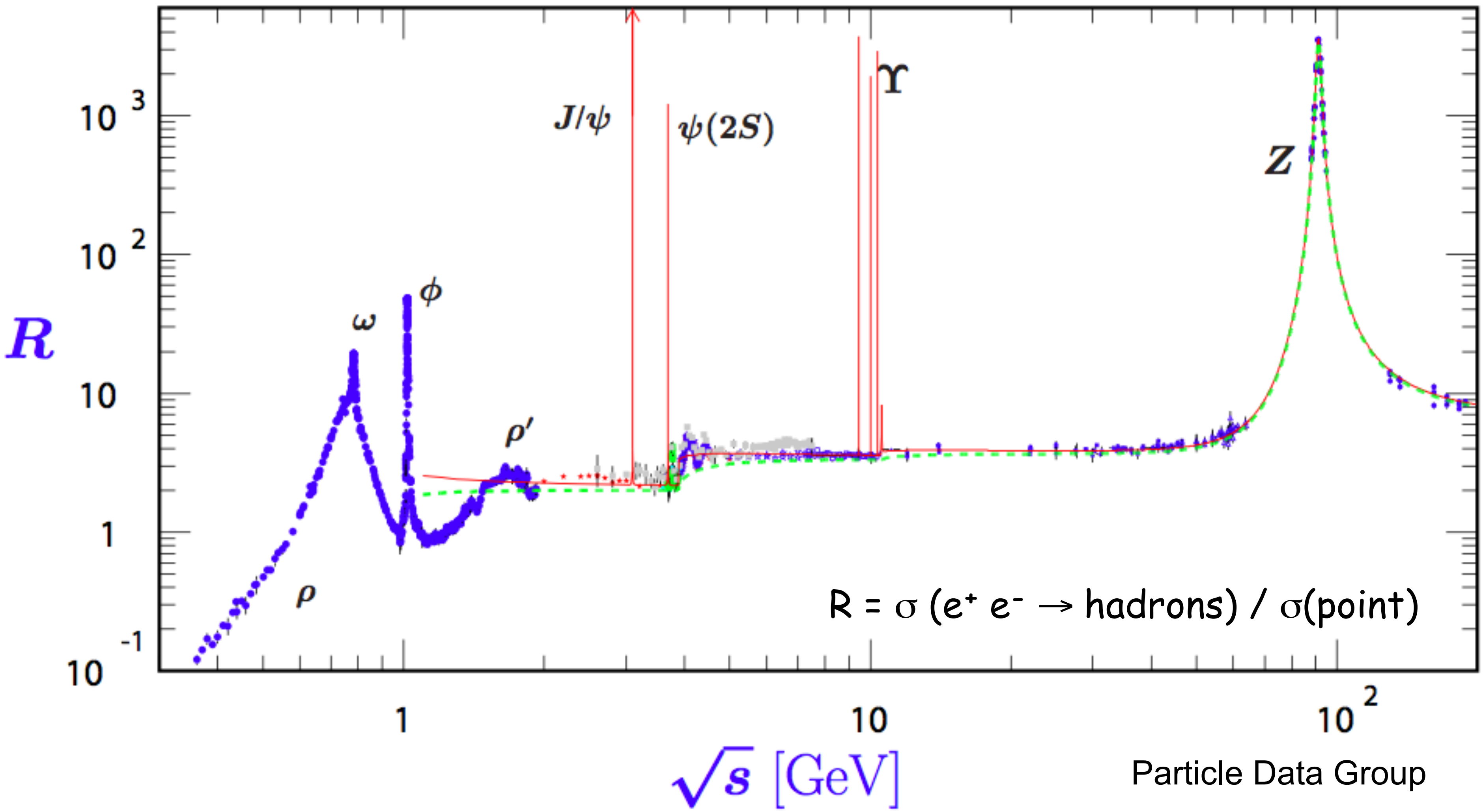
This ideal value of $R \approx 5700$ is reduced to $R \approx 3200$ by radiative corrections (initial state radiation)

