Physics 661

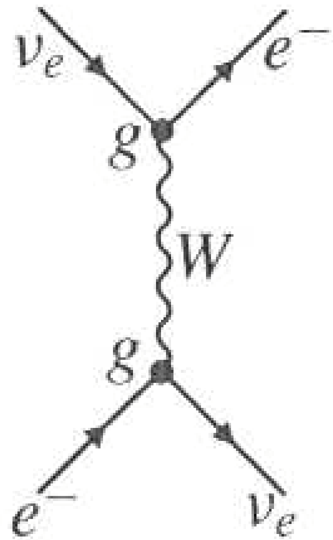
Particle Physics Phenomenology

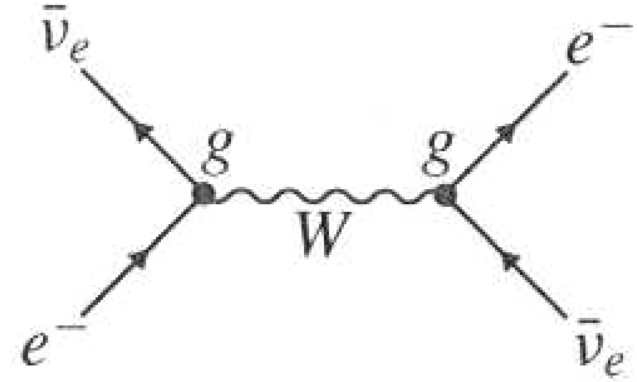
December 3, 2003

Lepton and Quark Scattering

- $e^+ e^- \to \mu^+ \mu^-$ ✓
- $e^+ e^-$ annihilation to hadrons ($e^+ e^- \to Q\bar{Q}$) ✓
- Electron-muon scattering, $e^- \mu^+ \to e^- \mu^+$ ✓
- Neutrino-electron scattering, $\nu_e e^- \to \nu_e e^-$
- Elastic lepton-nucleon scattering
- Deep inelastic scattering and partons
- Deep inelastic scattering and quarks
  - Electron-nucleon scattering
  - Neutrino-nucleon scattering
- Quark distributions within the nucleon
- Sum rules

Neutrino-electron scattering, $\nu_e\, e^- \to \nu_e\, e^-$





Consider only the charged-current reaction

$$M(\nu_e\, e^- \to \nu_e\, e^-) = (g/\sqrt{2})^2 \,/\, (q^2 + M_w^2)$$

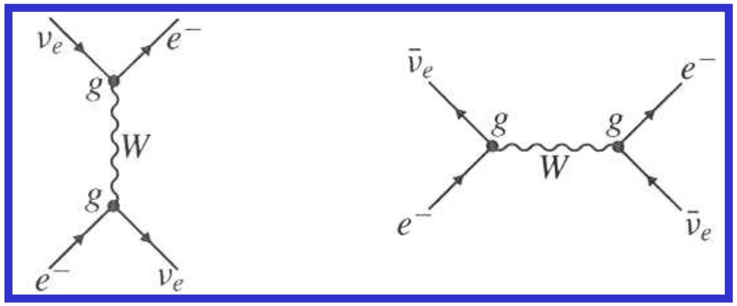
$$= g^2 \,/\, 2M_w^2 \quad \text{for } q^2 << M_w^2$$

Neutrino-electron scattering, $\nu_e\, e^- \to \nu_e\, e^-$

$$M(\nu_e\, e^- \to \nu_e\, e^-) = g^2/2M_W^2$$

$$G \equiv \sqrt{2}\, g^2/8M_W^2$$

$$M(\nu_e\, e^- \to \nu_e\, e^-) = 2\sqrt{2}\, G$$



$$\frac{d\sigma}{d\Omega} = \frac{W}{\phi_i} = \frac{W}{v_i} = \frac{2\pi}{\hbar}\frac{|M_{if}|^2}{v_i}\frac{1}{(2\pi\hbar)^3}p_f^2\frac{dp_f}{dE_0}$$

