Physics 661

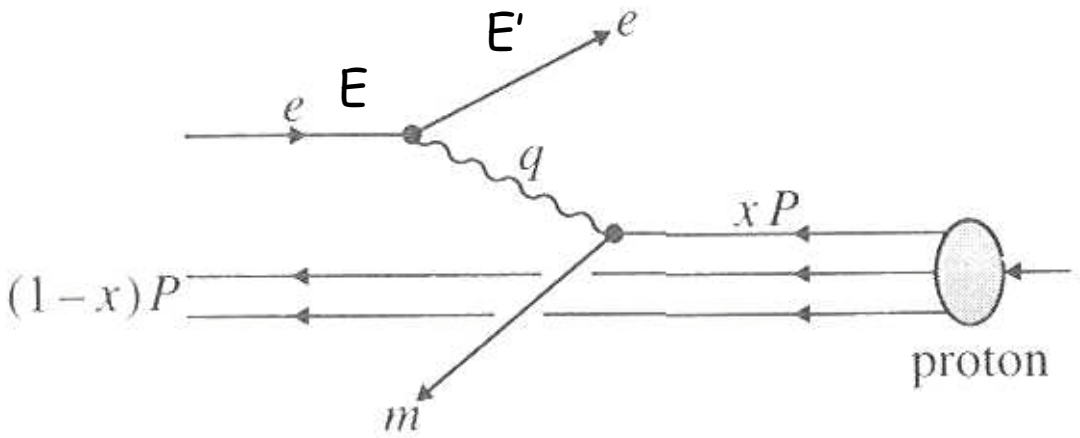
Particle Physics Phenomenology

December 4, 2003

Lepton and Quark Scattering

- $e^+ e^- \rightarrow \mu^+ \mu^-$ ✓
- $e^+ e^-$ annihilation to hadrons ($e^+ e^- \rightarrow Q\overline{Q}$) ✓
- Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$ ✓
- Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$ ✓
- Elastic lepton-nucleon scattering ✓
- Deep inelastic scattering and partons ✓
- Deep inelastic scattering and quarks ✓
  - Electron-nucleon scattering
  - Neutrino-nucleon scattering
- Quark distributions within the nucleon
- Sum rules

Deep Inelastic Scattering and Partons



$$x = q^2/2M\nu$$

(ν is the energy transfer in the lab)

$$\nu = E - E'$$

$$q^2 = (\vec{p} - \vec{p}')^2 - (E - E')^2$$

Deep Inelastic Electron-Nucleon Scattering and Quarks

$$\frac{d^2\sigma(en)}{dydx} = \frac{4\pi\,\alpha^2\,xs}{q^4}\,F_2^{en}(x)\,\left[\frac{1+(1-y)^2}{2}\right]$$

$$F_2^{ep}(x) = x\,\{4/9[u(x)+\bar{u}(x)]+1/9[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)]\}$$

