

Physics 661

Particle Physics Phenomenology

November 4, 2003

Invariance Principles and Conservation Laws

- Outline
 - Translation and rotation
 - Parity
 - Charge Conjugation
 - Charge Conservation and Gauge Invariance
 - Baryon and lepton conservation
 - CPT Theorem
 - CP violation and T violation
 - Isospin symmetry

Conservation Rules

<u>Conserved quantity</u>	Interaction		
	<u>strong</u>	<u>EM</u>	<u>weak</u>
energy-momentum	yes	yes	yes
charge			
baryon number			
lepton number			
CPT	yes	yes	yes
P (parity)	yes	yes	no
C (charge conjugation parity)	yes	yes	no
CP (or T)	yes	yes	10^{-3} violation
I (isospin)	yes	no	no

Discrete and Continuous Symmetries

- Continuous Symmetries
 - Space-time symmetries
 - Lorentz transformations
 - Poincare transformations
 - combined space-time translation and Lorentz T.
- Discrete symmetries
 - cannot be built up from succession of infinitesimally small transformations
- Other symmetries
 - Dynamical symmetries
 - Internal symmetries

Conservation Laws

- Continuous symmetries lead to additive conservation laws:
 - energy
- Discrete symmetries lead to multiplicative conservation laws
 - parity

Translation and Rotation

- Invariance of the energy of an isolated physical system under space translations leads to conservation of linear momentum
- Invariance of the energy of an isolated physical system under spatial rotations leads to conservation of angular momentum
- Noether's Theorem

E. Noether, "Invariante Variationsprobleme", Nachr. d. König. Gesellsch. d. Wiss. zu Göttingen, Math-phys. Klasse (1918), 235-257;

English translation M. A. Trapp, Transport Theory and Statistical Physics 1(3) 1971,183-207.

Parity

- Spatial inversion
 - $(x,y,z) \rightarrow (-x,-y,-z)$
 - discrete symmetry
- $P \psi(r) = \psi(-r)$
 - P is the parity operator
- $P^2 \psi(r) = P \psi(-r) = \psi(r)$,
 - therefore $P^2 = 1$ and the parity of an eigen-system is 1 or -1

Parity

- Example, the spherical harmonics:

$$Y_L^M$$

- The spherical harmonics describe a state in a spherically symmetric potential with definite ang. momentum

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$$

- $p = (-1)^L$

$$Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi}$$

Parity

- Parity is a multiplicative quantum number
- Composite system:
 - parity is equal to product of the parts
 - eg. two pions in an angular momentum L state
$$P = P(\pi_1) \cdot P(\pi_2) \cdot (-1)^L$$
- Intrinsic parity of particles:
 - proton and neutron (+1 by definition, could be -1)
 - parity of fermion and anti-fermion are opposite, and assignment of one is arbitrary
 - pion (-1, measured)

Pion spin and parity

- 1. Spin of the charged pion
 - $p + p \leftrightarrow \pi^+ + d$ ($|M_{if}|^2 = |M_{fi}|^2$)
 - then, "detailed balance" gives:
$$\sigma(pp \rightarrow \pi^+ d) \sim (2s_\pi + 1)(2s_d + 1)p_\pi^2$$
$$\sigma(\pi^+ d \rightarrow pp) \sim 1/2 (2s_p + 1)^2 p_p^2$$
(1/2 because protons are identical)
 - Measurements of σ 's reveals $s_\pi = 0$
- 2. Spin of the neutral pion
 - $\pi^0 \rightarrow \gamma\gamma$ shows it must be 0 (see following)

Pion spin and parity

- 2. Spin of the neutral pion
 - $\pi^0 \rightarrow \gamma\gamma$ shows it must be 0
 - along the flight path of the γ s in the π^0 rest frame, the total photon spin (S_z) must be 0 or 2
 - If $S_\pi = 1$, then S_z must be 0
 - If $S_\pi = 1$, $S_z = 0$, the two-photon amplitude must behave as $P_1^{m=0}(\cos \theta)$, which is antisymmetric under interchange ($=180^\circ$ rotation)
 - This corresponds to two right or left circularly polarized photons travelling in opposite directions, which must be symmetric by Bose statistics $\rightarrow$ forbidden $S_\pi \neq 1$
 - Therefore, $S_\pi = 0$ or $S_\pi \geq 2$ (which is ruled out by pion production statistics - all three charges produced equally)