$$d\sigma/dq^2 = G^2/\pi$$

$\sigma = G^2 s/\pi$ since $q^2_{max} = s$

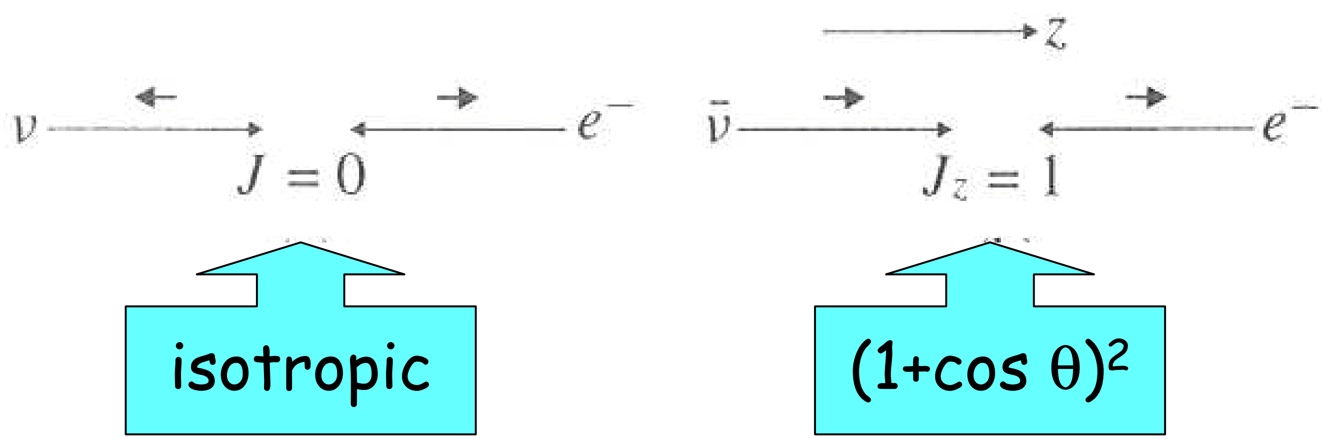
Neutrino-electron scattering, $\nu_e\, e^- \to \nu_e\, e^-$

- Leptons participate in weak interactions as longitudinally polarized particles

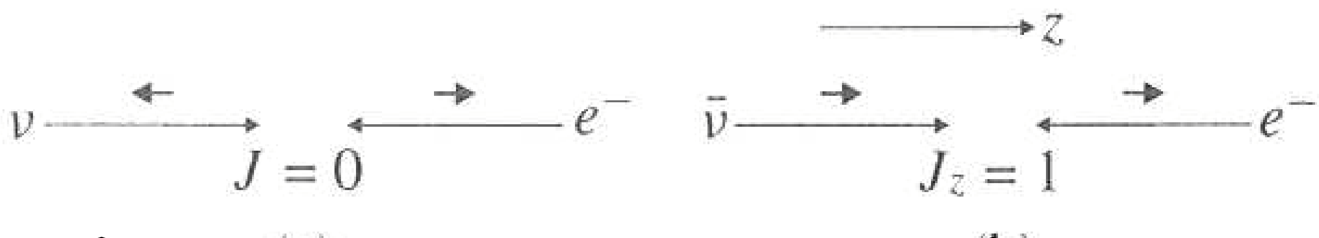
$$P = (I_+ - I_-)/(I_+ + I_-) = \alpha\, p/E = \alpha\, v/c$$

α = -1 for leptons

α = +1 for antileptons



Neutrino-electron scattering, $\nu_e\, e^- \rightarrow \nu_e\, e^-$



isotropic

$(1+\cos\theta)^2$

$\sigma = G^2 s/\pi$

$d\sigma/d\cos\theta = (G^2/8\pi)s\,(1+\cos\theta)^2$

$\sigma = (G^2/3\pi)s$

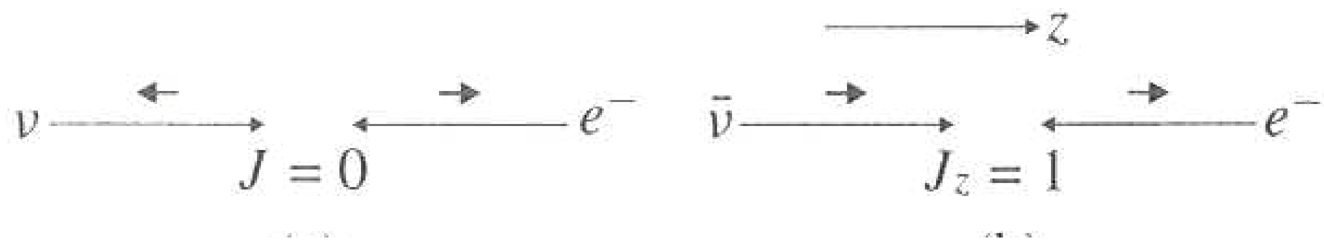
$$\sigma(\nu_e\, e^- \rightarrow \nu_e\, e^-) = 3\ \sigma(\overline{\nu}_e\, e^- \rightarrow \overline{\nu}_e\, e^-)$$