$q(x)\,dx$ = number of q quarks carrying momentum fraction $x \rightarrow x+dx$

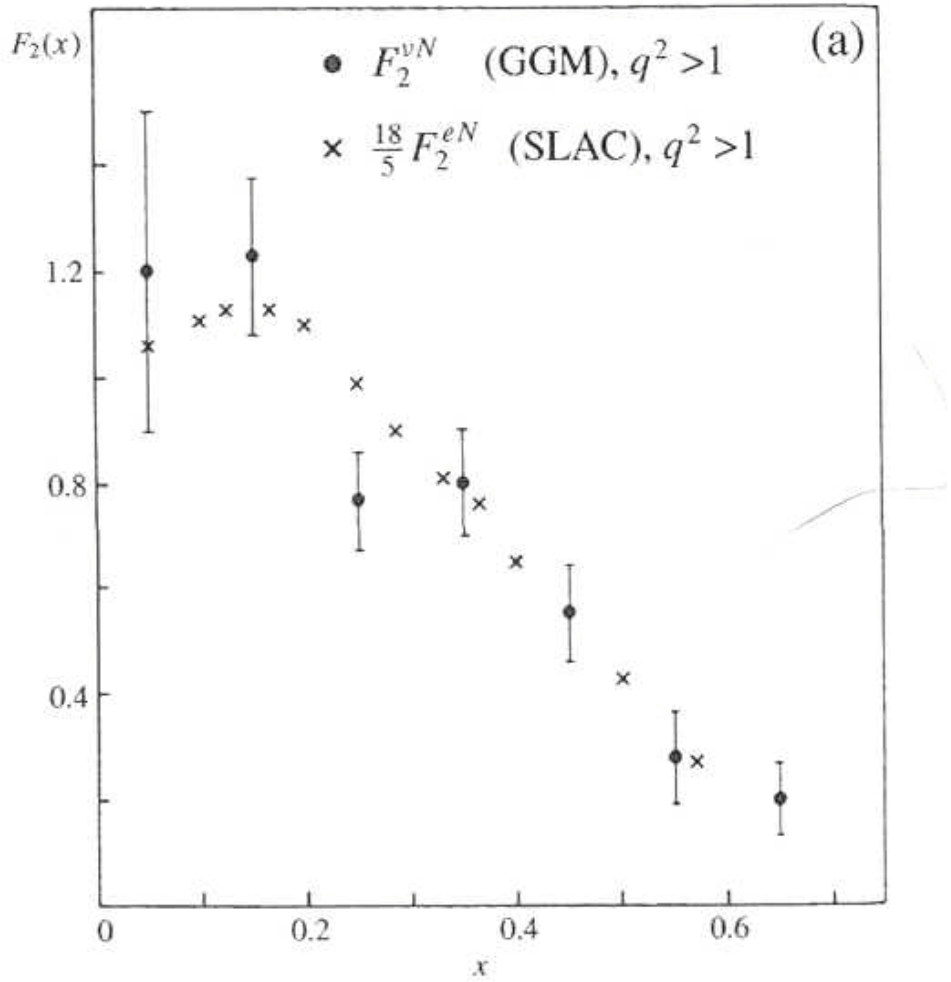
$$F_2^{en}(x) = x\,\{4/9[d(x)+\bar{d}(x)]+1/9[u(x)+\bar{u}(x)+s(x)+\bar{s}(x)]\}$$

by isospin invariance

for a target with equal number of protons and neutrons:

$$F_2^{eN}(x) = x\,\{5/18[d(x)+\bar{d}(x)+u(x)+\bar{u}(x)]+1/9[s(x)+\bar{s}(x)]\}$$

Deep Inelastic Scattering and Quarks



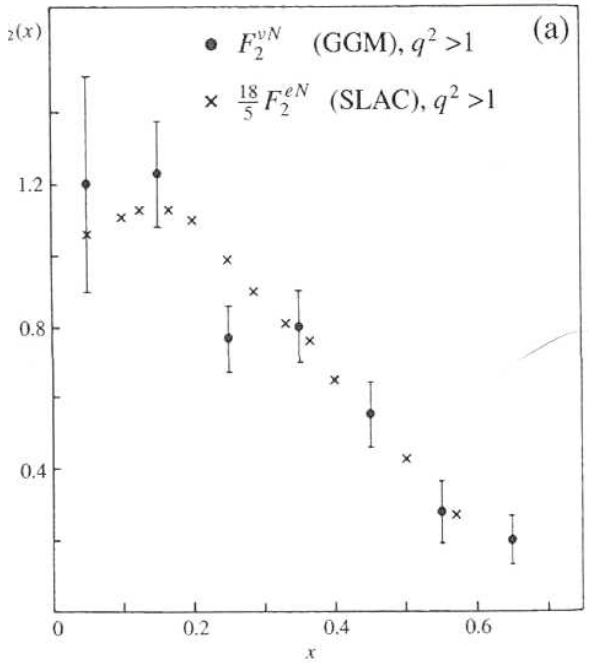
Deep Inelastic Neutrino-Nucleon Scattering and Quarks

Electron-nucleon scattering is sensitive to the parton charge, showing we have +2/3 and -1/3 quarks.

Neutrino nucleon scattering is not sensitive to charge.

Theory predicts:

$$F_2^{eN}(x) = 5/18\ F_2^{\nu N}(x)$$



Deep Inelastic Electron-Nucleon Scattering and Quarks

Consider a model of the partons

$u(x)\,dx$ = number of u quarks carrying momentum fraction $x \to x+dx$

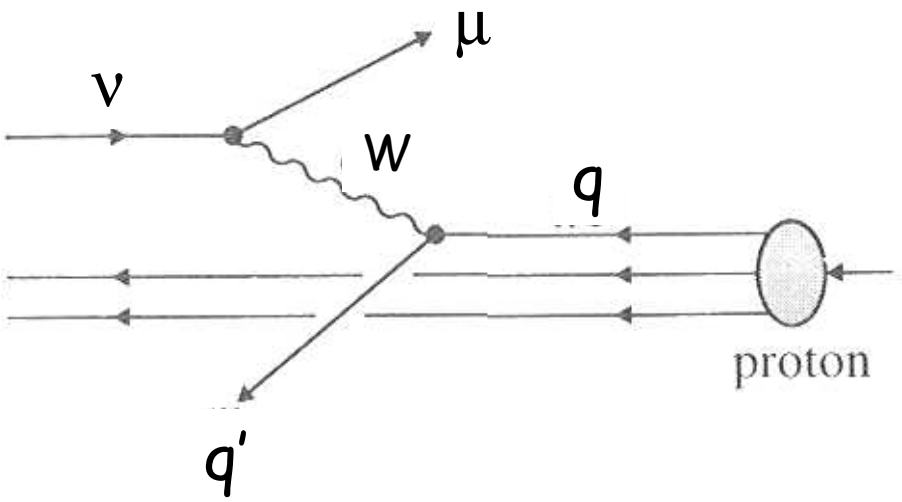
$d(x)\,dx$ = number of d quarks carrying momentum fraction $x \to x+dx$

Cross section depends on charge squared:

$$\frac{d^2\sigma(ep)}{dydx} = \frac{4\pi\,\alpha^2\,xs}{q^4}\,\frac{F_2^{ep}(x)}{x}\left[\frac{1+(1-y)^2}{2}\right]$$

$$F_2^{ep}(x) = x\{4/9[u(x)+\bar{u}(x)]+1/9[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)]\}$$

Deep Inelastic Neutrino-Nucleon Scattering and Quarks



Dominant charged current interactions

$\nu_\mu + d \rightarrow \mu^- + u$ (W^+)

$\nu_\mu + \bar{u} \rightarrow \mu^- + \bar{d}$ (W^+)

$\bar{\nu}_\mu + u \rightarrow \mu^+ + d$ (W^-)

$\bar{\nu}_\mu + \bar{d} \rightarrow \mu^+ + \bar{u}$ (W^-)

Deep Inelastic Neutrino-Nucleon Scattering and Quarks

left-handed right-handed

$$d\sigma(\nu p)/dydx = G^2xs/\pi\,[d(x) + \overline{u}(x)(1-y)^2]$$

- through isospin invariance:

$$d\sigma(\nu n)/dydx = G^2xs/\pi\,[u(x) + \overline{d}(x)(1-y)^2]$$

left-handed right-handed

- for isoscalar targets (# p = # n)

$$d\sigma(\nu N)/dydx = G^2xs/2\pi\,\{[u(x) + d(x)] + [\overline{u}(x) + \overline{d}(x)](1-y)^2]\}$$

- and

$$d\sigma(\bar{\nu}N)/dydx = G^2xs/2\pi\,\{[u(x) + d(x)](1-y)^2 + [\overline{u}(x) + \overline{d}(x)]]\}$$

Deep Inelastic Neutrino-Nucleon Scattering and Quarks

- Define the neutrino structure functions:

$F_2^{\nu N}(x) = x\,[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)]$

$F_3^{\nu N}(x) = x\,[u(x)-\bar{u}(x)+d(x)-\bar{d}(x)]$

Then

$$d\sigma^{\nu N,\,\bar{\nu}N}/dydx = G^2ME/\pi\left\{[F_2 \pm F_3]/2 + [F_2 \mp F_3](1-y)^2/2]\right\}$$

Defining:

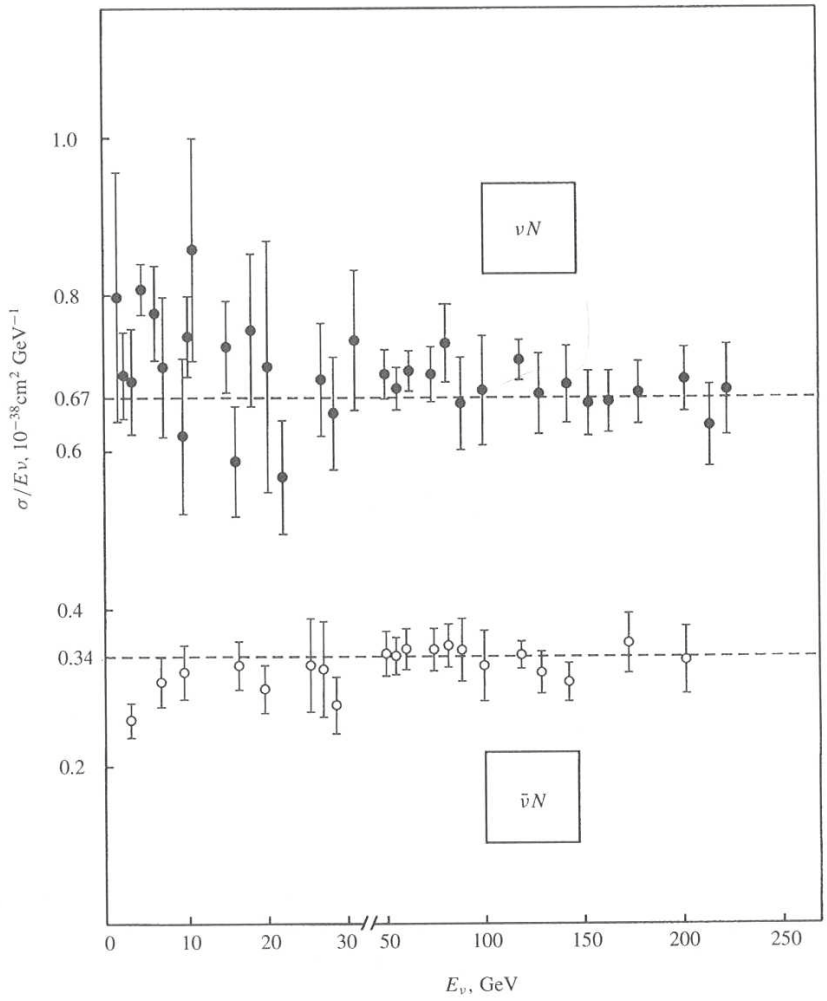
$Q = \int x[u(x)+d(x)]dx$ $\qquad$ $\bar{Q} = \int x[\bar{u}(x)+\bar{d}(x)]dx$

$$R = \sigma(\bar{\nu}N)/\sigma(\nu N) = (1+3\bar{Q}/Q)/(3+\bar{Q}/Q)$$

Quark distributions within the nucleon

- Several predictions of the nucleon model are validated
  - 1.) Total neutrino-nucleon cross section are proportional to energy

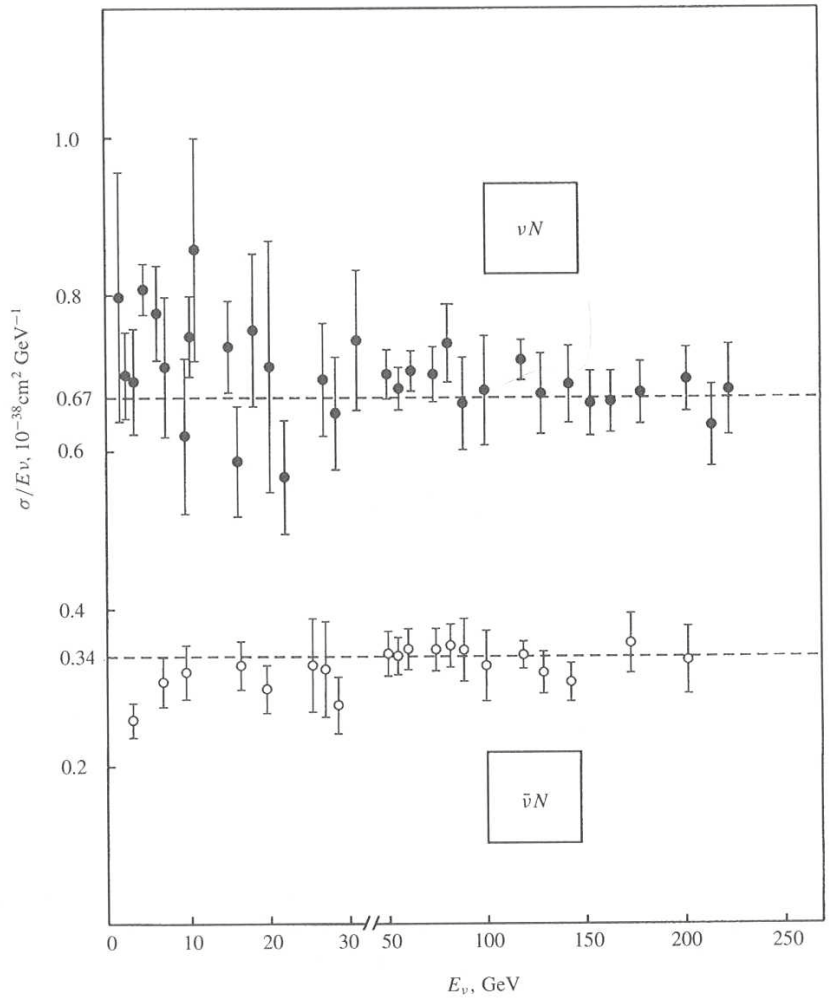
$$s = (E_\nu + m_p)^2 - E_\nu^2 = 2 E_\nu m_p$$