e⁺ e⁻ annihilation to hadrons

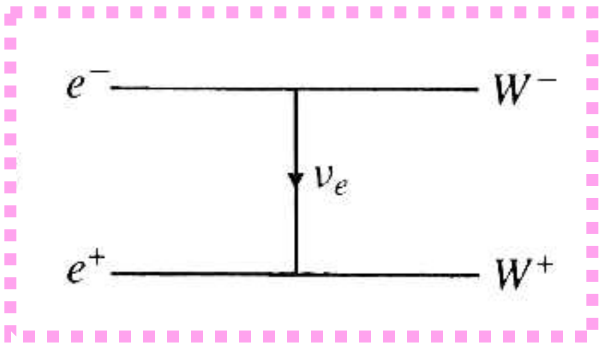


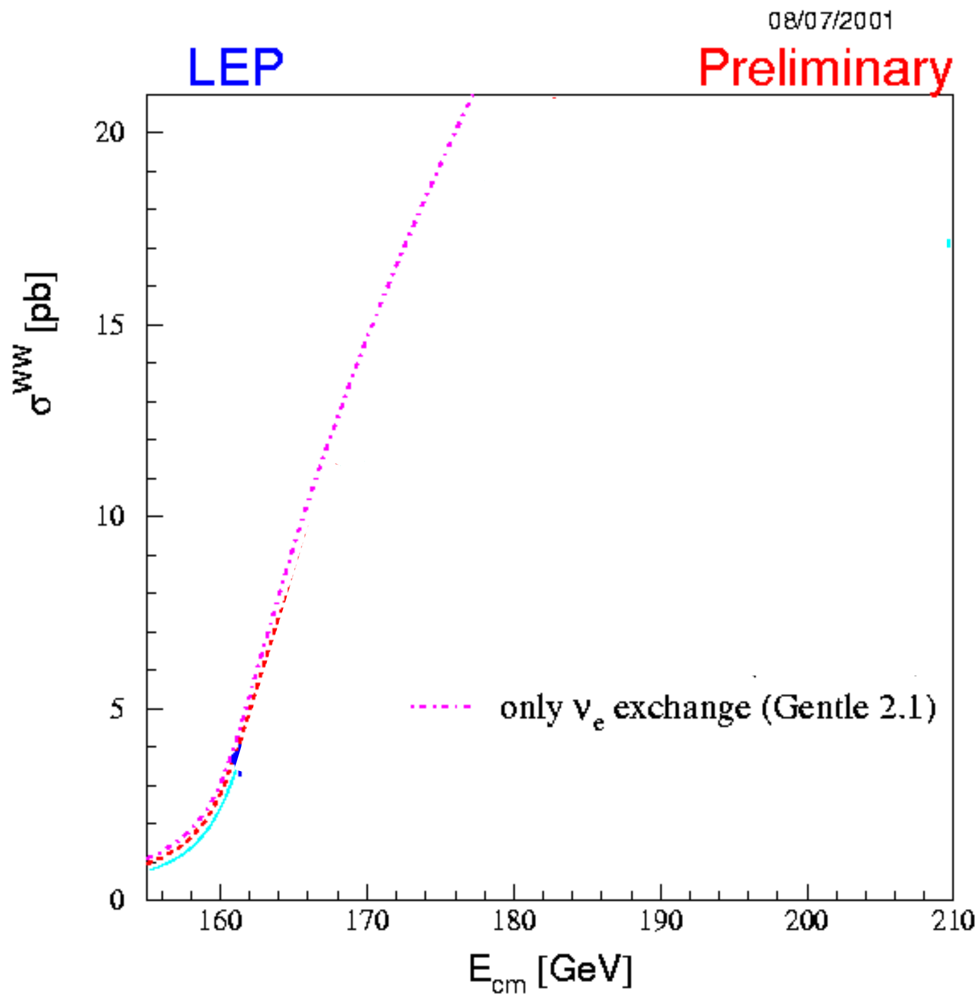
Particle Data Group

$e^+ e^-$ annihilation to hadrons

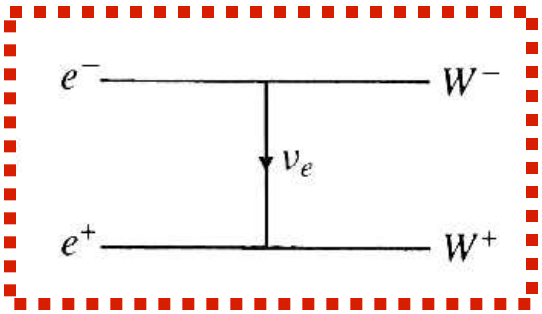


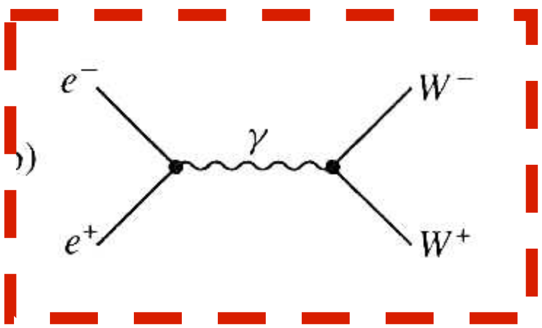
W pair production in e^+e^-

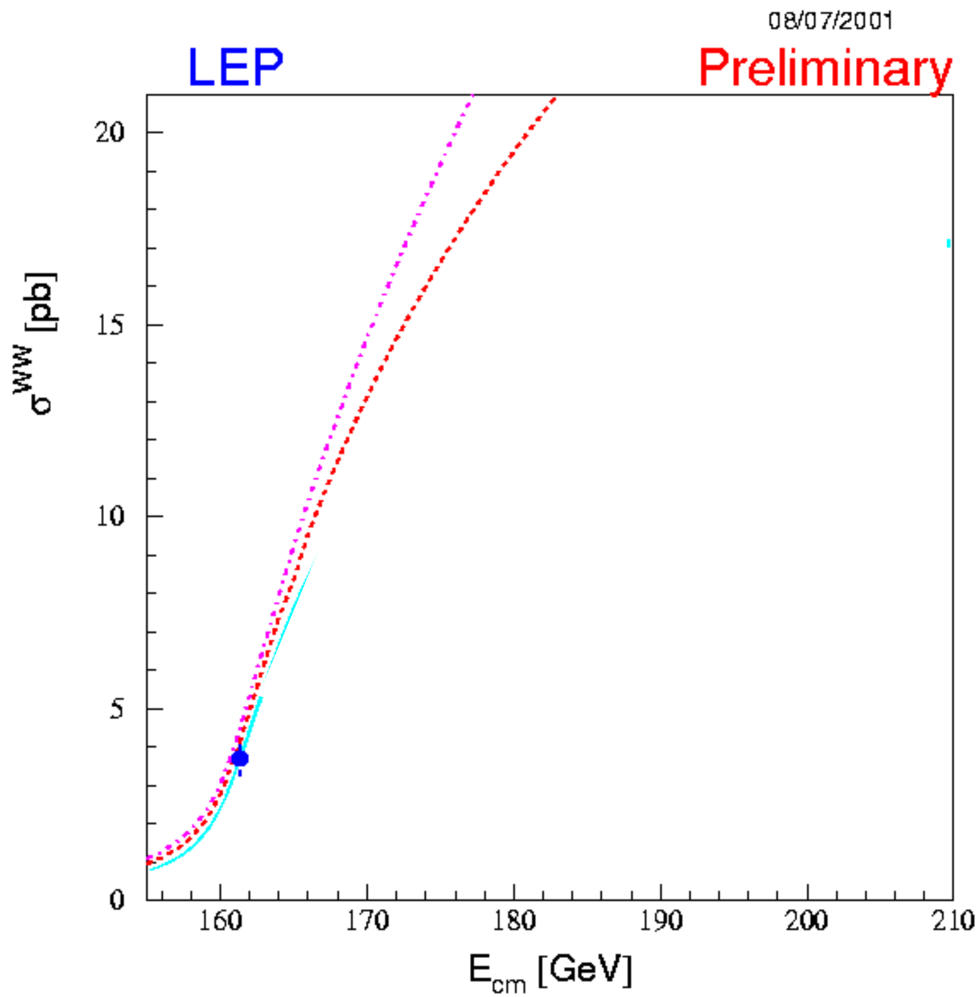




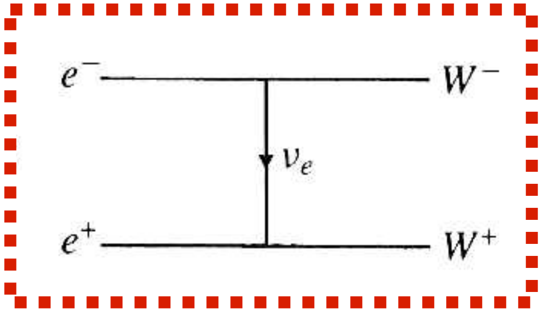
W pair production in e^+e^-

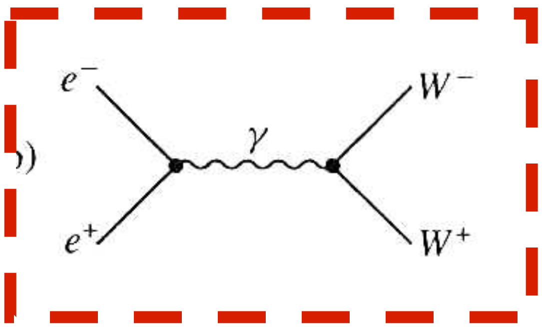


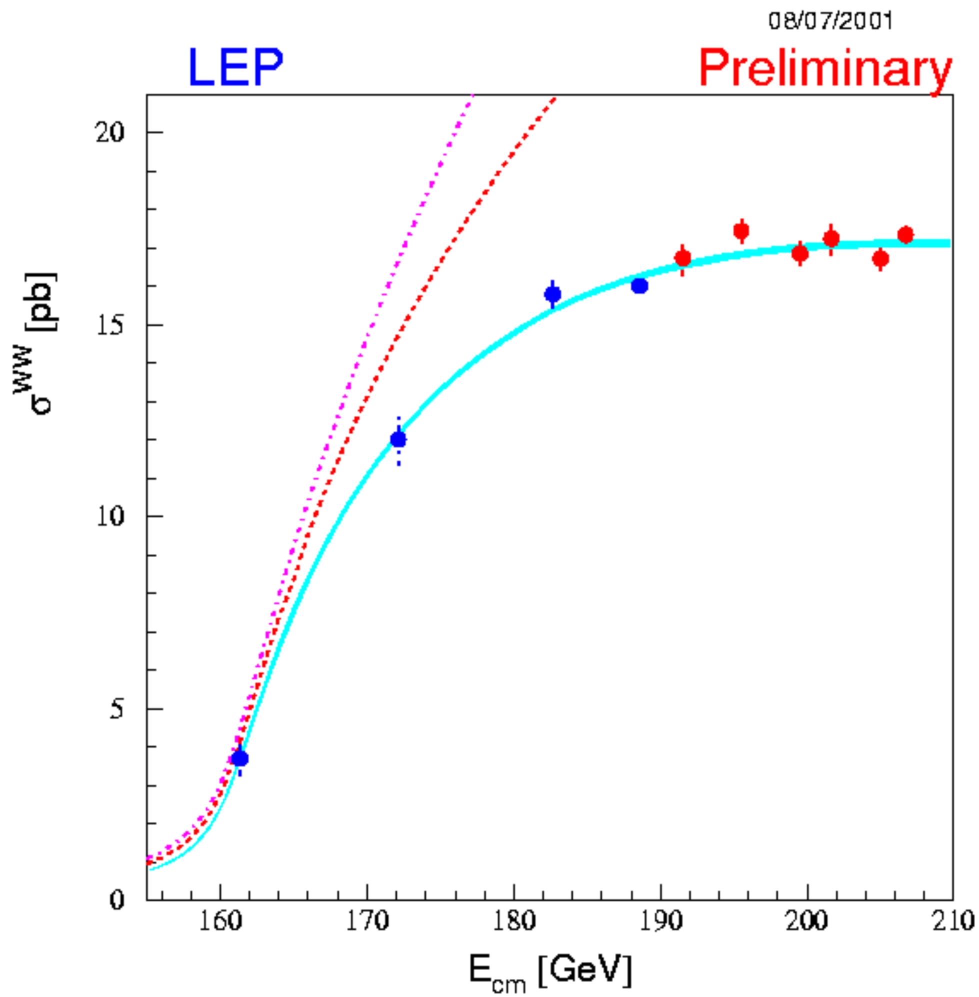




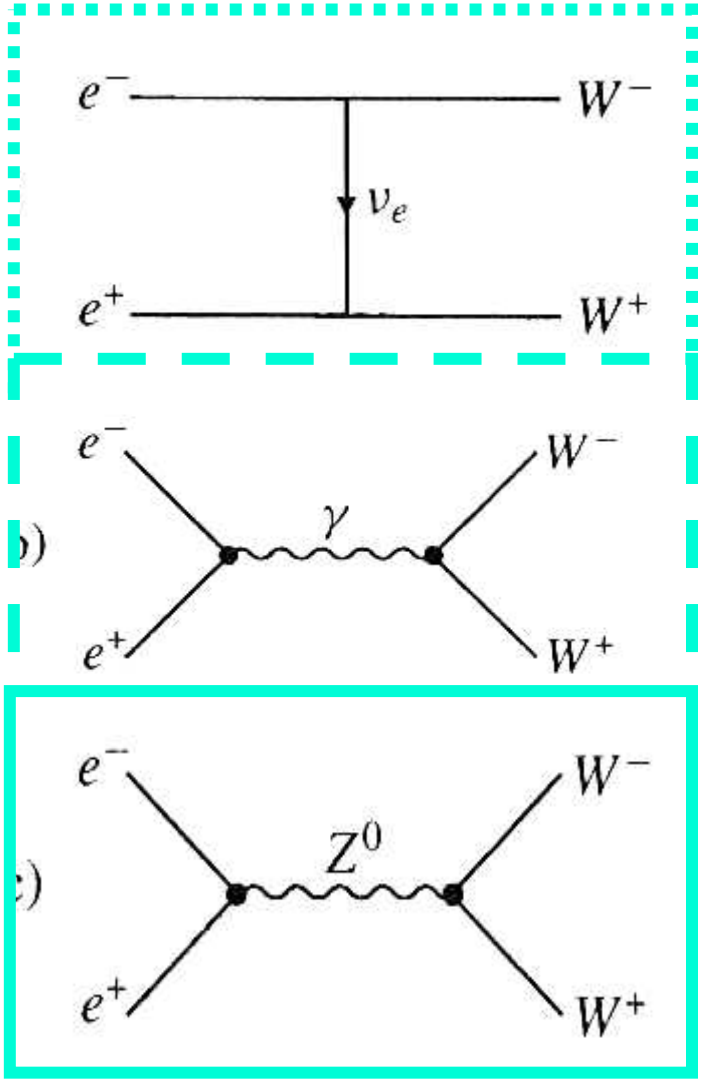
W pair production in e^+e^-

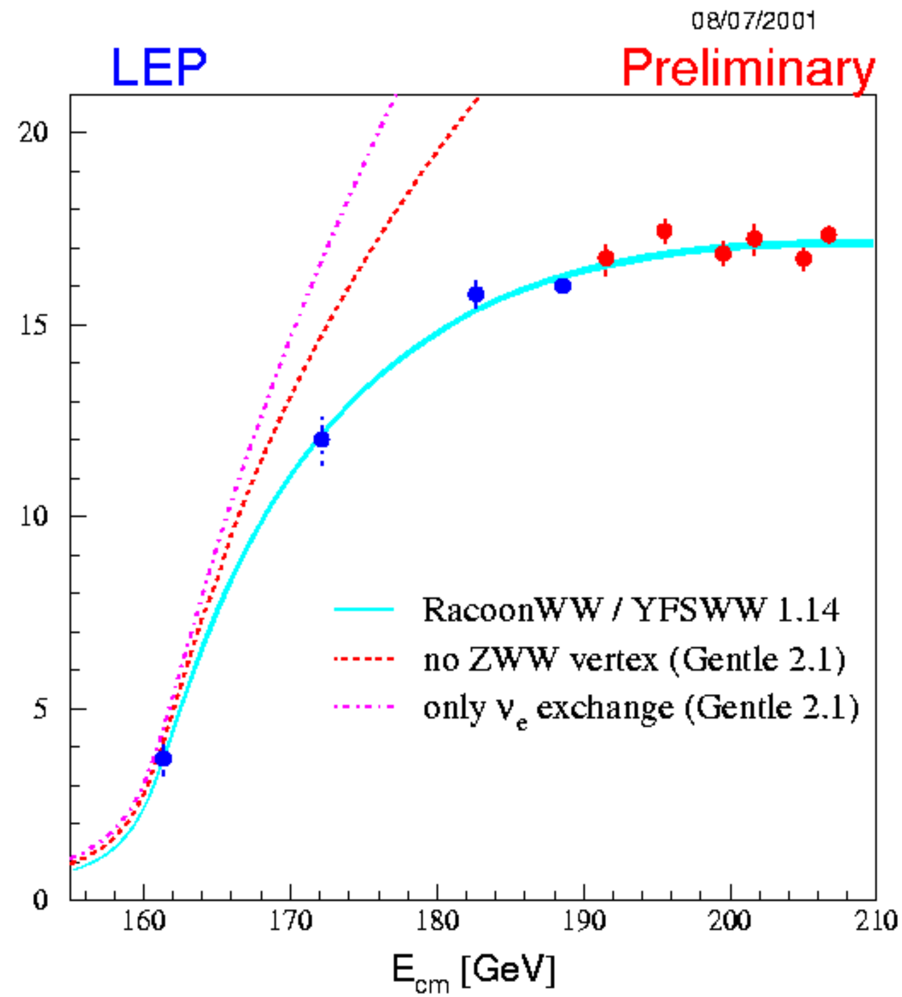






W pair production in e^+e^-

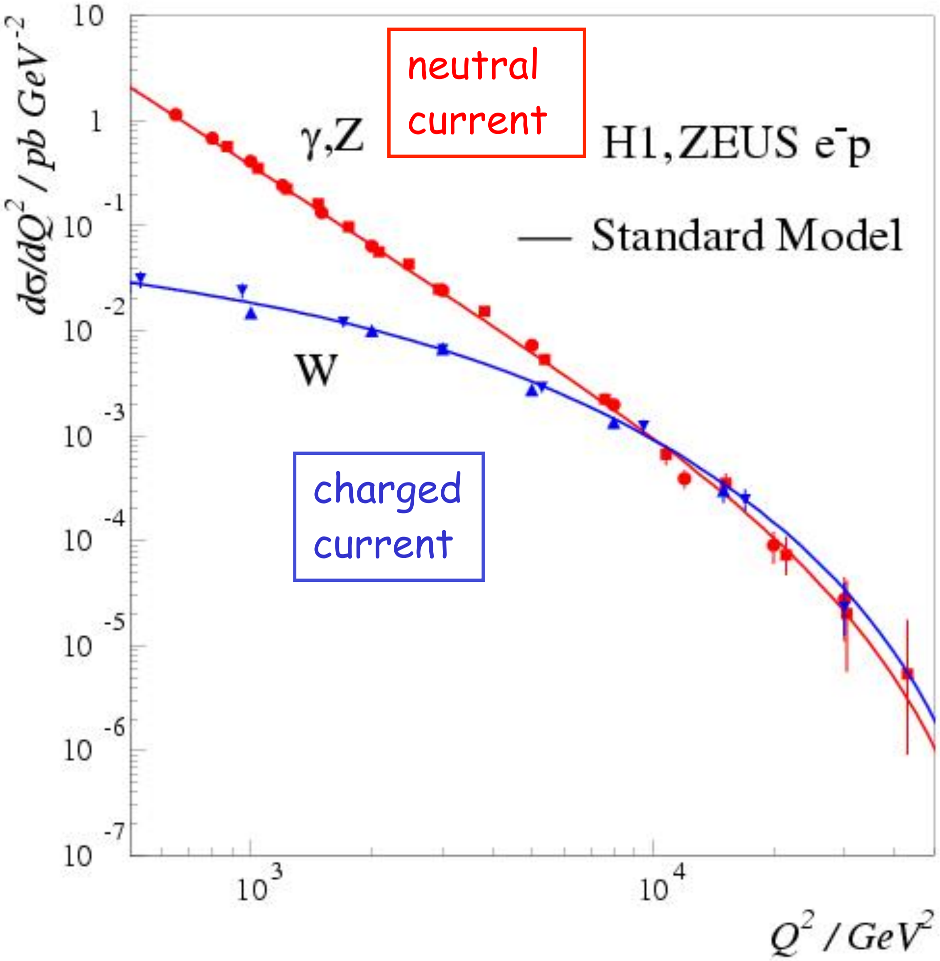




Gauge Boson Exchange in e^-p

$e^-p \rightarrow e^-X$

$\rightarrow \nu_e X$



Weak decays of quarks. The GIM model and the CKM matrix

$\Delta S = 1$ weak decays are suppressed relative to $\Delta S = 0$ weak decays by a factor of about 20

$$\Sigma^- \rightarrow n + e^- + \bar{\nu}_e \qquad (\Delta S = 1)$$

$$n \rightarrow p + e^- + \bar{\nu}_e \qquad (\Delta S = 0)$$

Fermi constant (G) deduced from neutron β-decay is slightly smaller than G deduced from muon decay ($\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$)

Can these facts be explained?

Cabibbo theory: d and s quarks interacting in weak force are not flavor eigenstates, but rotated by a mixing angle (θ_c)