Neutrino-electron scattering, $\nu_e e^- \to \nu_e e^-$

Consider the cross section in terms of $y = E_e/E_\nu$ in the lab:

as before, $(1+\cos\theta)^2 = 4(1-y)^2$

and $d\cos\theta = 2\,dy$



$\nu_e e^- \to \nu_e e^-$

$d\sigma/dy = G^2 s/\pi$

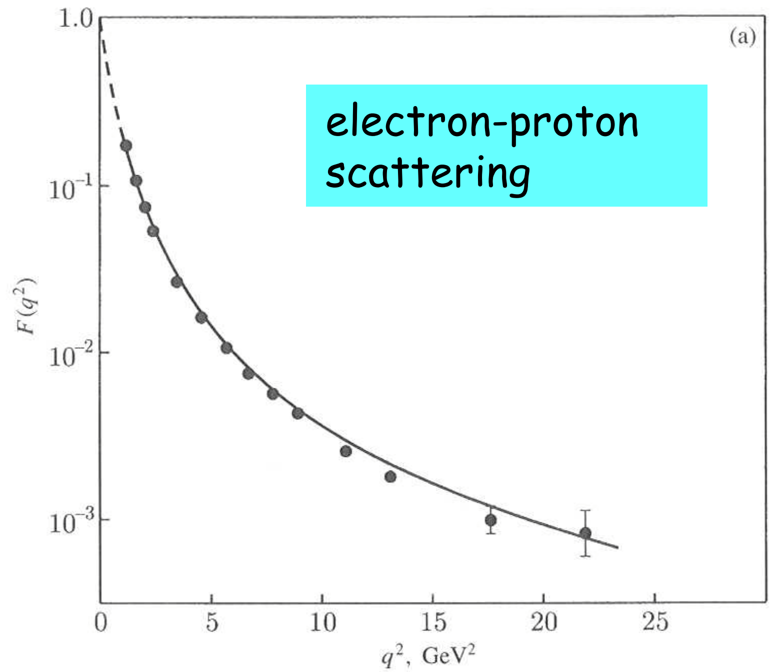
$\bar{\nu}_e e^- \to \bar{\nu}_e e^-$

$d\sigma/dy = (G^2/\pi)\, s\, (1-y)^2$

Elastic Lepton-Nucleon Scattering

If the nucleon were a point like particle the cross section would follow the $e\mu \rightarrow e\mu$ calc.

$$\frac{d\sigma}{dq^2} = \frac{2\pi\,\alpha^2}{q^4}\,[1 + (1-q^2/s)^2]$$



But this equation must be modified by the addition of a form factor

$$\frac{d\sigma}{dq^2} \rightarrow \frac{d\sigma}{dq^2}\,F(q^2)$$

therefore, the nucleon has structure

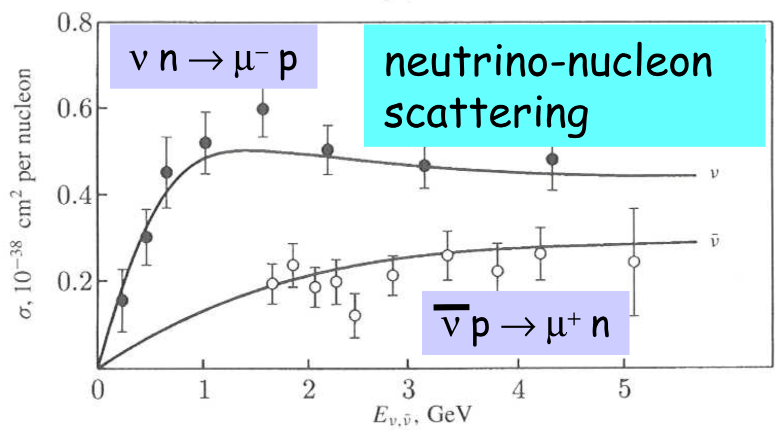
Elastic Lepton-Nucleon Scattering

Neutrino cross-section is expected to rise with s:

$$d\sigma/dy = G^2 s/\pi$$

However, elastic and quasi-elastic cross section saturate due to strong q^2 dependence of form factor

