Physics 661

Particle Physics Phenomenology

October 28, 2003

Interactions and Fields

- Quantum Picture of Interactions ✓
- Yukawa Theory ✓
- Boson Propagator ✓
- Feynman Diagrams ✓
- Electromagnetic Interactions ✓
- Renormalization and Gauge Invariance ✓
- Strong Interactions ✓
- Weak and Electroweak Interactions
- Gravitational Interactions
- Cross-sections
- Decays and resonances
- The Δ^{++} pion-proton resonance
- The Z^0 resonance
- Resonances in astrophysics

Weak and Electroweak Interactions

- The ratios of the decay rates give an approximate indication of the relative strengths of the interactions:

$$\{\tau[\Sigma^0\,(1192)\,]\,/\,\tau[\Sigma^+\,(1189)\,]\,\}^{1/2} \approx 10^{-5} \approx \alpha_{weak}\,/\,\alpha$$

- All leptons and quarks “feel” the weak force, but it is small that it is swamped by the strong and EM forces, unless they are forbidden by some conservation law

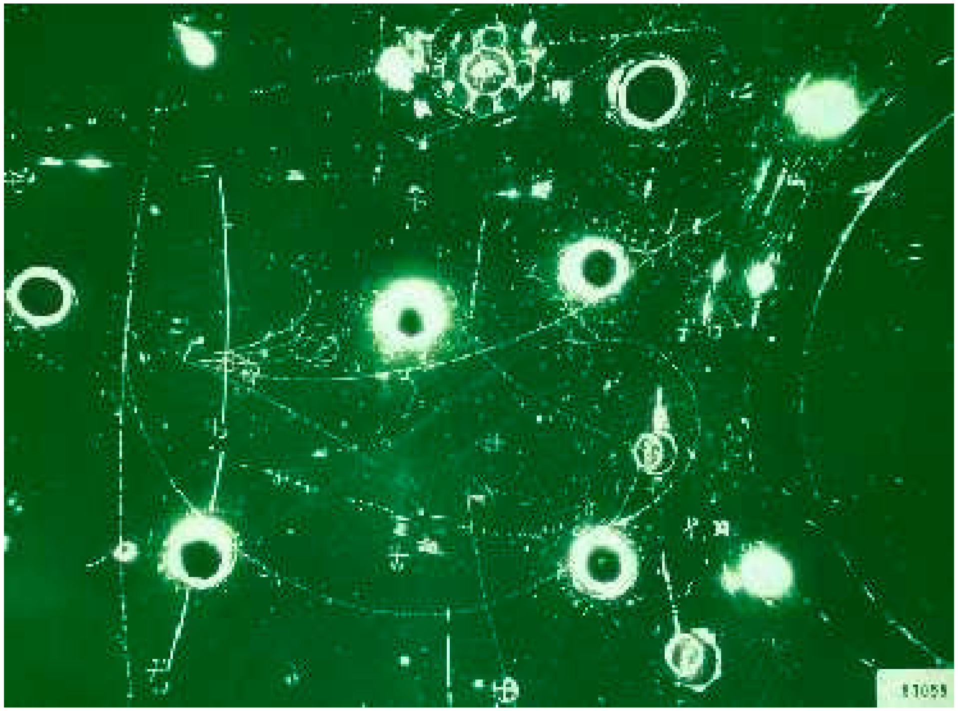
Weak and Electroweak Interactions

- Nuclear β-decay
- Lepton conservation
- Weak due to very heavy mediating vector bosons:

    $W^{\pm}$, 80 GeV/c^2

    Z^0 , 91 GeV/c^2

- charge-current
    - β-decay
- neutral-current



Weak and Electroweak Interactions

- Simplfied picture of the Weak Interaction:
  - Propagator:

    $f(q) = g^2 / (q^2 + M_{W,Z}{}^2)$
  - for $q^2 << M^2$, $f(q) = g^2 / M_{W,Z}{}^2$

  - Fermi's early theory of β-decay postulated an interaction with strength $G = 10^{-5}$, which we now recognize as $G = g^2 / M_{W,Z}{}^2 = 10^{-5}$

  - Once we recognize the origin of the weakness, we can predict the masses of the W and Z

    $M_{W,Z}{}^2 \sim e/\sqrt{G} \sim \sqrt{4\pi\alpha/G} \sim 90$ GeV
    - Glashow, Weinberg, Salam (1961-8)