Weak decays of quarks.
The GIM model and the CKM matrix

lepton doublets $\begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}$

quark doublet $\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$

These three quarks were the only known quarks at the time Cabibbo wrote down his theory

With this assumption the "effective" couplings will be:

$G \cos\theta_c$ for $\Delta S = 0$ ($d \rightarrow u$)

$G \sin\theta_c$ for $\Delta S = 1$ ($s \rightarrow u$)

experiment yields $\theta_c \approx 13^0$ ($\sin\theta_c \approx 0.22$)

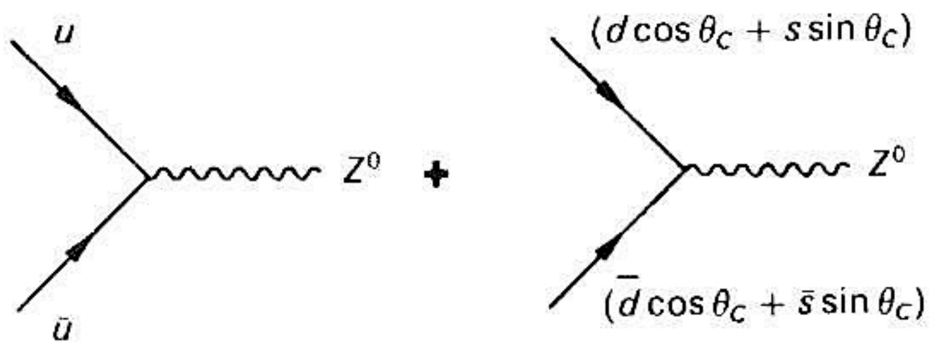
Weak decays of quarks.
The GIM model and the CKM matrix

Flavor-changing neutral currents

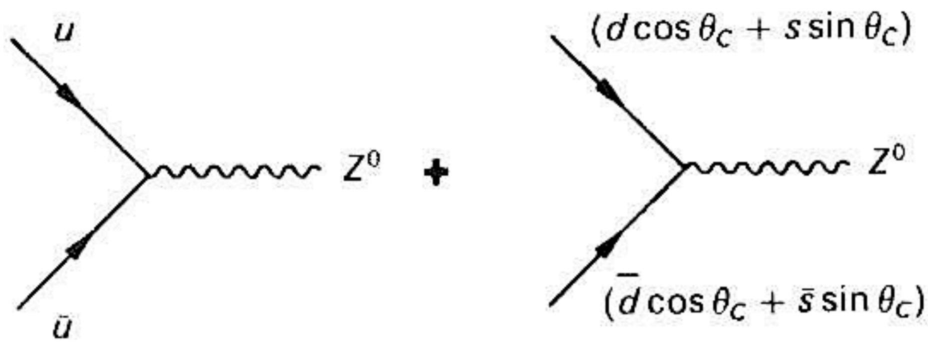
$$\frac{K^+ \rightarrow \pi^+ + \nu + \bar{\nu}}{K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu} \leq 10^{-5}$$

$$\frac{1.7 \pm 1.1 \times 10^{-10}}{3.353 \pm 0.034\ \%} = 4.8 \times 10^{-9}$$

2011



Weak decays of quarks.
The GIM model and the CKM matrix



Neutral current interaction should be:

$$\underbrace{u\bar{u} + (d\bar{d}\cos^2\theta_c + s\bar{s}\sin^2\theta_c)}_{\Delta S = 0} + \underbrace{(s\bar{d} + d\bar{s})\sin\theta_c\cos\theta_c}_{\Delta S = 1}$$

so flavor-changing neutral interactions should be allowed, with a slight suppression.

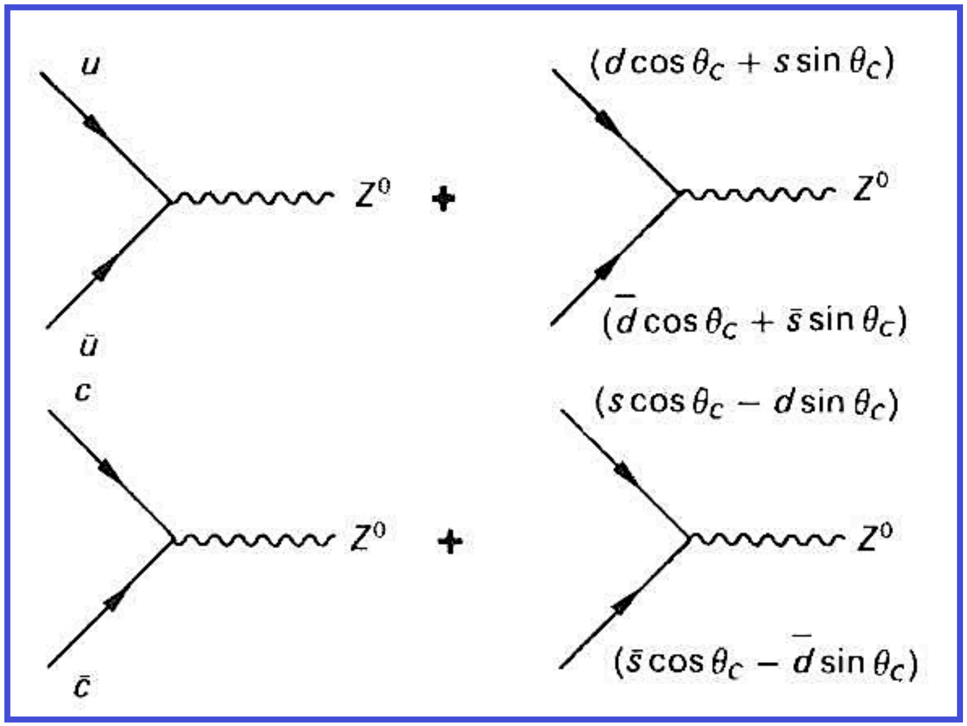
Why were they so rare?