This appears to represent an exponential charge distribution



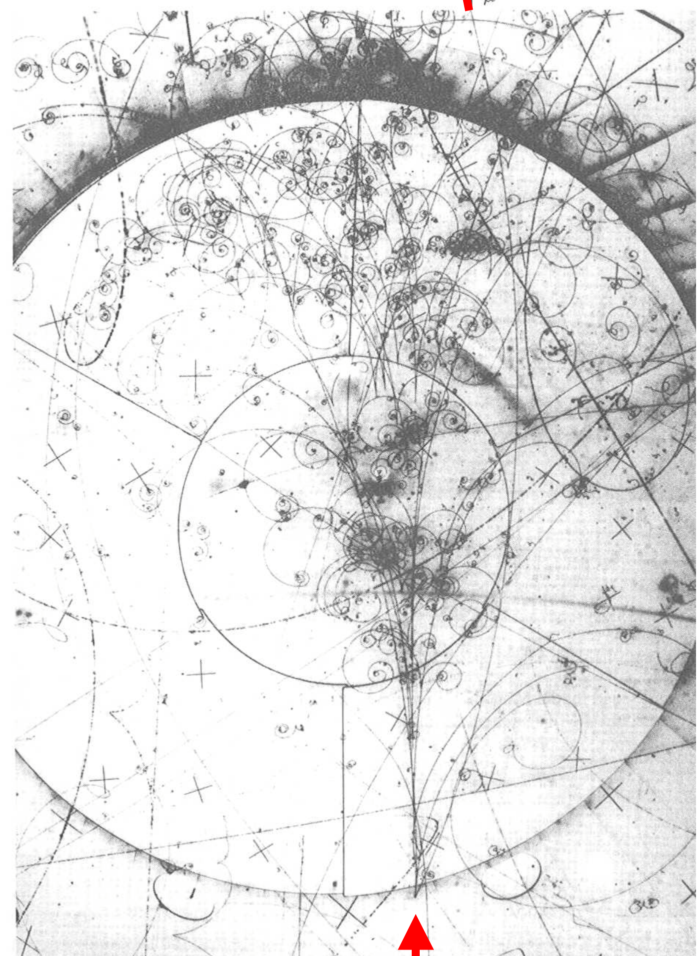
Deep Inelastic Scattering and Partons

Consider <u>inclusive</u> processes

lepton + nucleon
$\rightarrow$ lepton' + anything

Example of such an interaction with neutrinos

μ

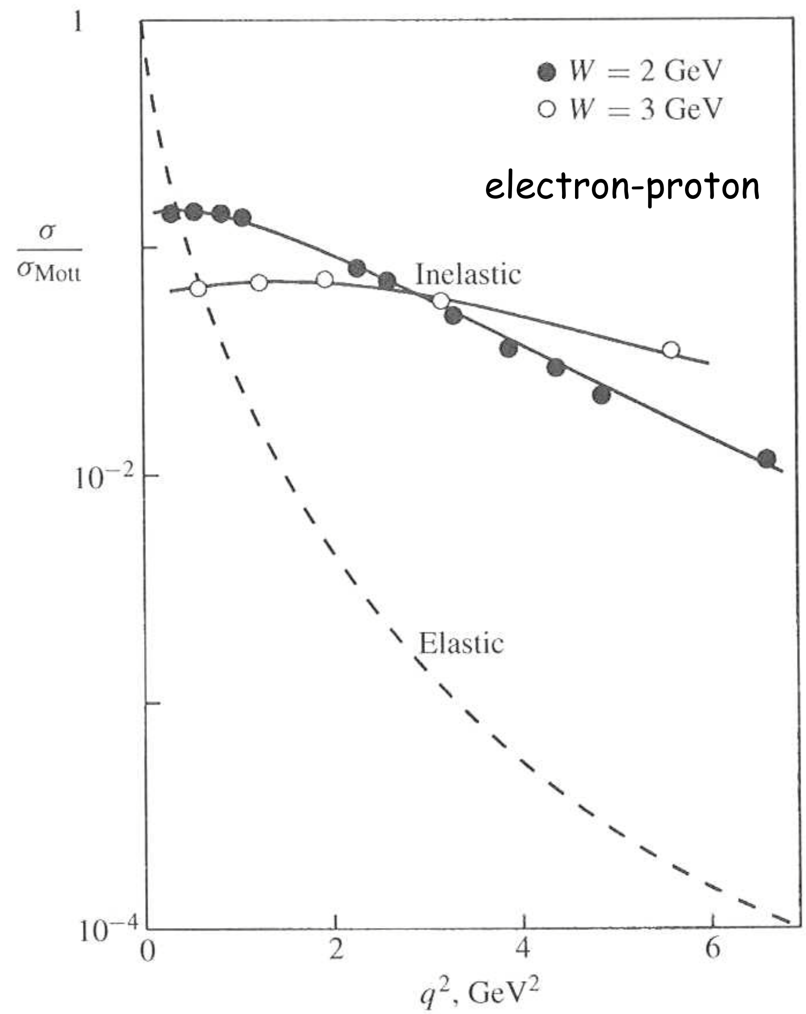


ν_μ

Deep Inelastic Scattering and Partons

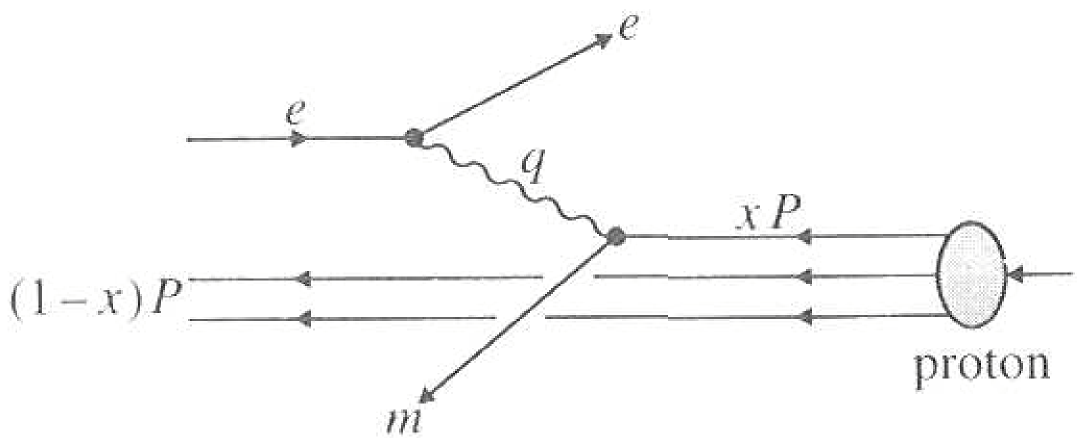