Gravitational Interactions

- Gravity is one of the four forces of Nature, but its strength is so small that is not important in accelerator experiments:
  - Force between two equal point masses M:
    - $F_N = G_n M^2 / r^2$
    - compared to EM $F_{EM} = e^2 / r^2$
    - so gravitational constant to be compared to fine structure constant is

      $G_n M^2 / 4\pi\hbar c = 5.3 \times 10^{-40}$ for $M = 1$ GeV/c^2
  - The coupling constant would approach unity for $M > (\hbar c/G_N)^{1/2} = 1.22 \times 10^{19}$ GeV/c^2, the Planck mass

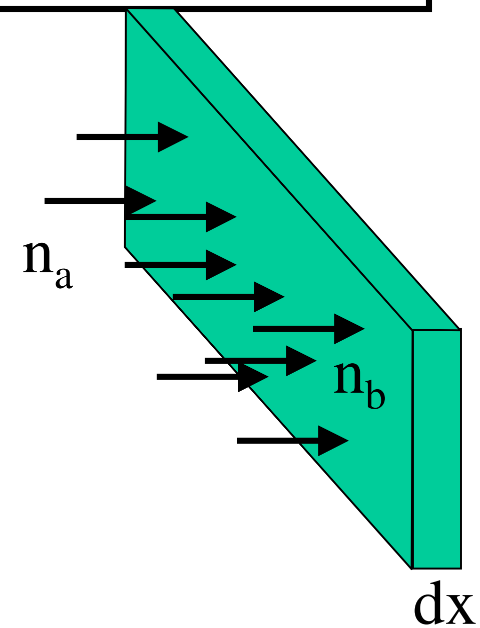
Gravitational Interactions

- Quantum gravitational effects become important at lengths such that the gravitational energy is comparable to the mass
  - $V_N = G_n M_P^2 / r = M_P c^2$
  - $r_P = G_n M_P / c^2 = \hbar / M_P c \approx 2 \times 10^{-20}$ fm $= 2 \times 10^{-35}$ m

The Interactions

	Gravity	Electromag.	Weak	Strong
field boson	graviton	photon	$W^{\pm}$,Z	gluon
spin-parity	2^+	1^-	1^-, 1^+	1^-
mass, GeV	0	0	M_W=80.2 M_Z=91.2	0
range, m	∞	∞	10^{-18}	$\leq 10^{-15}$
source	mass	electric charge	'weak' charge	'color' charge
coupling constant	5×10^{-40}	1/137	1.2×10^{-5}	≤ 1
typical cross-section, m^2		10^{-33}	10^{-39}	10^{-30}
typical lifetime, s		10^{-20}	10^{-10}	10^{-23}

Cross-sections



- Reaction, such as $a + b \rightarrow c + d$
- Flux = $n_a v_i$
  - n_a = density of particles in beam
  - v_i = velocity of particles in beam
- Target
  - n_b = density of particles in target
  - dx = thickness of target
- Reaction rate per target particle (W) = [Flux] • [cross section(σ)]
  - σ = W / Flux
  - 1 barn $\equiv 10^{-28}$ m^2
- Reactions per time per area of beam
  - [Flux] n_b dx [cross section(σ)]

Cross-sections

- barn = 10^{-28} m^2
- millibarn = mb = 10^{-31} m^2
- microbarn = μb = 10^{-34} m^2
- nanobarn = nb = 10^{-37} m^2
- picobarn = pb = 10^{-40} m^2
- femtobarn = fb = 10^{-43} m^2

Cross-sections

- Fermi's Second Golden Rule (non-rel QM) gives the reaction rate, W:

$$W = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_f$$

where $M_{if} = \int \psi_f^* U \psi_i \, dV$

and $\rho_f = dN/dE$, is the density of final states

Cross-sections

- The phase space available to a particle (neglecting spin) is:

$$dN = \frac{V}{(2\pi\hbar)^3} p^2 dp d\Omega$$

The volume factor cancels with the normalization of the wavefunctions:

$$\frac{d\sigma}{d\Omega} = \frac{W}{\phi_i} = \frac{W}{v_i} = \frac{2\pi}{\hbar} \frac{|M_{if}|^2}{v_i} \frac{1}{(2\pi\hbar)^3} p_f^2 \frac{dp_f}{dE_0}$$