Parity

- Parity of the charged pion
 - the observation of the reaction
$$\pi^- + d \rightarrow n + n$$
is evidence that the charged pion has $p = -1$
- Capture takes place from an s-state, $L_i=0$ ($S_d=1$)
 - (X-ray emissions following capture)
- In the two neutron system, $L+S$ must be even by the antisymmetric requirement on identical fermions
 - thus, $(2S+1)J_L = {}^3P_1$ state with $p = (-1)^L = -1$
 - since initial state is $p_d p_\pi (-1)^{L_i} = (+1) p_\pi$, $p_\pi = -1$

Parity

- Parity of the neutral pion

$\pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-)$ (rare decay of the pion)

- the planes of the pairs follow the E vectors of the internally converted photons ($\pi^0 \rightarrow \gamma\gamma$)

- even system of two photons (bosons)

$$A \sim (\mathbf{E}_1 \cdot \mathbf{E}_2) \quad (P=+1)$$

$$\text{or } A \sim (\mathbf{E}_1 \times \mathbf{E}_2) \cdot \mathbf{k} \quad (P=-1)$$

- Intensities or rates $\sim |A|^2$

$$|A|^2 \sim \cos^2 \phi$$

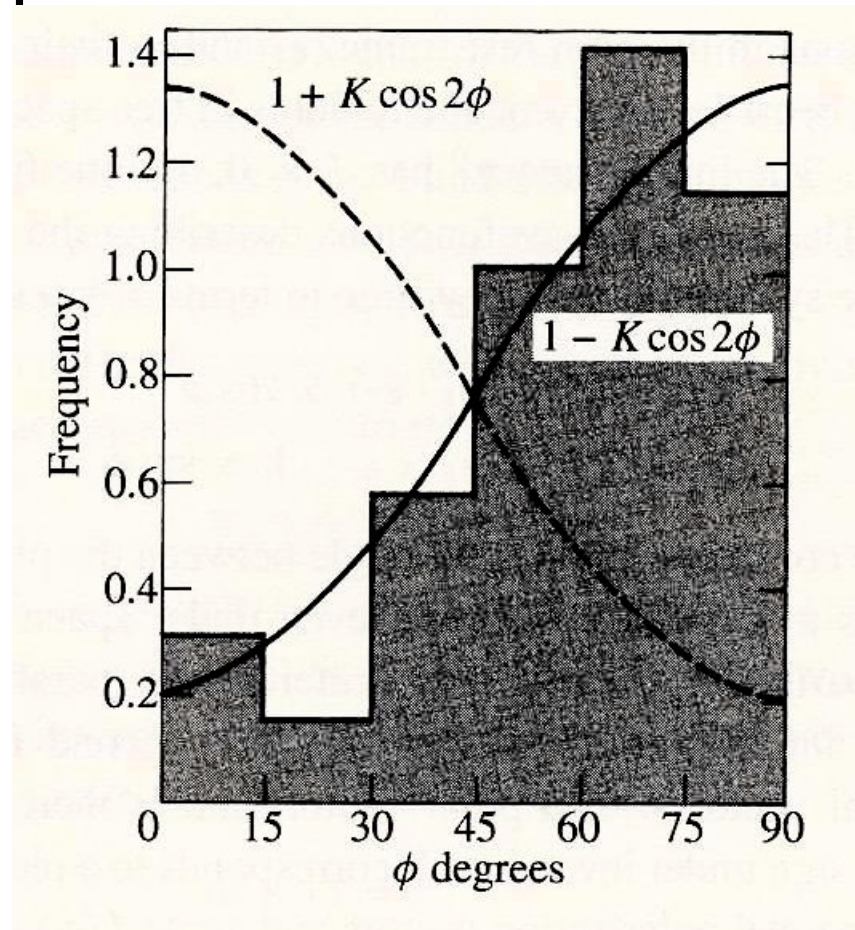
$$\text{or } |A|^2 \sim \sin^2 \phi$$

Parity

- Parity of the neutral pion

$$\pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-)$$

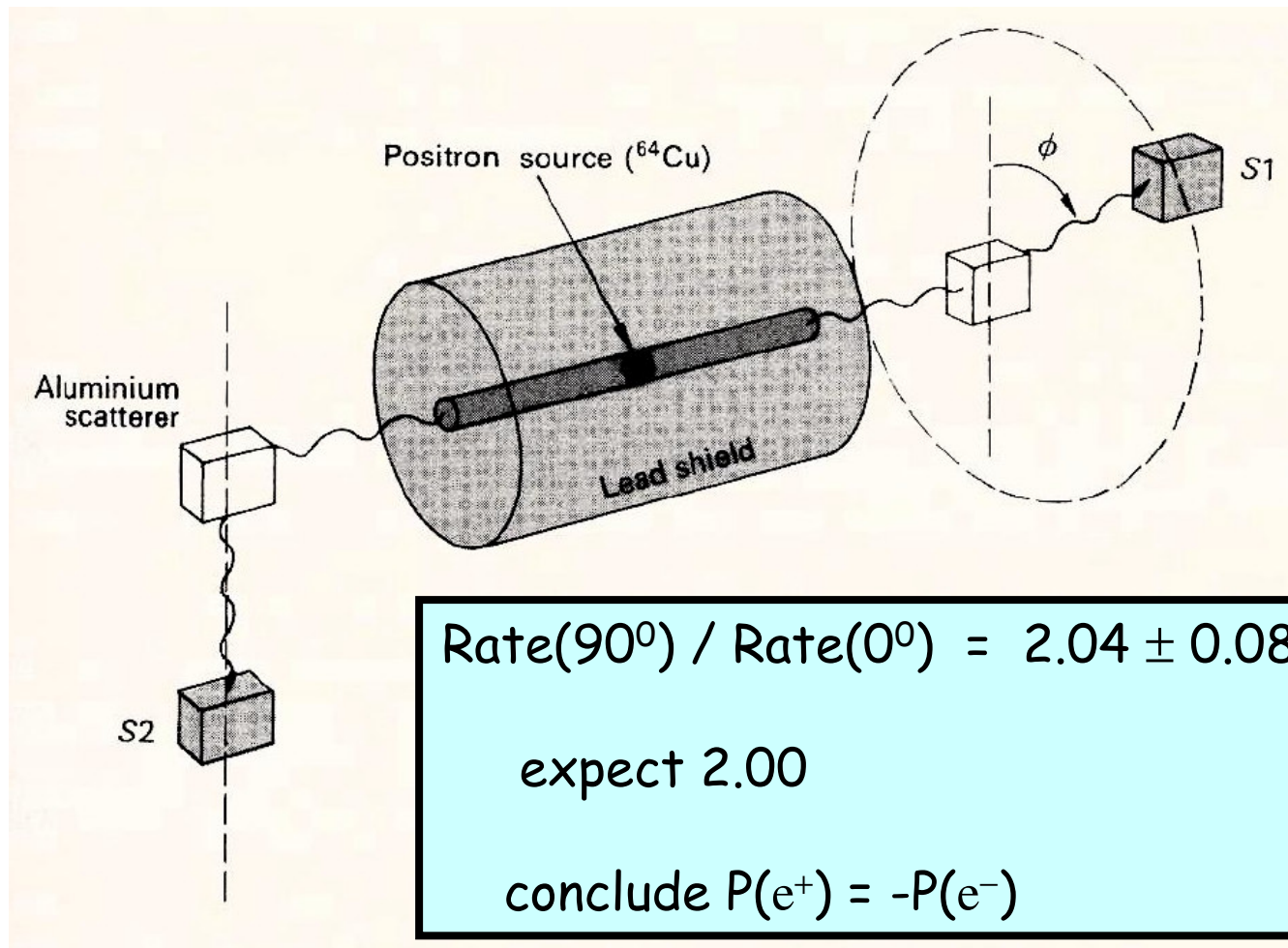
scalar(0^+) would follow dashed line,
while pseudoscalar(0^-) would follow solid line