GIM mechanism (1970)

Weak decays of quarks.
The GIM model and the CKM matrix

GIM mechanism (1970): introduce fourth quark (charm)

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}, \qquad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$



Neutral current interaction becomes:

$$\underbrace{u\bar{u} + c\bar{c} + (d\bar{d} + s\bar{s})\cos^2\theta_c + (s\bar{s} + d\bar{d})\sin^2\theta_c}_{\Delta S = 0}$$

$$\underbrace{+(s\bar{d} + \bar{s}d - \bar{s}d - s\bar{d})\sin\theta_c\cos\theta_c}_{\Delta S = 1}$$

Weak decays of quarks. The GIM model and the CKM matrix

The quark mixing,

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}, \qquad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$

can be expressed in matrix form:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

The weak interactions involving the four light quarks are consistent with this model, being defined by a unique value of θ_c

Weak decays of quarks.
The GIM model and the CKM matrix

Six quark model, mixing matrix (CKM matrix) is 3x3:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

NxN unitary matrix has n(n-1)/2 real parameters (Euler angles) and (N-1)(N-2)/2 non-trivial phase angles

so this 3x3 matrix has 3 Euler angles and 1 phase
- this phase results in T-violation (or CP-violation)

(CKM = Cabibbo, Kobayashi, Maskawa)

Weak decays of quarks.
The GIM model and the CKM matrix

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

V_{ud} (=cos θ_c) determined by comparing nuclear β-decay and μ-decay rates

most precise from superallowed 0+ → 0+ nuclear beta decays, pure vector transitions:

$|V_{ud}| = 0.97425 \pm 0.00022$ (avg of 20 meas.)

V_{us} determined from semi-leptonic decays of strange particles,

eg. $K_L^0 \to \pi e \nu,\ K_L^0 \to \pi \mu \nu,\ K^\pm \to \pi^0 e^\pm \nu,$
$K^\pm \to \pi^0 \mu^\pm \nu$ and $K_S^0 \to \pi e \nu$

$|V_{us}| = 0.2252 \pm 0.0009$

Weak decays of quarks. The GIM model and the CKM matrix

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

V_{ub} measured by selecting B semi-leptonic decays to non-strange particles (inclusive $B \rightarrow X_u \, l \, \nu$ decays) for example, at the end-point

$$|V_{ub}| = (3.89 \pm 0.44) \times 10^{-3}$$

V_{cb} from exclusive and inclusive semileptonic decays of B mesons to charm (eg. $B \rightarrow Dl\nu$)

$$\Gamma(b \rightarrow cl\nu) = \frac{R(b \rightarrow cl\nu)}{\tau_B} = \frac{G^2 m_b^5}{192\pi^3} |V_{cb}|^2 f$$

$$|V_{cb}| = (40.6 \pm 1.3) \times 10^{-3}$$

Weak decays of quarks.
The GIM model and the CKM matrix

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

V_{cd} from ν_μ di-μ events

$$\nu_\mu + d \to \mu^- + c$$
$$\hookrightarrow s + \mu^+ + \nu_\mu$$

and $D \to \pi\, l\, \nu$

$|V_{cd}| = 0.230 \pm 0.011$

V_{cs} from semileptonic D decays (eg. $D^+ \to K^0\, e^+\, \nu_e$)
or leptonic D_s decays (eg. $D_s^+ \to \mu^+ \nu_\mu$)

$|V_{cs}| = 1.023 \pm 0.036$

Weak decays of quarks. The GIM model and the CKM matrix

V_{td}, V_{ts} from B–Bbar oscillations mediated by box diagrams with top quarks, or loop-mediated rare K and B decays

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

$$|V_{td}| = (8.4 \pm 0.6) \times 10^{-3}$$

$$|V_{ts}| = (38.7 \pm 2.1) \times 10^{-3}$$

V_{tb} from top decay branching fractions

$$R = B(t \to Wb)/B(t \to Wq)$$
$$= |V_{tb}|^2/(\sum|V_{tq}|^2) = |V_{tb}|^2, \text{ where } q = b,s,d.$$

$$|V_{tb}| = 0.88 \pm 0.07$$

Weak decays of quarks.
The GIM model and the CKM matrix

$$V_{CKM} = \begin{vmatrix} V_{ud} = 0.97425 \pm 0.00022 & V_{us} = 0.2252 \pm 0.0009 & V_{ub} = 0.00389 \pm 0.00044 \\ V_{cd} = 0.230 \pm 0.011 & V_{cs} = 1.023 \pm 0.036 & V_{cb} = 0.0406 \pm 0.0013 \\ V_{td} = 0.0084 \pm 0.0006 & V_{ts} = 0.0387 \pm 0.0021 & V_{tb} = 0.88 \pm 0.07 \end{vmatrix}$$

Diagonal terms are close to unity
Off-diagonal terms are small

Particle Data Group

Unitarity of the CKM Matrix

$$V_{CKM} = \begin{vmatrix} V_{ud} = 0.9742 & V_{us} = 0.2252 & V_{ub} = 0.0039 \\ V_{cd} = 0.23 & V_{cs} = 1.02 & V_{cb} = 0.041 \\ V_{td} = 0.0084 & V_{ts} = 0.039 & V_{tb} = 0.88 \end{vmatrix}$$

From the independently measured CKM elements, the unitarity of the CKM matrix can be checked.

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999 \pm 0.0006$ (1st row)
$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.101 \pm 0.074$ (2nd row)
$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1.002 \pm 0.005$ (1st column)
$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.098 \pm 0.074$ (2^{nd} column)

Weak decays of quarks. The GIM model and the CKM matrix

"Standard" parametrization of CKM matrix (PDG)

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$ $s_{ij} = \sin\theta_{ij}$
(unitarity requirement limits to 3 angles and one phase)

Can we approximate?

$$V_{CKM} = \begin{vmatrix} V_{ud} = 0.9742 & V_{us} = 0.2252 & V_{ub} = 0.0039 \\ V_{cd} = 0.23 & V_{cs} = 1.02 & V_{cb} = 0.041 \\ V_{td} = 0.0084 & V_{ts} = 0.039 & V_{tb} = 0.88 \end{vmatrix}$$

Diagonal terms are close to unity
Off-diagonal terms are small: $s_{13} \ll s_{23} \ll s_{12}$

Weak decays of quarks.
The GIM model and the CKM matrix

Diagonal terms are close to unity

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

Wolfenstein Parametrization

$\lambda = s_{12}$, introduce A, ρ, η

$$V_{\mathrm{CKM}} = \begin{vmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{vmatrix}$$

Global fit in the Standard Model

- The CKM matrix elements can be most precisely determined by a global fit that uses all available measurements and imposes the SM constraints (i.e., three generation unitarity).

$$\lambda = 0.2253 \pm 0.0007\,, \qquad A = 0.808^{+0.022}_{-0.015}\,,$$

$$\bar{\rho} = 0.132^{+0.022}_{-0.014}\,, \qquad \bar{\eta} = 0.341 \pm 0.013\,.$$

$$\bar{\rho} = \rho(1 - \lambda^2/2 + \ldots)$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

Particle Data Group

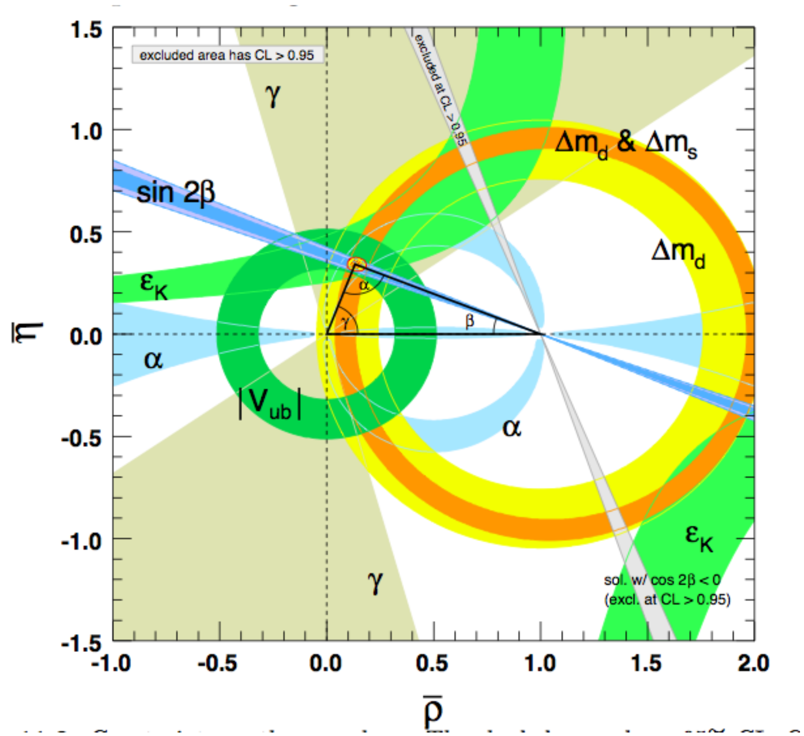
Weak decays of quarks.
The GIM model and the CKM matrix

Wolfenstein parametrization
plot on $\eta-\rho$ plane

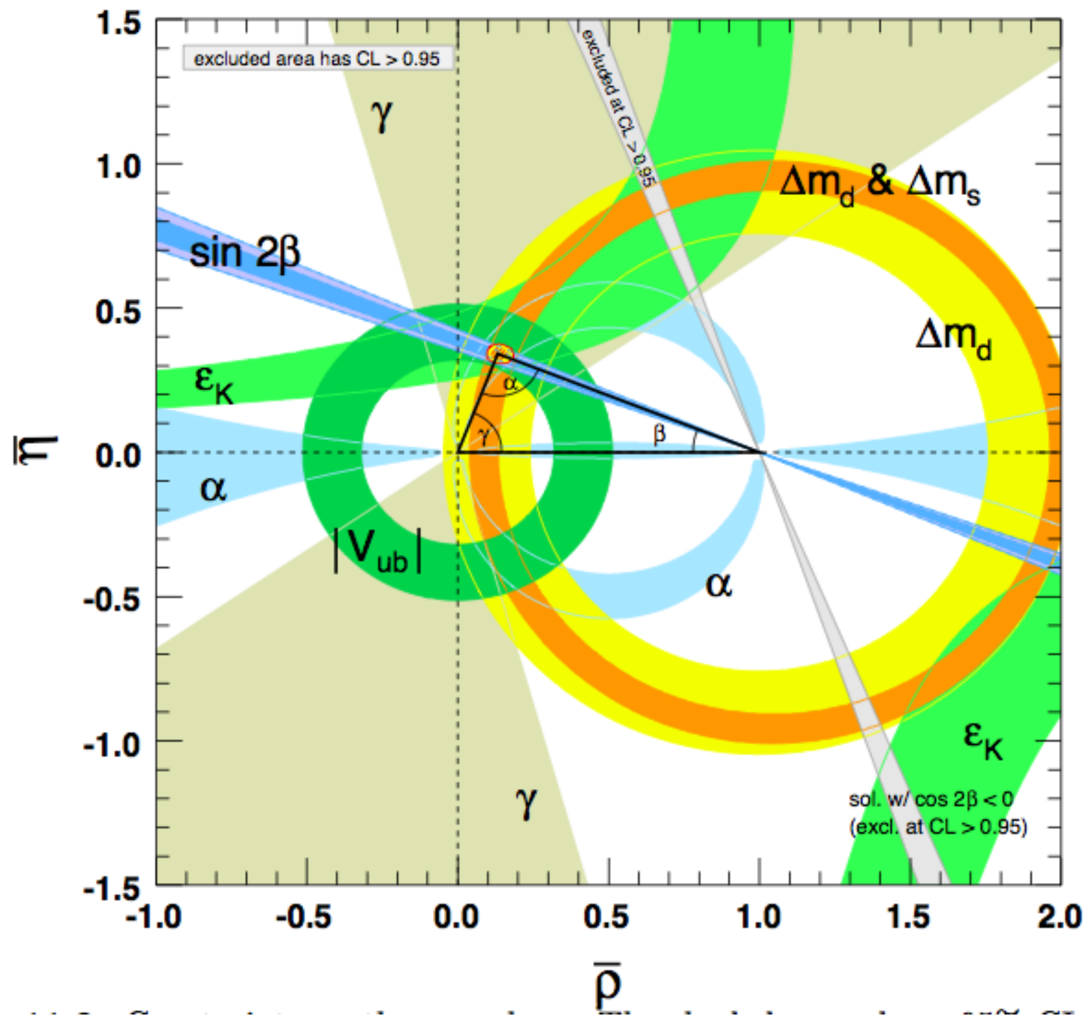
$$V_{CKM} = \begin{vmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{vmatrix}$$