Quark distributions within the nucleon

- 2.) Ratio of antineutrino to neutrino cross sections indicates nucleon contains antiquarks with $\overline{Q}/Q \approx 0.15$

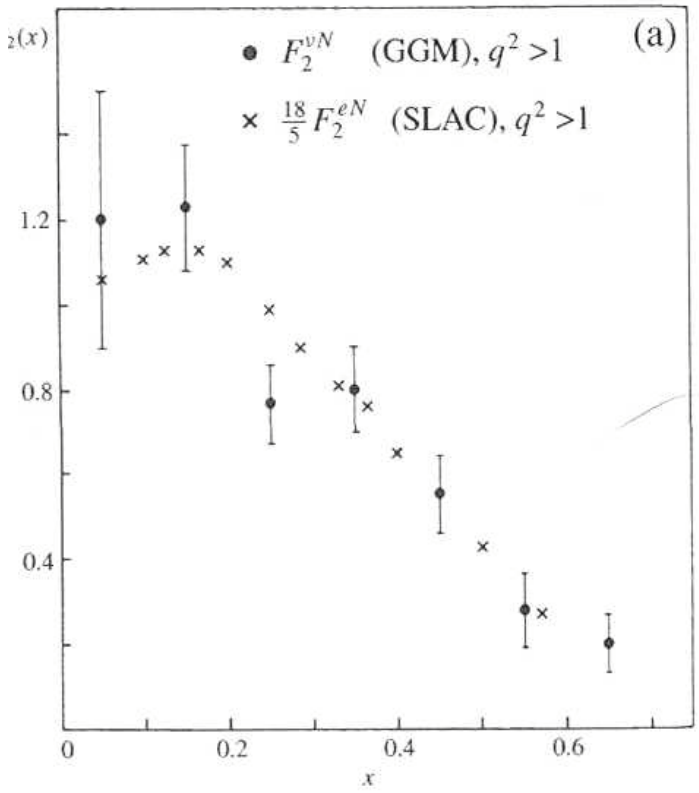
$$R = \frac{1 + 3\,\overline{Q}/Q}{3 + \overline{Q}/Q} \approx 0.45$$



Quark distributions within the nucleon

- 3.) Comparison of neutrino-nucleon cross sections with electron-nucleon cross sections establish fractionally charged quarks are real dynamical entities