Cross-sections

- Phase space factor
  - density of states in 1D:
    - $N = L\,p / h$
  - in 3D:
    - $N = \int (dx)^3 (dp)^3 / (h)^3 = V / (h)^3 \int p^2\, dp\, d\Omega$
    -

$$dN = \frac{V}{(2\pi\hbar)^3} p^2 dp d\Omega$$

Cross-sections

- Considering $a + b \to c + d$, energy conservation in the CMS says

$$\sqrt{p_f^2 + m_c^2} + \sqrt{p_f^2 + m_d^2} = E_0$$

$1/(2\,E_c)\,2\,p_f\,dp_f + 1/(2\,E_d)\,2\,p_f\,dp_f = dE_0$

$p_f\,dp_f\,[1/(2E_c) + 1/(2\,E_d)\,] = dE_0$

$dp_f/dE_0 = E_cE_d/[p_f(E_c+E_d)] = E_cE_d/(p_fE_0)$

$$\frac{dp_f}{dE_0} = \frac{E_cE_d}{E_0p_f} = \frac{1}{v_f}$$

Cross-sections

$$\frac{d\sigma}{d\Omega}(a+b\to c+d)=\frac{1}{4\pi^2\hbar^4}|M_{if}|^2\frac{p_f^2}{v_i v_f}$$

- Now let's consider the spin of the particles
- We must average over the number of initial state spin configurations:

$$g_i = (2s_a+1)(2s_b+1)$$

- and sum over the final states:

$$g_f = (2s_c+1)(2s_d+1)$$

$$\frac{d\sigma}{d\Omega}(a+b\to c+d)=\frac{1}{4\pi^2\hbar^4}|M_{if}|^2\frac{p_f^2\, g_f}{v_i v_f\, g_i}$$

Cross-sections

- "Crossed reactions"

$$a + b \rightarrow c + d$$

$$a + \bar{c} \rightarrow \bar{b} + d$$

$$a + \bar{d} \rightarrow c + \bar{b}$$

$$a \rightarrow \bar{b} + c + d$$

$$c + d \rightarrow a + b$$

- The rates are described by the same matrix element but different kinematics

Decays and resonances

- Consider the decay of a particle:
- From uncertainty principle, natural width and lifetime are related:

$$\Gamma = \frac{\hbar}{\tau} = \hbar W = 2\pi |M|^2 \int \rho_f d\Omega$$

$$\Gamma = -\hbar \frac{dN_A}{dt} \frac{1}{N_A}$$

Decays and resonances

- Total width:

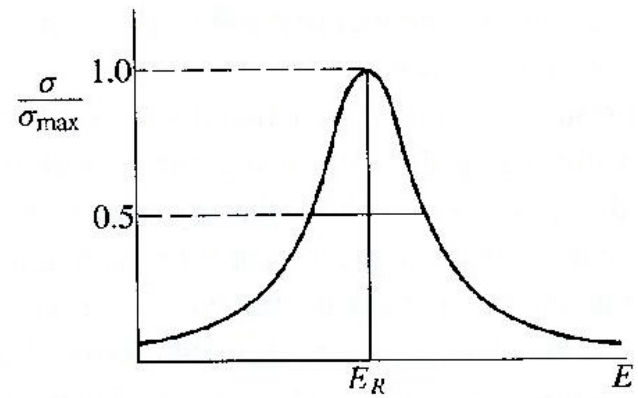
$$\Gamma = \sum_i \Gamma_i$$

- Particle survival with time:

$$N_A(t) = N_A(0)\exp\left(-\frac{\Gamma t}{\hbar}\right)$$

Breit-Wigner Distribution

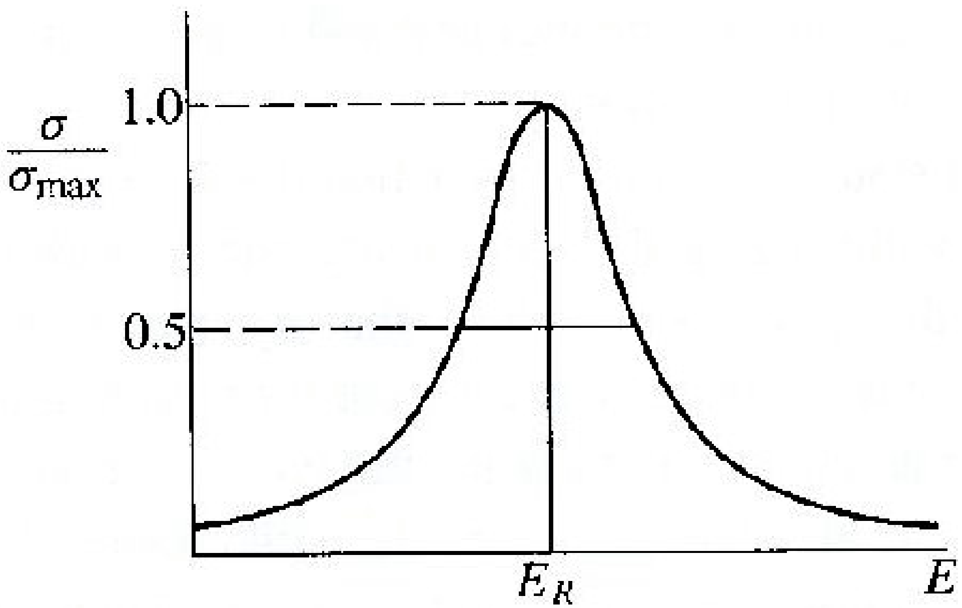
- When two particles collide, they can form an unstable, broad state, known as a "resonance".
- The cross section which measures the probability of forming the resonant state follows the Breit-Wigner distribution:

