

Physics 661

Particle Physics Phenomenology

October 15, 2003

Quarks and Leptons

- Elem. Particles and High Energy Physics ✓
- Fixed Targets and Colliding Beams ✓
- The Standard Model ✓
 - fundamental fermions, interactions, time scales
- Limitations of the Standard Model ✓
- Fermions and Bosons ✓
- Particles and Antiparticles ✓
- Free Particles and their wave equations
- Helicity States
- Lepton Flavors
- Quark Flavors

Free particles and their wave equations

- Bosons (Klein-Gordon equation)

$$\partial^2 \psi / \partial t^2 = (\nabla^2 - m^2) \psi$$

this equation follows from $E^2 = p^2 + m^2$,
from the substitutions of the operators

$$E = i\hbar \partial / \partial t, \quad p = -i\hbar \nabla$$

Free particles and their wave equations

- Fermions (Dirac equation)

$$E\psi = (\alpha \cdot \mathbf{p} + \beta m) \psi, \text{ where } E \text{ \& } p \text{ are op.}$$

$$\alpha_k = \begin{bmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{bmatrix} \quad \beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

notice this equation is linear in both space and time, unlike the Schrödinger equation

Free particles and their wave equations

- The Dirac equation equation can be re-expressed in covariant form:

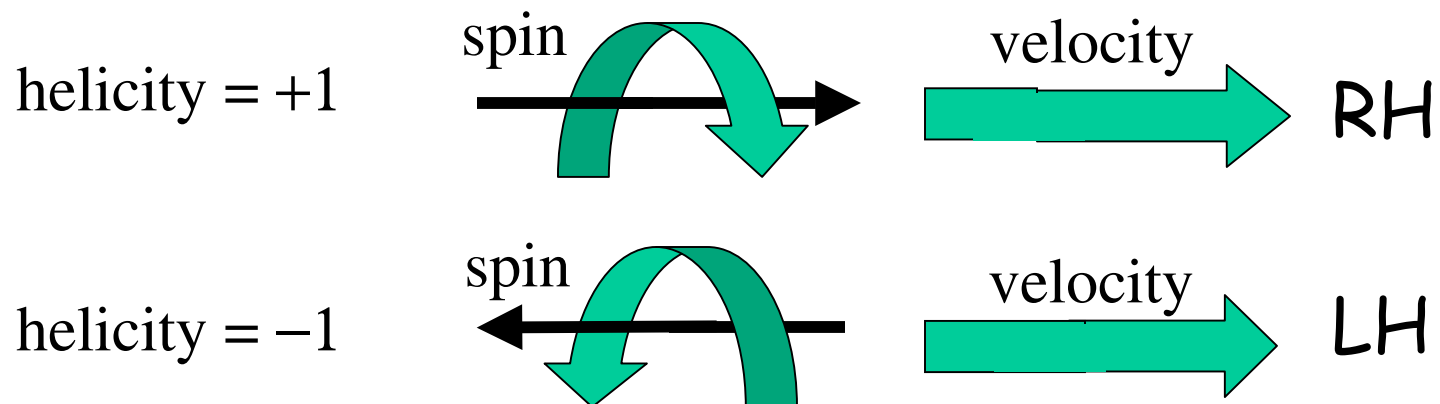
$$(i\gamma_\mu \partial/\partial x_\mu - m) \psi = 0$$

$$\gamma_k = \beta \alpha_k = \begin{bmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{bmatrix}, \quad k = 1, 2, 3$$