Parity of particles and antiparticles

- Dirac theory required fermions and antifermions to have opposite intrinsic parity
- This was checked in decays of the spin singlet ground state of positronium
$$e^+e^- (^1S_0) \rightarrow 2 \gamma$$
$$P = (+1) (-1) (-1)^0 = -1$$
- This is exactly the case of the π^0 decay
 - the photon polarizations must have a $\sin^2\phi$ form

Parity of particles and antiparticles

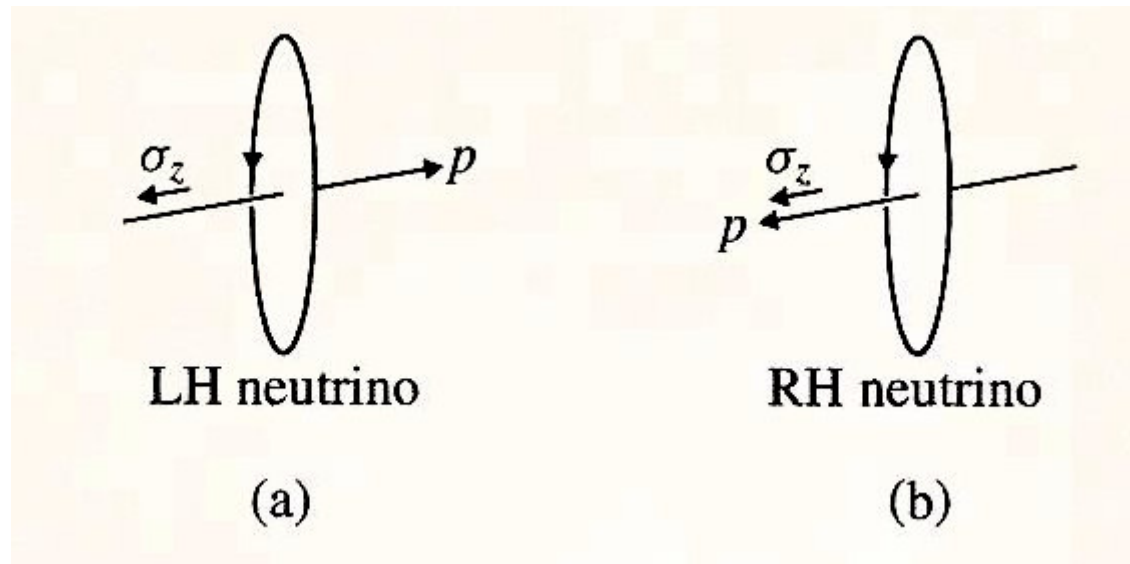


Parity of particles and antiparticles

- Two-particle systems
 - Fermion-antifermion
 - $p = (-1)^{L+1}$
 - eg, pion which is q-antiq with $L=0$ has $p = -1$
 - Boson-antiboson
 - same parity
 - $p = (-1)^L$
 - eg. $\rho \rightarrow \pi^+ \pi^-$
 $1^- \rightarrow 0^- 0^-$, $L=1$

Tests of parity conservation

- Strong and EM interactions conserve parity, but weak interactions do not



- LH neutrino observed, but RH neutrino is not
 - this is a maximally violated symmetry

Tests of parity conservation

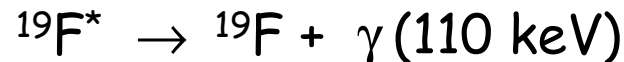
- In interactions dominated by the strong or the EM interaction, some parity violation may be observed due to the small contribution of the weak int. to the process

$$H = H_{\text{strong}} + H_{\text{EM}} + H_{\text{weak}}$$

- In nuclear transitions, for example, the degree of parity violation will be of the order of the ratio of the weak to the strong couplings ($\sim 10^{-7}$)

Tests of parity conservation

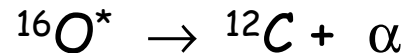
- Examples of nuclear transitions
 - fore-aft asymmetry in gamma emission



$$J^P: 1/2^- \rightarrow 1/2^+$$

$$\Delta \sim 10^{-4}$$

- very narrow decay



$$J^P: 2^- \rightarrow 2^+$$

$$\Gamma = 10^{-10} \text{ eV} \quad (\text{note } ^{16}\text{O}^* \rightarrow ^{16}\text{O} + \gamma, \Gamma = 3 \times 10^{-3} \text{ eV})$$