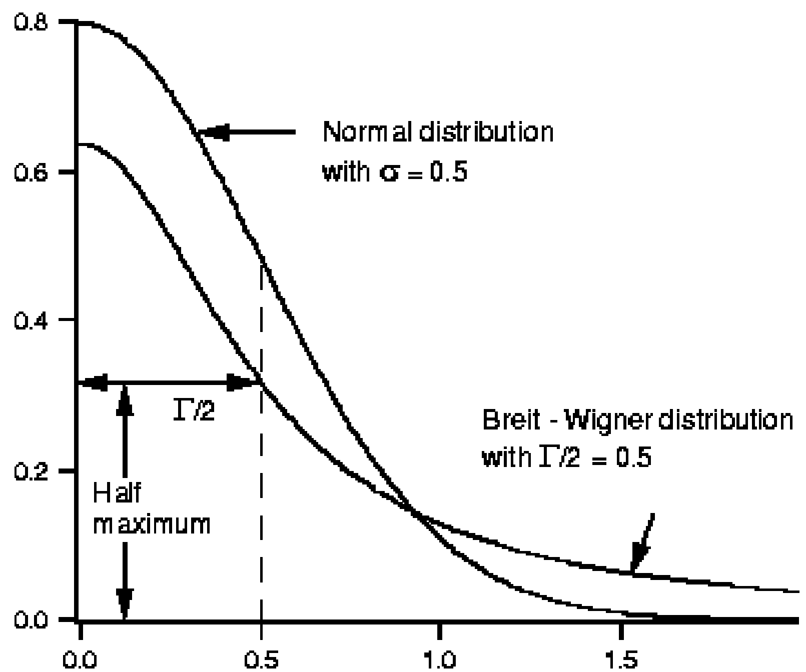


$$\sigma(E) = \sigma_{max} \frac{\Gamma^2/4}{(E - E_R)^2 + \Gamma^2/4}$$

Breit-Wigner Distribution

$$\sigma(E) = \sigma_{\max} \frac{\Gamma^2/4}{(E - E_R)^2 + \Gamma^2/4}$$





Breit-Wigner Distribution

- $\psi(t) = \psi_0 \, e^{-\Gamma/2t} \, e^{-iEo\, t}$
- Fourier Transform:
- $\psi(E) = \int \psi(t)\, e^{iEt}\, dt = \int \psi_0\, e^{-\Gamma/2t}\, e^{-iEo\, t}\, e^{iEt}\, dt$
  $= \int \psi_0\, e^{i[(E - Eo) + i\Gamma/2]t}\, dt$
- $\psi(E) \sim 1/[E-E_0 + i\Gamma/2]$
- $|\, \psi(E)\, |^2 \sim 1/[(E-E_0)^2+\Gamma^2/4]$

Breit-Wigner Distribution

- When spin and angular momentum are taken into consideration

$$\sigma = \frac{4\pi \lambda\!\!\!^{-2}(2J+1)}{(2s_a+1)(2s_b+1)} \frac{\Gamma^2/4}{[(E-E_R)^2+\Gamma^2/4]}$$

where λ is the wavelength of the scattered and scattering particle in their common CMS

Breit-Wigner Distribution

$$\sigma = \frac{4\pi\bar{\lambda}^2(2J+1)}{(2s_a+1)(2s_b+1)} \frac{\Gamma^2/4}{[(E-E_R)^2+\Gamma^2/4]}$$

- Elastic scattering

$$\sigma = \frac{4\pi\bar{\lambda}^2(2J+1)}{(2s_a+1)(2s_b+1)} \frac{\Gamma_{el}^2/4}{[(E-E_R)^2+\Gamma^2/4]}$$