$\lambda = 0.2253 \pm 0.0007$
$A = 0.808 + 0.022 - 0.015$

$$V = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$$

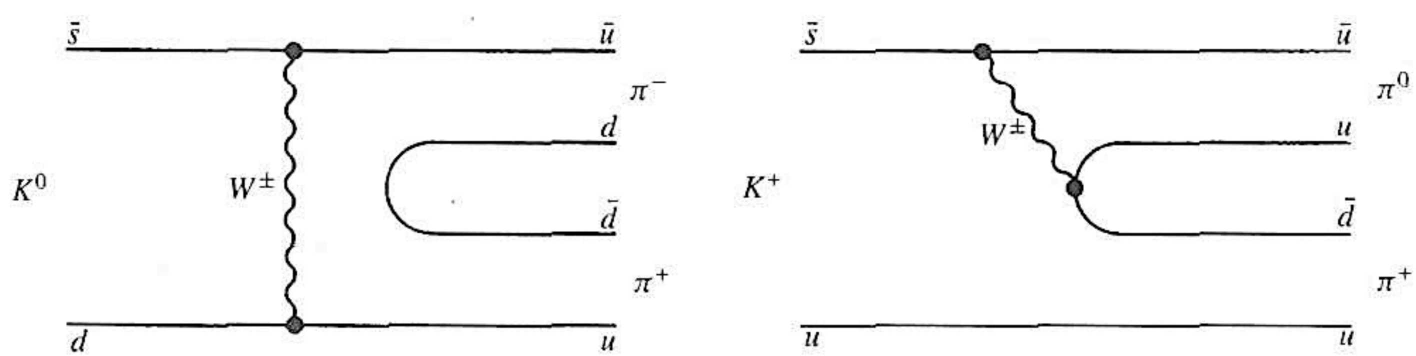


Weak decays of quarks.
The GIM model and the CKM matrix



Neutral K mesons

I	I_3	S	Meson	Quark combination	Decay	Mass, MeV
$\frac{1}{2}$	$\frac{1}{2}$	$+1$	K^+	$u\bar{s}$	$K^+ \to \mu\nu$	494
$\frac{1}{2}$	$-\frac{1}{2}$	$+1$	K^0	$d\bar{s}$	$K^0 \to \pi^+\pi^-$	498
$\frac{1}{2}$	$-\frac{1}{2}$	-1	K^-	$\bar{u}s$	$K^- \to \mu\nu$	494
$\frac{1}{2}$	$\frac{1}{2}$	-1	$\bar{K}^0$	$\bar{d}s$	$\bar{K}^0 \to \pi^+\pi^-$	498



Neutral K mesons

K^0 and $\overline{K^0}$ are particle and antiparticle

related by charge conjugation

reversal of I_3 and change of strangeness $\Delta S = 2$

These particles are clearly distinctive in strong interaction,

since SI conserves isospin and strangeness

However, in propagating through free space, mixing occurs:

$$\begin{tikzcd}
 & 2\pi & \\
K^0 \arrow[ur] \arrow[dr] & & \bar{K}^0 \arrow[ul] \arrow[dl] \\
 & 3\pi &
\end{tikzcd}$$

a pure K^0 state at t=0 will become a mixed state:

$$|K(t)\rangle = \alpha(t)|K^0\rangle + \beta(t)|\bar{K}^0\rangle$$

Neutral K mesons

CP eigenstates can be defined for the K^0 mesons

K^0 and $\overline{K^0}$ are not CP eigenstates:

$$CP|K^0\rangle \rightarrow \eta|\bar{K}^0\rangle, \qquad CP|\bar{K}^0\rangle \rightarrow \eta'|K^0\rangle$$

we can form the linear combinations:

$$|K_S\rangle = \sqrt{\tfrac{1}{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right), \qquad CP = +1$$

$$|K_L\rangle = \sqrt{\tfrac{1}{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right), \qquad CP = -1$$

resulting in:

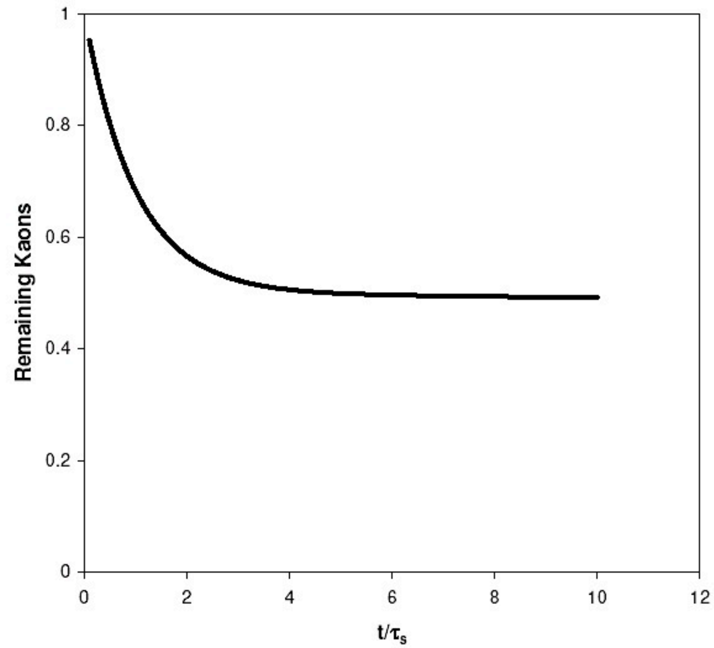
$$CP|K_S\rangle \rightarrow |K_S\rangle, \qquad CP|K_L\rangle \rightarrow -|K_L\rangle$$

Neutral K mesons

Consider K^0 production and subsequent decay

Why don't the K^0's decay away with a single exponential?

K^0's are mixtures of K_S and K_L



$$K_S \rightarrow 2\pi(CP = +1), \qquad \tau_S = 0.893 \times 10^{-10}\ \mathrm{s}$$
$$K_L \rightarrow 3\pi(CP = -1), \qquad \tau_L = 0.517 \times 10^{-7}\ \mathrm{s}$$

$0.8953 \pm 0.0005 \times 10^{-10}$ s
$0.5292 \pm 0.0009 \times 10^{-7}$ s

2011

Neutral K mesons

It can be shown that the 2π state has CP = +1,
and the 3π state has CP = -1

Thus, if CP is conserved,

$$K_S = \sqrt{\tfrac{1}{2}}(K^0 + \bar{K}^0) \to 2\pi(CP = +1),$$
$$K_L = \sqrt{\tfrac{1}{2}}(K^0 - \bar{K}^0) \to 3\pi(CP = -1),$$

Since the 2 and 3-pion decays have different Q values,
the decay rates are different:

$$K_S \to 2\pi(CP = +1), \qquad \tau_S = 0.893 \times 10^{-10}\ \mathrm{s}$$
$$K_L \to 3\pi(CP = -1), \qquad \tau_L = 0.517 \times 10^{-7}\ \mathrm{s}$$

which is why the CP=+1 state is called K-short(K_S)
and the CP=-1 state is called K-long (K_L)

Neutral K mesons

Strangeness oscillations

$$A_S(t) = A_S(0)e^{-(\Gamma_S/2+im_S)t}$$

$$A_L(t) = A_L(0)e^{-(\Gamma_L/2+im_L)t}$$

$$I(K^0) = \tfrac{1}{2}[A_S(t) + A_L(t)][A_S^*(t) + A_L^*(t)]$$
$$= \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

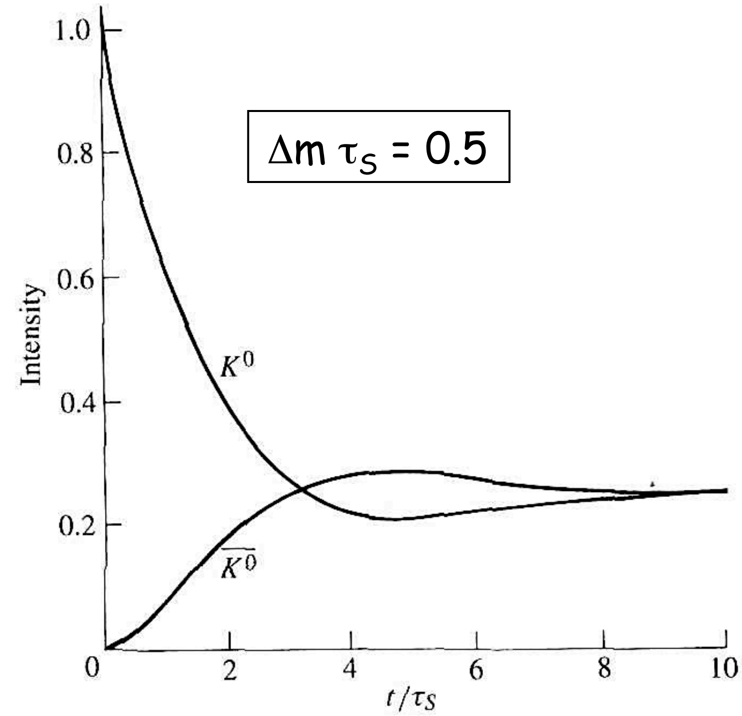
Assuming an initially pure K^0 beam, so
$A_S(0) = A_L(0) = 1/\sqrt{2}$

$$\Delta m = m_L - m_S$$

$$I(\bar{K}^0) = \tfrac{1}{4}[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t]$$

$$\boxed{\Delta m = (3.491 \pm 0.009) \times 10^{-12}\ \text{MeV}}$$

Old value in Perkins



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$\Delta m = 0.5292 \pm 0.0009 \times 10^{10}\,\hbar\ s^{-1}$
$= 3.475 \pm 0.006 \times 10^{-12}$ MeV