$$F_2^{eN}(x) = 5/18\ F_2^{\nu N}(x)$$



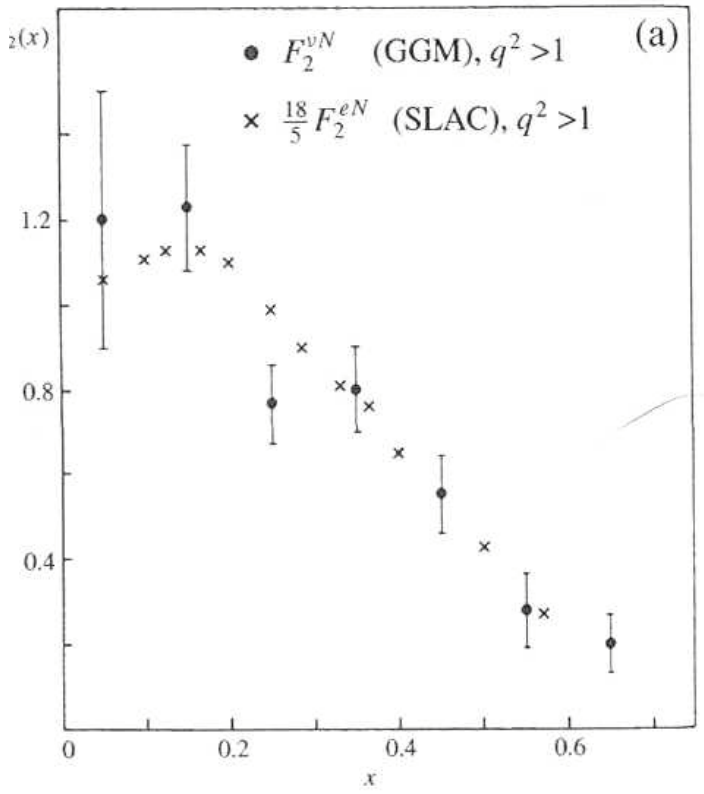
Quark distributions within the nucleon

- 4.) Total momentum fraction carried by quarks and anti-quarks is ≈ 0.5

$$\int F_2^{\nu N}(x)\,dx = \tfrac{18}{5}\int F_2^{eN}(x)\,dx \approx 0.5$$

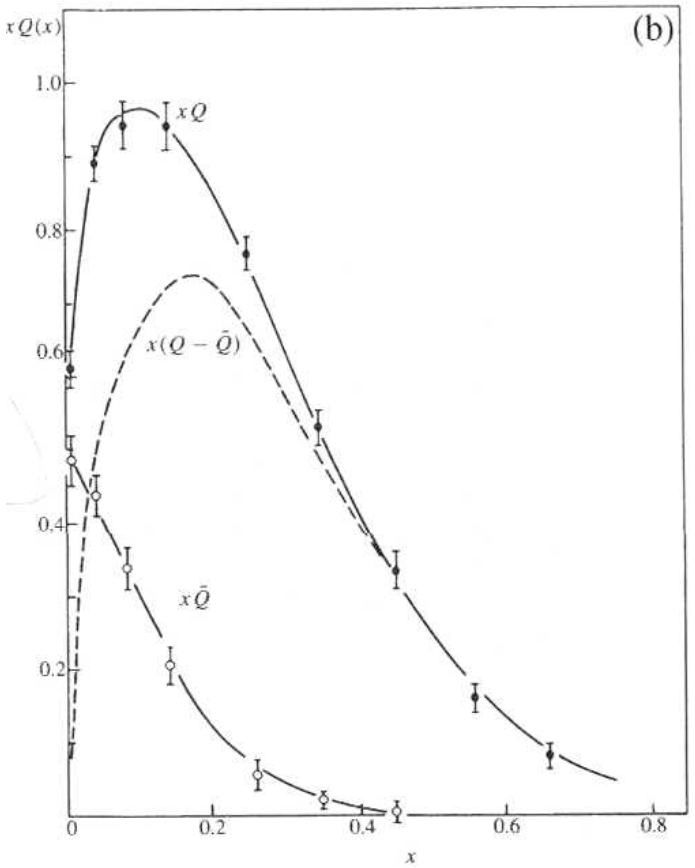
$$= \int x[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)]\,dx$$

Therefore 50% is carried by partons which do not couple to weak or EM forces (gluons)



Quark distributions within the nucleon

Quark and antiquark distributions can be determined from F_2 and F_3



Sum Rules

- Gottfried Sum Rule

$$\int [F_2^{ep}(x) - F_2^{en}(x)]\, dx/x = \tfrac{1}{3} + \tfrac{2}{3}\int [\overline{u}(x) - \overline{d}(x)]\, dx$$

$$= 0.24 \pm 0.03$$

The "sea" is not flavor symmetric,
the proton contains more anti-d than anti-u

Sum Rules

- Gross Llewellyn Smith Sum Rule

$$\int x F_3^{\nu N}(x)\, dx/x = \int [u_V(x)+d_v(x)]\, dx = 3$$

$u_v = u - \overline{u}$ "valence" quarks

this rule is satisfied to about 10%

Sum Rules

- Bjorken Sum Rule (spin structure)
  - Polarized probes

$F_1(x) \equiv \frac{1}{2} \Sigma z^2[q\uparrow(x) + \overline{q}\uparrow(x) + q\downarrow(x) + \overline{q}\downarrow(x)]$

$g_1(x) \equiv \frac{1}{2} \Sigma z^2[q\uparrow(x) + \overline{q}\uparrow(x) - q\downarrow(x) - \overline{q}\downarrow(x)]$

where ze is the quark charge

and the sum is over all quark flavors

$$\int (g_1^p - g_1^n)\, dx = \frac{1}{6} \left(\frac{g_A}{g_V}\right) \left(1 - \frac{\alpha_s}{\pi} + \ldots\right)$$

Good to about 10%