$$\gamma_4 = \beta$$

Helicity States

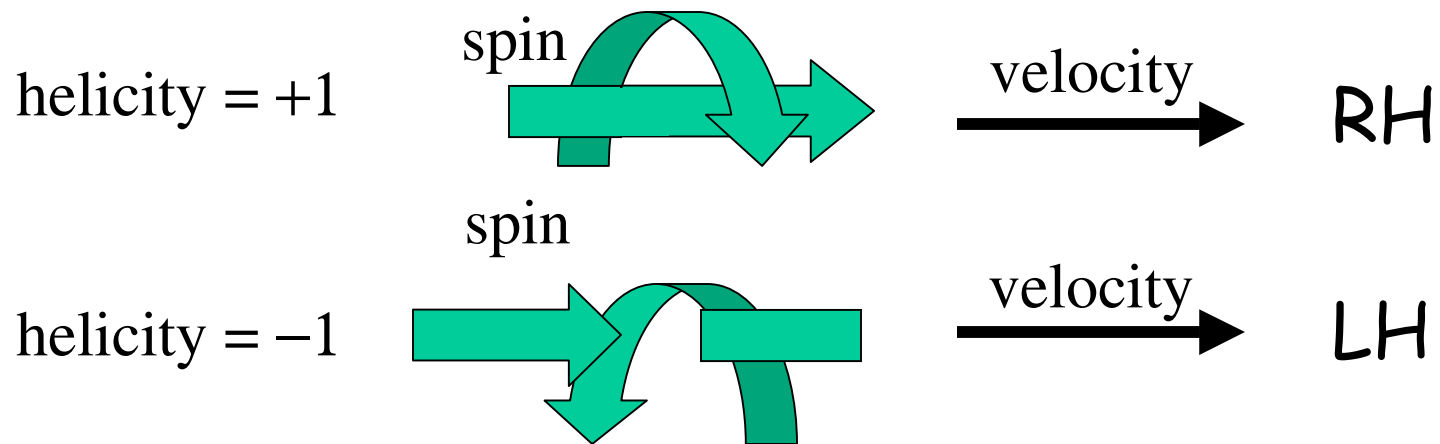
- For a massless fermion of positive energy, the helicity is a good quantum number



- Massive particles are admixtures of LH and RH components
 - at high energy, mass becomes less significant
 - neutrinos are nearly massless

Helicity States

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Helicity States

- Interactions involving vector (eg. EM) or axial vector (1^+) conserve helicity in the relativistic limit
- Since Strong, EM, and Weak all involve vector or axial vector fields, helicity will be conserved in the high energy limit

Leptons Flavors

	Strong	EM	Weak	Gravity
e		✓	✓	✓
μ		✓	✓	✓
τ		✓	✓	✓
ν_e			✓	✓
ν_μ			✓	✓
ν_τ			✓	✓

Leptons Flavors

- The Standard Model includes massless neutrinos
 - left-handed neutrinos and RH antineutrinos
- Lepton flavors are conserved in interactions
 - L_e, L_μ, L_τ
 - $\text{mass}(\mu) = 106 \text{ MeV}/c^2$
 - $\text{mass}(e) = 0.5 \text{ MeV}/c^2$
 - However $\mu \rightarrow e \gamma$ is forbidden (exp: $\text{BR} < 1.2 \times 10^{-11}$)
- Neutrino oscillations are indications that the neutrinos have small masses and that flavor conservation will be violated at a small level

Quark Flavors

	Strong	EM	Weak	Gravity
u	✓	✓	✓	✓
d	✓	✓	✓	✓
c	✓	✓	✓	✓
s	✓	✓	✓	✓
t	✓	✓	✓	✓
b	✓	✓	✓	✓

Quark Flavors

	Quantum Number	Rest Mass, GeV
up	-	$m_u \approx m_d \approx 0.31$
strange	$S = -1$	$m_s \approx 0.50$
charm	$C = +1$	$m_c \approx 1.60$
bottom	$B = -1$	$m_b \approx 4.6$
top	$T = +1$	$m_t \approx 175$

Particles comprised of these quarks will have masses determined roughly by the rest masses of the quarks (internal interactions alter the mass some)