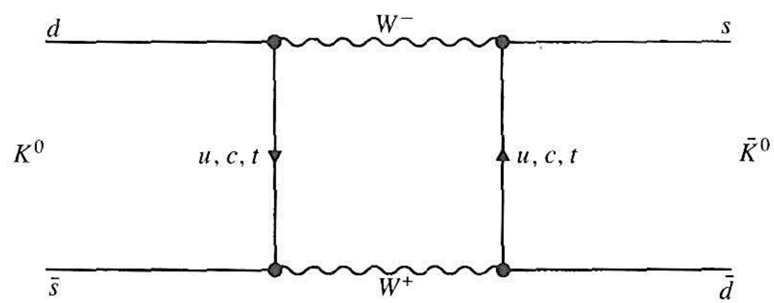
Neutral K mesons

$\Delta m = 0.5292 \pm 0.0009 \times 10^{10} \hbar\, s^{-1}$
$= 3.475 \pm 0.006 \times 10^{-12}$ MeV

This mass difference corresponds to the rate at which K^0 oscillates into $\bar{K}^0$.

This oscillation proceeds through the second-order weak interaction

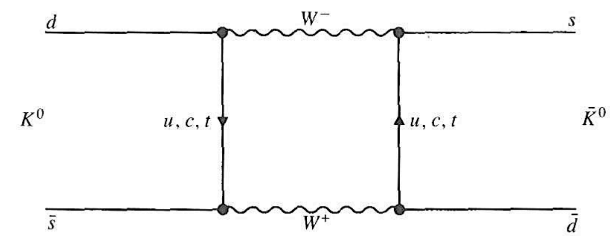
The c quark dominates (CKM matrix)



$$\Delta m = \frac{G^2}{4\pi} m_K f_K^2 m_c^2 \cos^2\theta_c \sin^2\theta_c$$

Neutral K mesons

$$\Delta m = \frac{G^2}{4\pi} m_K f_K^2 m_c^2 \cos^2\theta_c \sin^2\theta_c$$



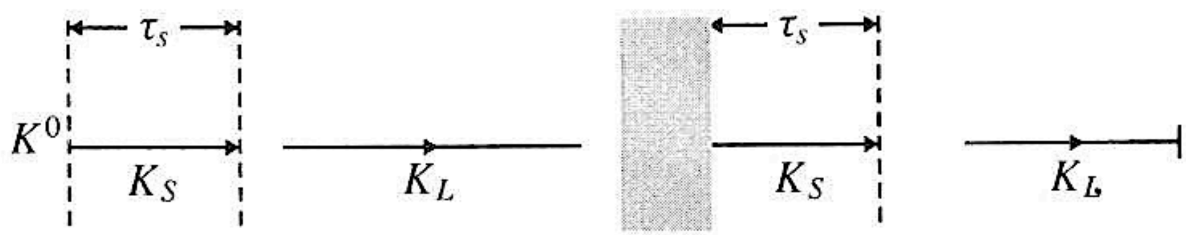
f_K is the called the "kaon decay constant"
$f_K \approx 1.2\ m_\pi$

$m_K f_K^2$ relates to $|\psi(0)|^2$, the wavefunction at the origin, or the point at which the quarks interact

measurement of Δm allowed the mass of the charmed quark to be estimated ($m_c \approx 1.5$ GeV) before it was discovered.

Neutral K mesons

K^0 Regeneration



$$K_L = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)$$

K^0 and $\overline{K}^0$ are absorbed differently

$$^{1}/_{\sqrt{2}}\,(fK^0 - \bar{f}\overline{K}^0) = {}^{1}/_{2}\,(f+\bar{f})K_L + {}^{1}/_{2}\,(f-\bar{f})K_S$$

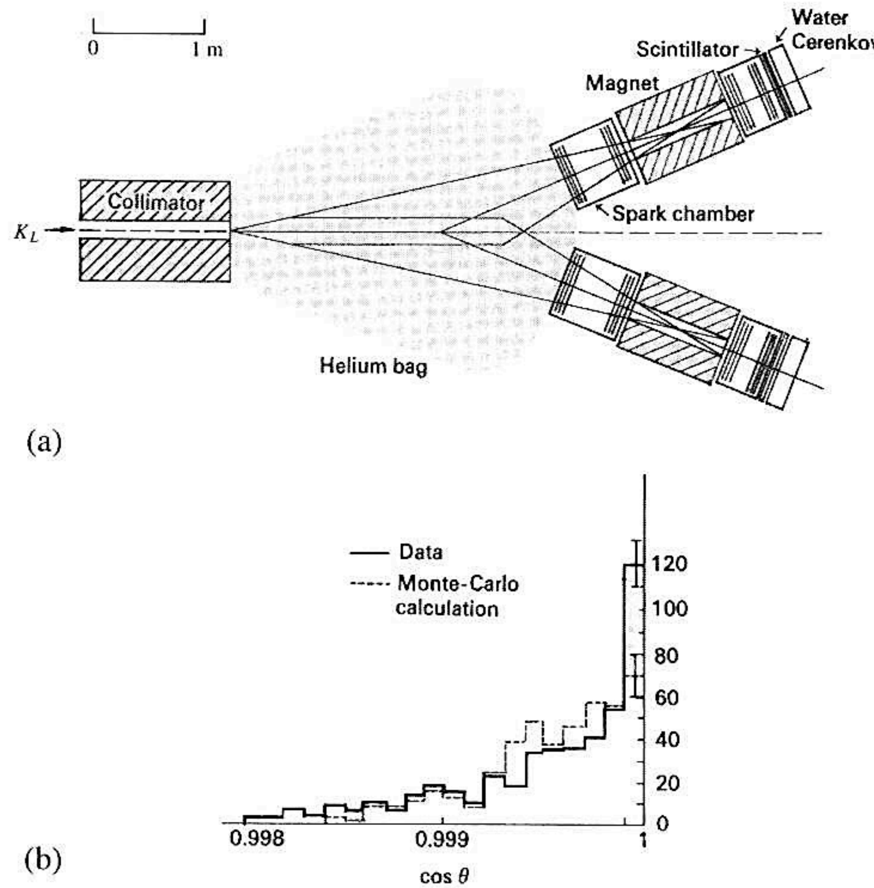
CP violation in the neutral kaon system

K_L is the CP=-1 eigenstate, and is not supposed to decay to two pions, just to three pions

1964 - experiment finds that K_L does decay to two pions with BR = 0.003

$$K_L \to \pi^+\pi^- \quad (0.002)$$
$$K_L \to \pi^0\pi^0 \quad (0.001)$$

This violates CP since K_L was is the CP=-1 eigenstate, but the CP of two pions is +1



(a)

(b)

CP violation in the neutral kaon system

Let K_1 represent the CP = +1 state and K_2 the CP = -1 state

$$K_L = \frac{1}{\sqrt{1+|\varepsilon|^2}}(K_2 + \varepsilon K_1)$$
$$K_S = \frac{1}{\sqrt{1+|\varepsilon|^2}}(K_1 - \varepsilon K_2)$$

ε is a small parameter quantifying the CP violation, which must be determined experimentally

$$|\eta_{+-}| = \frac{\text{ampl}(K_L \to \pi^+\pi^-)}{\text{ampl}(K_S \to \pi^+\pi^-)} = (2.29 \pm 0.02) \times 10^{-3}$$
$$|\eta_{00}| = \frac{\text{ampl}(K_L \to \pi^0\pi^0)}{\text{ampl}(K_S \to \pi^0\pi^0)} = (2.28 \pm 0.02) \times 10^{-3}$$

$|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$
$|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3}$
$|\eta_{00}| = (2.221 \pm 0.011) \times 10^{-3}$
2011

CP violation in the neutral kaon system

Interference between K_S and K_L Amplitudes

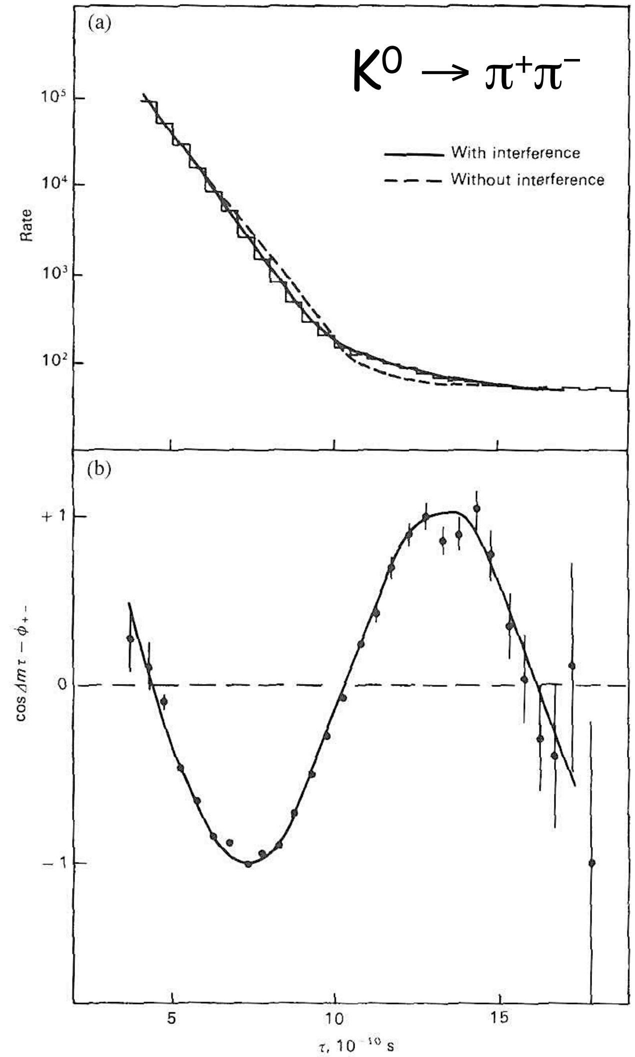
We've seen, starting from a pure K^0 beam:

$$I(K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

Now, K_L will contribute 2π with amplitude η_{+-}

$$\frac{I_{2\pi}(t)}{I_{2\pi}(0)} = e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2|\eta_{+-}|\, e^{-[(\Gamma_L+\Gamma_S)/2]t}\cos(\Delta m\, t + \phi_{+-})$$

where ϕ is the phase angle between the K_s and K_L amplitudes

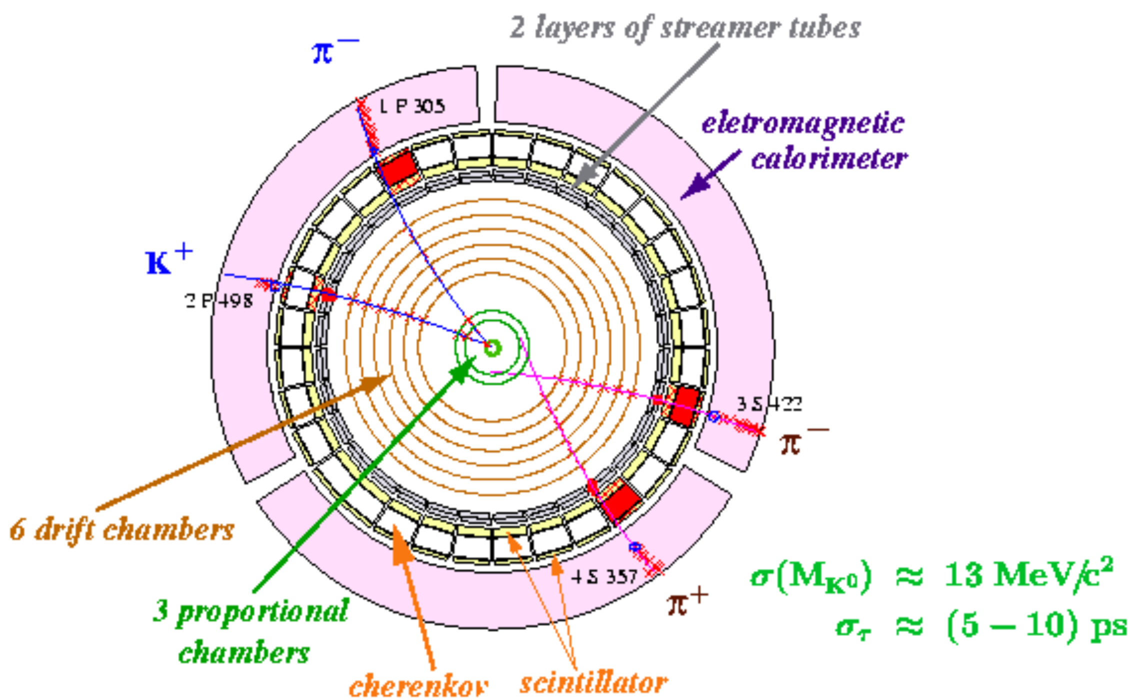


CP violation in the neutral kaon system

CPLEAR

$$p\bar{p} \to K^-\pi^+K^0$$
$$p\bar{p} \to K^+\pi^-\bar{K}^0$$

Measured asymmetry between K^0 and K^0



CP violation in the neutral kaon system

Interference between K_S and K_L Amplitudes

$$\frac{I_{2\pi}(t)}{I_{2\pi}(0)} = e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2|\eta_{+-}|\, e^{-[(\Gamma_L+\Gamma_S)/2]t} \cos(\Delta m\, t + \phi_{+-})$$

where ϕ is the phase angle between the K_s and K_L amplitudes

CPLEAR

$$p\bar{p} \to K^- \pi^+ K^0$$
$$p\bar{p} \to K^+ \pi^- \bar{K}^0$$

Measured asymmetry between K^0 and K^0

$$A_{+-}(t) = \frac{2|\eta_{+-}| e^{[(\Gamma_S-\Gamma_L)/2]t} \cos(\Delta m\, t + \phi_{+-})}{1 + |\eta_{+-}|^2 e^{(\Gamma_S-\Gamma_L)t}}$$

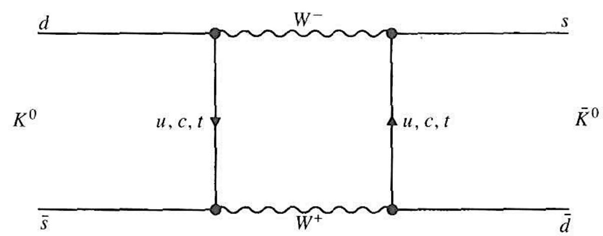
$$\phi_{+-} = 43.7^\circ \pm 0.6^\circ \qquad \phi_{00} = 43.5^\circ \pm 1.0^\circ$$

$\phi_{+-} = (43.51 \pm 0.05)^\circ$ $\phi_{00} = (43.52 \pm 0.06)^\circ$ (2010 PDG)

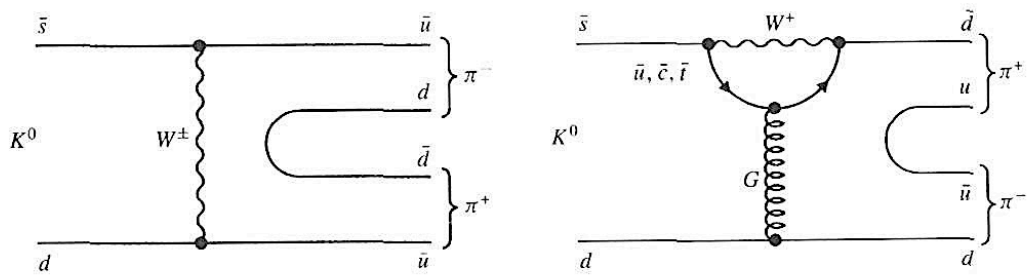
CP violation in the neutral kaon system

There are two possible sources of CP violation in the K^0 decay:

1. Indirect CP violation - admixture of ε



2. Direct CP violation $\Rightarrow$ ε', from interference



Penguin diagram

$$\eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}} \simeq \varepsilon + \varepsilon'$$
$$\eta_{00} = |\eta_{00}| e^{i\phi_{00}} \simeq \varepsilon - 2\varepsilon'$$

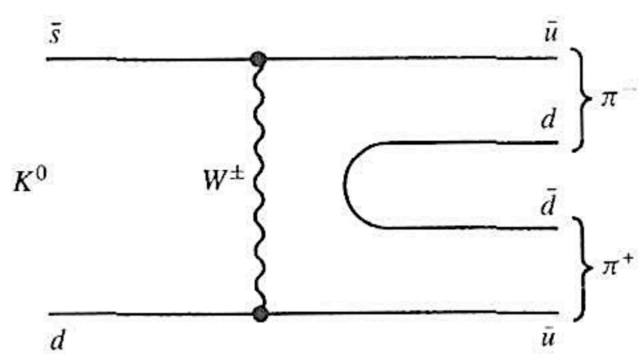
CP violation in the neutral kaon system

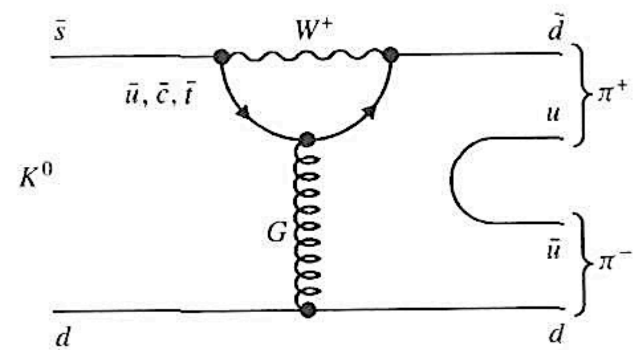
Measuring precisely ε'/ε has been a major effort at CERN and Fermilab for many years

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.65 \pm 0.26) \times 10^{-3} \qquad \text{(2010 PDG)}$$

Direct CP violation is established.

This is a puzzle for the Standard Model because the interference (and therefore ε') should be small with m_t = 175 GeV





CP violation in the neutral kaon system

CP Violation in leptonic decays

$$K_L \rightarrow e^+ + \nu_e + \pi^-$$
$$K_L \rightarrow e^- + \bar{\nu}_e + \pi^+$$

These decays transform into one another under the CP operation, so CP-invariance demands they be equal

$$\Delta = \frac{\text{rate}(K_L \rightarrow e^+ + \nu_e + \pi^-) - \text{rate}(K_L \rightarrow e^- + \bar{\nu}_e + \pi^+)}{\text{rate}(K_L \rightarrow e^+ + \nu_e + \pi^-) + \text{rate}(K_L \rightarrow e^- + \bar{\nu}_e + \pi^+)}$$
$$= (0.327 \pm 0.012) \times 10^{-2}$$

Since e^+ must come from K^0 and e^- from $\overline{K^0}$

$$\Delta = \frac{|1+\varepsilon|^2 - |1-\varepsilon|^2}{|1+\varepsilon|^2 + |1-\varepsilon|^2} \simeq 2\,\mathrm{Re}\,\varepsilon$$

$\Longrightarrow$ $\Delta(\varepsilon' = 0) = 2\eta\cos\phi = (0.332 \pm 0.004) \times 10^{-2}$

CP violation in the neutral kaon system

Superweak Interaction?