These *inclusive* reactions are observed to be only weakly q^2 dependent, unlike the case for *exclusive* reactions

This is a sign of elastic scattering from point-like constituents within the nucleon



Deep Inelastic Scattering and Partons

Infinite momentum frame (Feynman)



Very large proton momentum

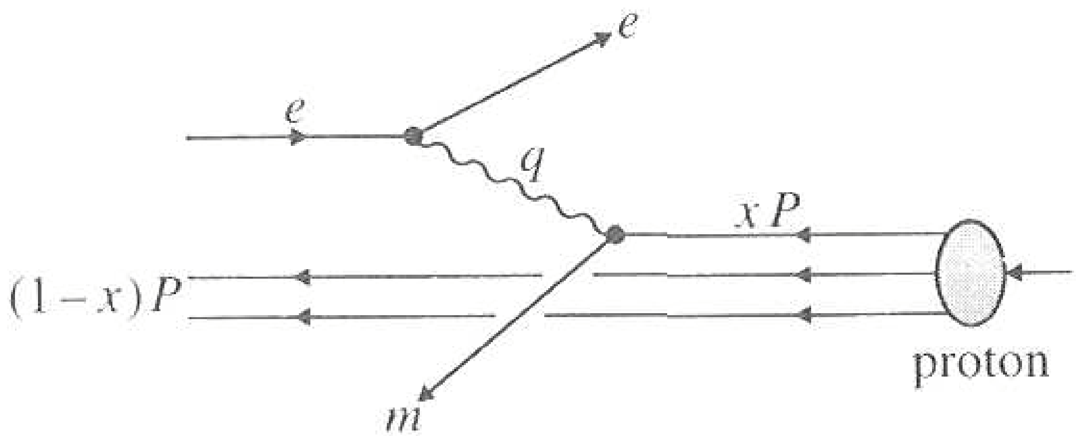
proton   P = (p,0,0,ip)   as mass can be neglected