- General scattering (i $\rightarrow$ j)

$$\sigma = \frac{4\pi\bar{\lambda}^2(2J+1)}{(2s_a+1)(2s_b+1)} \frac{\Gamma_i\Gamma_j/4}{[(E-E_R)^2+\Gamma^2/4]}$$

The Δ^{++} pion-proton resonance

$$\pi^+ p \rightarrow \Delta^{++}(1232) \rightarrow \pi^+ p$$

- $J = 3/2$
- $\Gamma = 120$ MeV
- $\sigma_{max} = 8\pi ƛ^2$

since $s_a = 1/2$,

$s_b = 0$, and $J = 3/2$:

$$\frac{(2J+1)}{(2s_a+1)(2s_b+1)} = 2$$

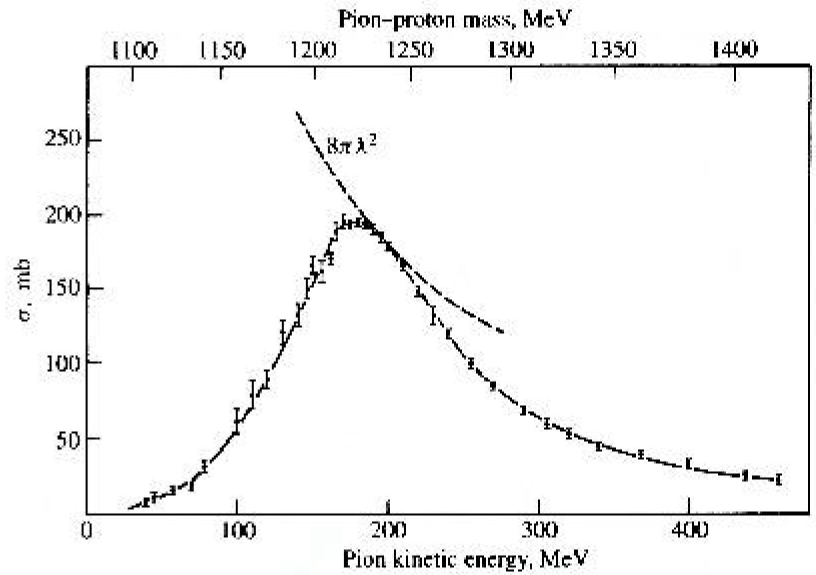


Fig. 2.11. The $\pi^+ p$ elastic scattering cross-section in the region of the $\Delta^{++}(1232)$ resonance. The central mass is 1232 MeV and the width is $\Gamma = 120$ MeV. Note that the formula (2.28) holds strictly for a narrow resonance. For a broad resonance with width comparable with the central mass, such as the Δ^{++}, the final-state phase-space factor varies appreciably over the width so that, in comparison with Figure 2.10, the resonance curve appears asymmetric.