The strong interaction conserves each of the flavor quantum numbers

Quark Flavor Conservation

- Strange production and decay is a good example of how the quark flavor conservation law works
- Strange particles are always produced in pairs in strong interactions:

$$\pi^- p \rightarrow \Lambda K^0$$

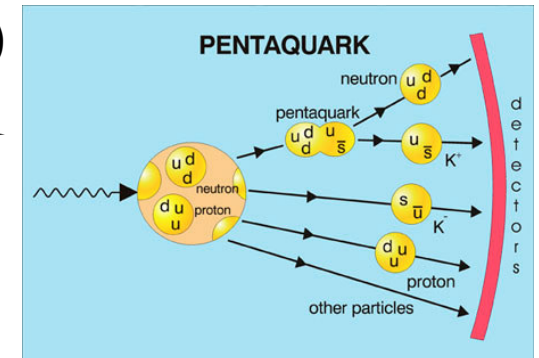
- $S(\Lambda) = -1$, contains strange quark
 - $S(K) = +1$, contains antistrange quark
 - this is a strong interaction and therefore has a large cross section
- In the decays of strange particles, the strangeness may vanish:
$$\Lambda \rightarrow p \pi^- \quad (\{S = -1\} \rightarrow \{S = 0\})$$
 - this process is not allowed by the strong interaction, therefore must involve the weak interaction, and is therefore slow.

Baryons and Mesons

- Quarks have never been observed in isolation in nature, only in bound states of three quarks or a quark and an antiquark

- baryon = QQQ {three quark state}
- meson = $Q\bar{Q}$ {quark antiquark pair}
- (and recently, perhaps, in pentaquark states)

- Baryons and mesons are collectively referred to as hadrons and comprise all of the strongly interaction particles



- Baryons have half-integer spin and mesons integer spin

Quark Mass

- There are two ways of describing the quark mass:
 - constituent quark mass (used in Perkins)
 - $m(u) \approx 0.31 \text{ GeV}$ $m(d) \approx 0.31 \text{ GeV}$
 - current quark mass (table in earlier lecture)
 - $m(u) \approx 0.003 \text{ GeV}$ $m(d) \approx 0.006 \text{ GeV}$
- Consider the mass of the two light mesons, the pion and the rho, both composed of ud
 - $m(\pi) = 0.14 \text{ GeV}$
 - $m(\rho) = 0.77 \text{ GeV}$
 - constituent quark picture requires large spin-orbit interaction energy to explain difference

Interactions and Fields

- Quantum Picture of Interactions
- Yukawa Theory
- Boson Propagator
- Feynman Diagrams
- Electromagnetic Interactions
- Renormalization and Gauge Invariance
- Strong Interactions
- Weak and Electroweak Interactions
- Gravitational Interactions
- Cross-sections
- Decays and resonances
- The Δ^{++} pion-proton resonance
- The Z^0 resonance
- Resonances in astrophysics

Quantum Picture of Interactions

- Quantum Theory views action at a distance through the exchange of quanta associated with the interaction
- These exchanged quanta are virtual and can “violate” the conservation laws for a time defined by the Uncertainty Principle:
 - $\Delta E \Delta t \cong \hbar$

Yukawa Theory

- During the 1930's, Yukawa was working on understanding the short range nature of the nuclear force ($R \approx 10^{-15}\text{m}$)
- He postulated that this was due to the exchange of massive quanta which obey the Klein-Gordon eqtn:

$$\partial^2 \psi / \partial t^2 = c^2 (\nabla^2 - m^2 c^2 / \hbar^2) \psi$$



Yukawa Theory

$$\partial^2 \psi / \partial t^2 = c^2 (\nabla^2 - m^2 c^2 / \hbar^2) \psi$$

for a static potential, this becomes:

$$\nabla^2 \psi = (m^2 c^2 / \hbar^2) \psi$$

We can interpret ψ as the potential $U(r)$ and solve for U :

$$U(r) = g_0 e^{-r/R} / 4\pi r,$$

$$\text{where } R = \hbar/mc,$$

and g_0 is a constant (the strength)

Yukawa Theory

The range of the nuclear force was known,

$$R \approx 10^{-15} \text{m}$$

Therefore, the mass of this new exchange particle could be predicted:

$$R = \hbar/mc,$$

$$mc^2 = \hbar c/R \approx 200 \text{ MeV-fm}/1 \text{ fm} \approx 200 \text{ MeV}$$

- The pion with mass $140 \text{ MeV}/c^2$ was discovered in 1947! (the muon was discovered in 1937 and misidentified as Yukawa's particle, the "mesotron")
- We now realize that this interaction is actually a residual interaction, so Yukawa was a bit fortunate to find a particle with the predicted mass