each parton has 4-momentum xp

now scatter by absorbing q

$$(xP + q)^2 = -m^2 \approx 0$$

Deep Inelastic Scattering and Partons



$$(xP + q)^2 = -m^2 \approx 0$$

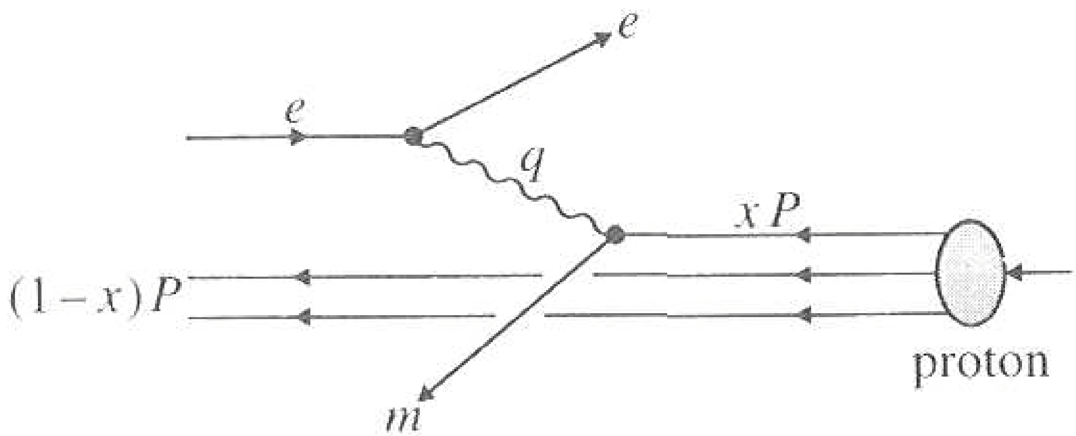
$$x^2 P^2 + q^2 + 2xPq \approx 0$$

If $|x^2 P^2| = x^2 M^2 \ll q^2$,

$$x = -q^2/2Pq = q^2/2M\nu$$

(ν is the energy transfer in the lab.)

Deep Inelastic Scattering and Partons



This model assumes the process occurs in two stages:

1. One parton is scattered

   collision time $t_1 \sim \hbar/\nu$

2. Partons recombine to form final state (M=W)

   time $t_2 \sim \hbar/W$

   $t_2 \gg t_1$

Deep Inelastic Electron-Nucleon Scattering and Quarks

Evidence that partons are quarks:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2 \sin^4(\theta/2)} \left[1 + \cos^4(\theta/2)\right]$$

$e^- \mu^+ \rightarrow e^- \mu^+$

1. ang. dist. characterized by two terms:
   a. $\cos^2(\theta/2)$ - electric (Mott)
   b. $\sin^4(\theta/2)$ - magnetic (spin-flip)

$$[1 + \cos^4(\theta/2)] = 2\cos^2(\theta/2) + \sin^4(\theta/2)$$

Data reveals just this dependence

Deep Inelastic Electron-Nucleon Scattering and Quarks

Consider a model of the partons

$u(x)\,dx$ = number of u quarks carrying momentum fraction $x \to x+dx$

$d(x)\,dx$ = number of d quarks carrying momentum fraction $x \to x+dx$

Cross section depends on charge squared:

$$\frac{d^2\sigma(ep)}{dydx} = \frac{4\pi\,\alpha^2\,xs}{q^4}\,\frac{F_2^{ep}(x)}{x}\,\frac{[1+(1-y)^2]}{2}$$

$$F_2^{ep}(x) = x\,\{4/9[u(x)+\bar{u}(x)]+1/9[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)]\}$$

Deep Inelastic Electron-Nucleon Scattering and Quarks

What about electron-neutron scattering?

$$\frac{d^2\sigma(en)}{dydx} = \frac{4\pi\,\alpha^2\,xs}{q^4}\,\frac{F_2^{en}(x)}{x}\,\left[\frac{1 + (1-y)^2}{2}\right]$$

$$F_2^{en}(x) = x\,\{4/9[d(x)+\bar{d}(x)]+1/9[u(x)+\bar{u}(x)+s(x)+\bar{s}(x)]\}$$

by isospin invariance

for a target with equal number of protons and neutrons:

$$F_2^{eN}(x) = x\,\{5/18[d(x)+\bar{d}(x)+ u(x)+\bar{u}(x)]+1/9[s(x)+\bar{s}(x)]\}$$