The Z^0 resonance

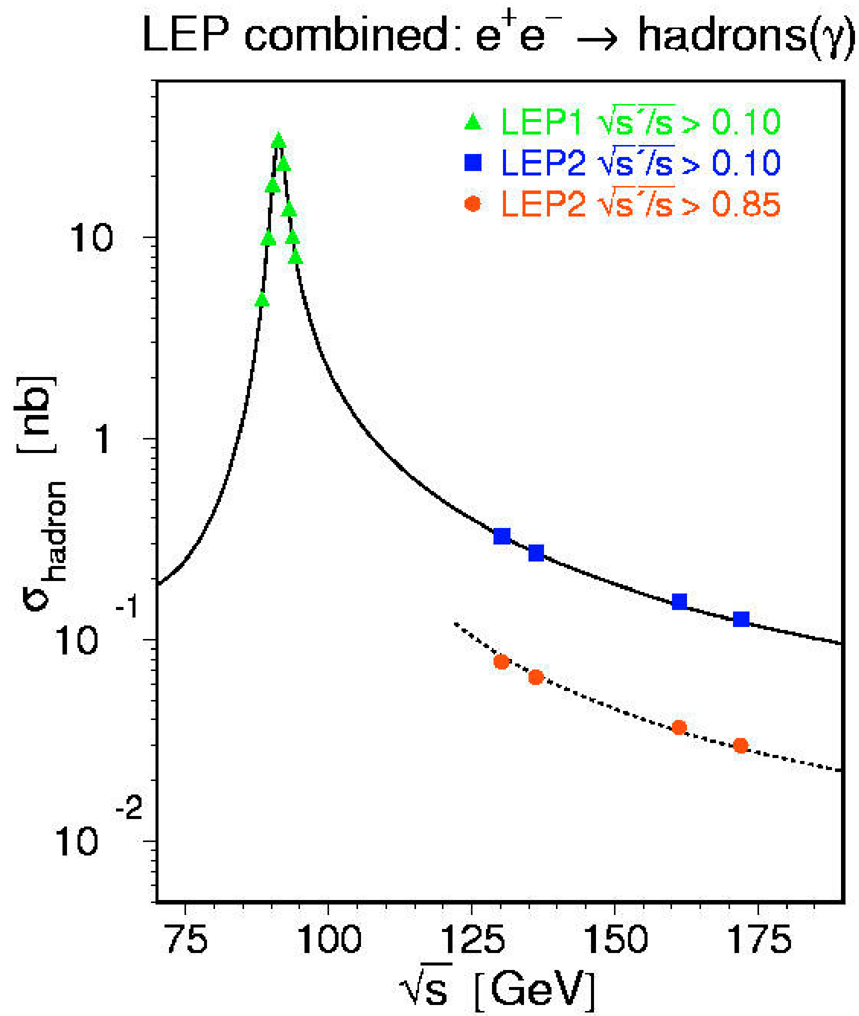
- M_Z = 91 GeV
- Γ_Z = 2.5 GeV

- The Z decays to many different final states:

  q anti-q

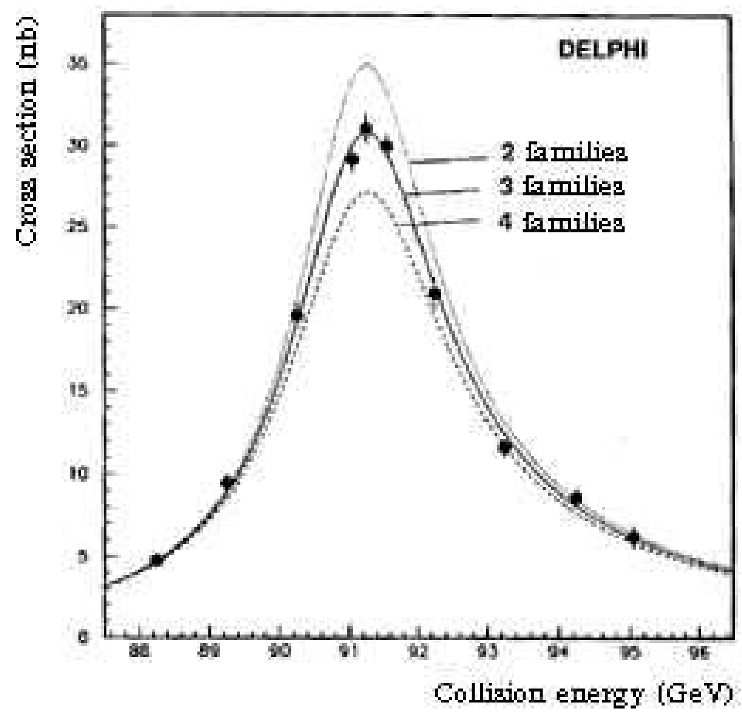
  $l^+ \ l^-$

  ν anti-ν



The Z^0 resonance

- Neutrino counting:
  - the more neutrino families there are, the large the cross section and the broader the resonance
  - $N_\nu = 2.99 \pm 0.01$



Resonances in Astrophysics

- ^{12}C production in stars
  - helium burning red giant stars produce carbon through the triple alpha process:

    $3\,\alpha \rightarrow {}^{12}C$
  - Competing processes

    $\alpha + \alpha \rightarrow {}^{8}Be$
    - lifetime of ^{8}Be is 10^{-16} sec

    $\alpha + {}^{8}Be \rightarrow {}^{12}C^{*}$ (7.654 MeV resonance with $\Gamma \approx 10$ eV)
    - $^{12}C^{*}$ may decay back to $\alpha + {}^{8}Be$
    - or with small prob (4×10^{-4}) decay to ^{12}C (ground state)
  - ^{12}C production depends crucially on the existence of this resonance (Hoyle, 1953)

Resonances in Astrophysics

- ^{12}C production in stars
  - in principle carbon could be burnt up in stars during a further stage of stellar evolution through the process

$$\alpha + {}^{12}C \rightarrow {}^{16}O$$

  - the absence of an appropriate resonance in oxygen preserves the carbon in the universe