Wolfenstein postulated (1964) that CP violation arises from a new superweak interaction ($\Delta S = 2$)

an alternative to the six quark model explanation

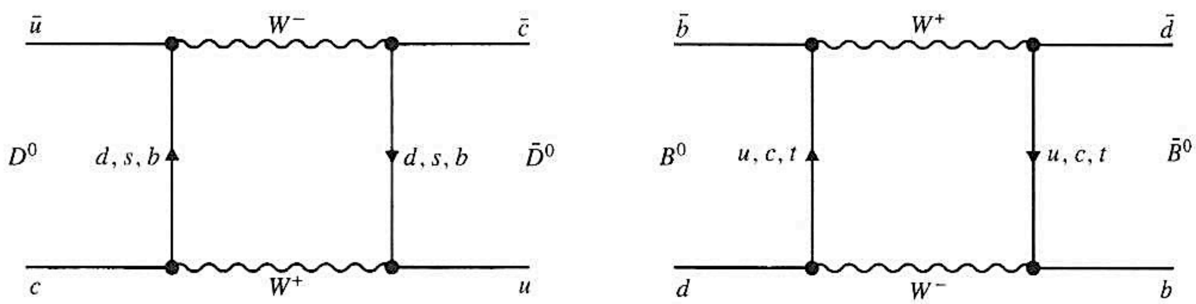
Prediction:

$\varepsilon' = 0$ (no Direct CP violation)

which now has been ruled out

D^0-$\overline{D^0}$ and B^0-$\overline{B^0}$ mixing

Mixing which is seen for Kaons, can also occur in charm and bottom mesons



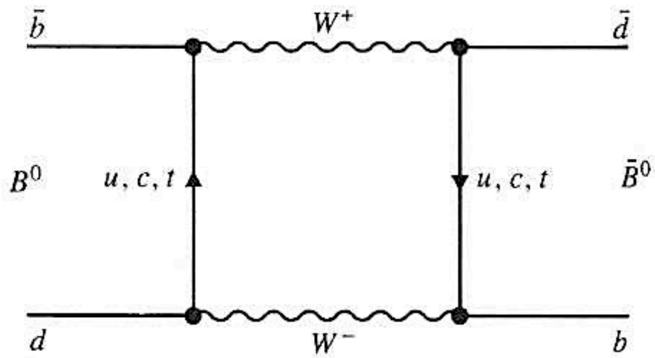
Mixing in the charm meson is expected to be very small:

the b quark diagram in D mixing depends on $|V_{cb}V_{ub}|^2 \approx |0.004 \times 0.04|^2 \approx |0.00016|^2 \approx 3 \times 10^{-8}$

and the s quark diagram is suppressed by the small value of the s quark mass

D^0-$\overline{D^0}$ and B^0-$\overline{B^0}$ mixing

Mixing in the bottom meson is dominated by the t quark exchange, and is substantial



for kaons we had $P(t)dt = \frac{\Gamma e^{-\Gamma t}}{2}\left[\frac{e^{-y\Gamma t}}{2} + \frac{e^{+y\Gamma t}}{2} \pm \cos x\Gamma t\right] dt$

$\Gamma = (\Gamma_S + \Gamma_L)/2 \approx \Gamma_S/2$; $y = (\Gamma_S - \Gamma_L) / 2\Gamma \approx 1$; $x = \Delta m / \Gamma \approx 0.95$

For Bs, $\Gamma = (\Gamma_S + \Gamma_L)/2$

$y = (\Gamma_S - \Gamma_L) / 2\Gamma \approx 0$ (B^0 and $\overline{B^0}$ decay to common channels)

$x = \Delta m / \Gamma$

$$P(t)dt = \frac{\Gamma e^{-\Gamma t}}{2}\left[1 \pm \cos x\Gamma t\right] dt$$

$\Rightarrow \quad x = 0.72$

D^0-$\overline{D^0}$ and B^0-$\overline{B^0}$ mixing

CKM matrix should be unitary

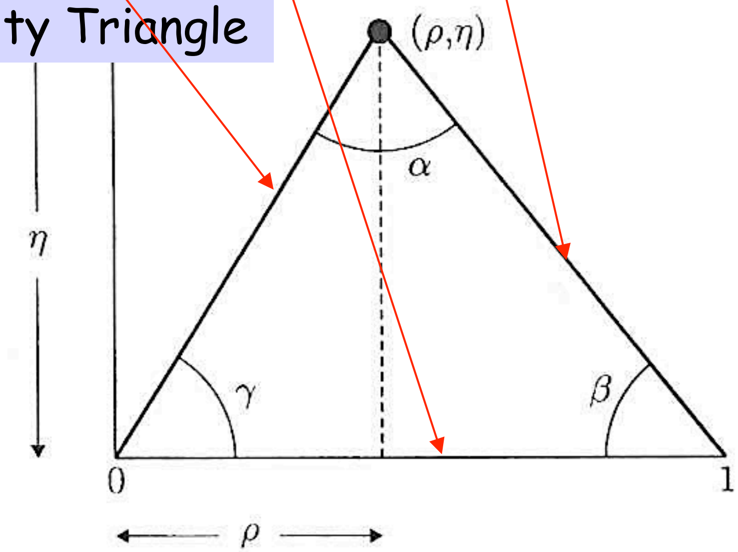
$$V_{\mathrm{CKM}} = \begin{vmatrix} V_{ud} = 0.975 & V_{us} = 0.221 & V_{ub} = 0.005 \\ V_{cd} = 0.221 & V_{cs} = 0.974 & V_{cb} = 0.04 \\ V_{td} = 0.01 & V_{ts} = 0.041 & V_{tb} = 0.999 \end{vmatrix}$$

Therefore off-diagonal terms will vanish, such as:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Unitarity Triangle

$$V_{\mathrm{CKM}} = \begin{vmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{vmatrix}$$



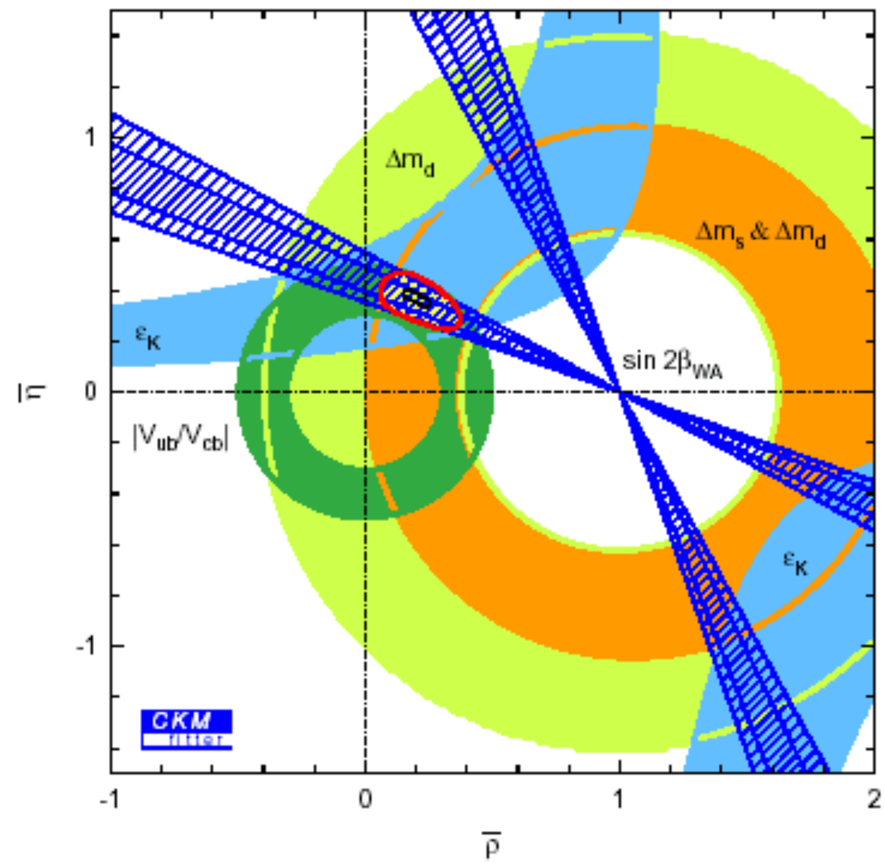
CP Violation in the B System

CP violation shows up as large asymmetries in decay modes with very small BRs

eg. $B^0 \rightarrow J/\psi \; K_S^0$

$$R \propto e^{-t/\tau}(1 \pm \sin 2\beta \sin \Delta m\, t)$$

sin 2β = 0.679 ± 0.020
http://www.slac.stanford.edu/xorg/hfag/



D^0 mixing revisited

CP eigenstates

$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$, $|q/p| = 1$

$M_{1,2} = M \pm \Delta M$

$\Gamma_{1,2} = \Gamma \pm \Delta\Gamma$

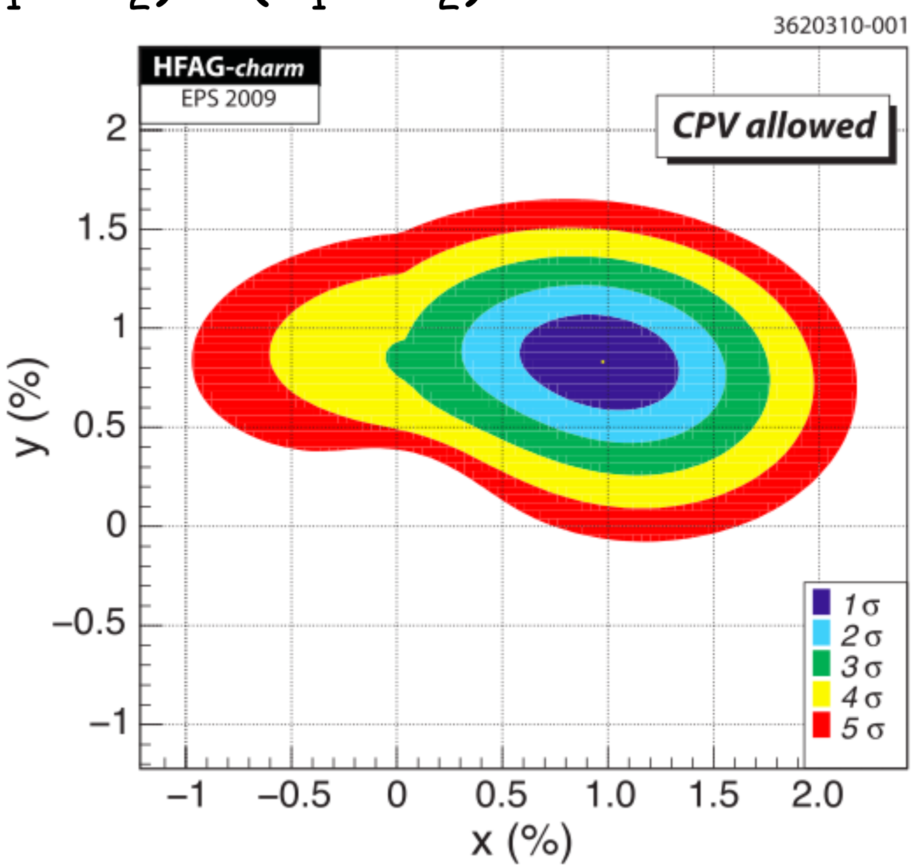
$x = 2\,\Delta M / (\Gamma_1 + \Gamma_2)$

$y = (\Gamma_1 - \Gamma_2) / (\Gamma_1 + \Gamma_2)$

D^0 mixing revisited

$x = 2\,\Delta M / (\Gamma_1 + \Gamma_2)$

$y = (\Gamma_1 - \Gamma_2) / (\Gamma_1 + \Gamma_2)$



Cosmological CP violation

1966 - Sakharov

showed that three conditions are required for the baryon asymmetry of the Universe to have developed

1) B-violating interactions
2) a non-equilibrium situation
3) CP and C violation

However, the CP violation observed in the K and B systems is not Sufficiently large to explain the baryon asymmetry

additional CP violation must occur
could it be from the lepton sector?
or possibly from very high mass new physics?