Physics 661

Particle Physics Phenomenology

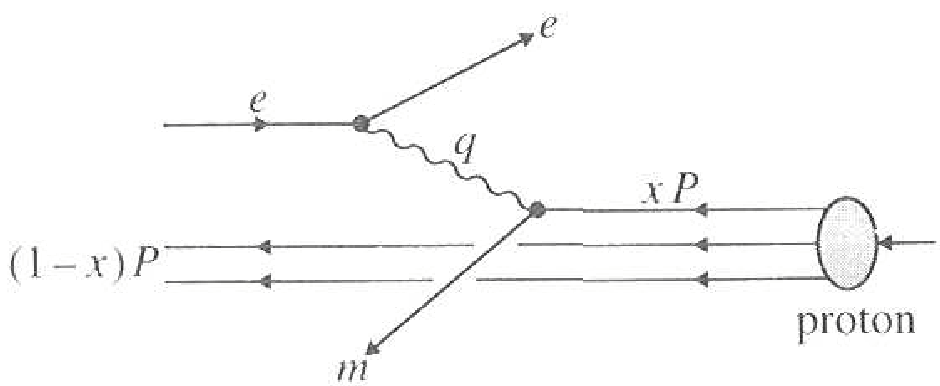
December 2, 2003

Lepton and Quark Scattering

- $e^+ e^- \rightarrow \mu^+ \mu^-$
- $e^+ e^-$ annihilation to hadrons ($e^+ e^- \rightarrow Q\overline{Q}$)
- Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$
- Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$
- Elastic lepton-nucleon scattering
- Deep inelastic scattering and partons
- Deep inelastic scattering and quarks
  - Electron-nucleon scattering
  - Neutrino-nucleon scattering
- Quark distributions within the nucleon
- Sum rules

The "discovery" of quarks

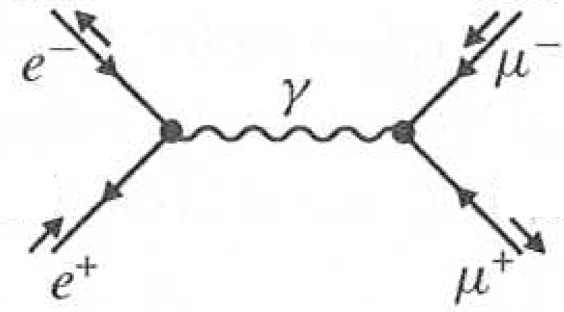
- deep inelastic lepton-nucleon scattering revealed dynamical understanding of quark substructure



- leptoproduction of hadrons could be interpreted as elastic scattering of the lepton by a pointlike constituent of the nucleon, the quark
- theory of scattering of two spin-1/2, pointlike particles required

$e^+ e^- \to \mu^+ \mu^-$

- Dominated by single-photon exchange
- $M_{if} = e^2/q^2 = 4\pi\alpha/q^2$
- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} f(\theta)$$

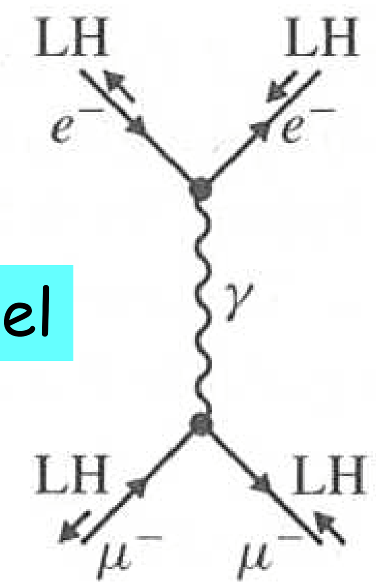


- What about spins?
  - The conservation of helicity at high energy for the EM interaction means only LR and RL states will interact

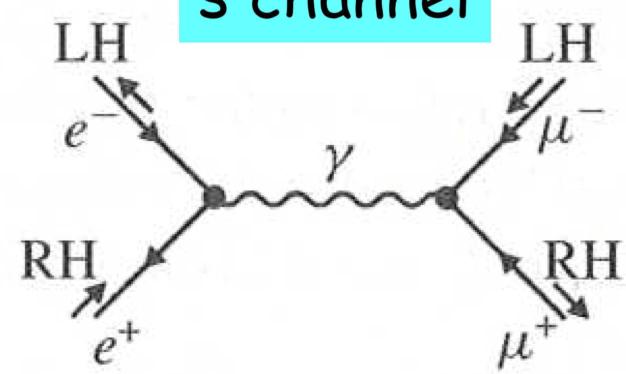
$e^+ e^- \to \mu^+ \mu^-$

- Conservation of helicity
  - consider crossed diagrams
    - (recall, helicity is conserved in vector int. at high energy)

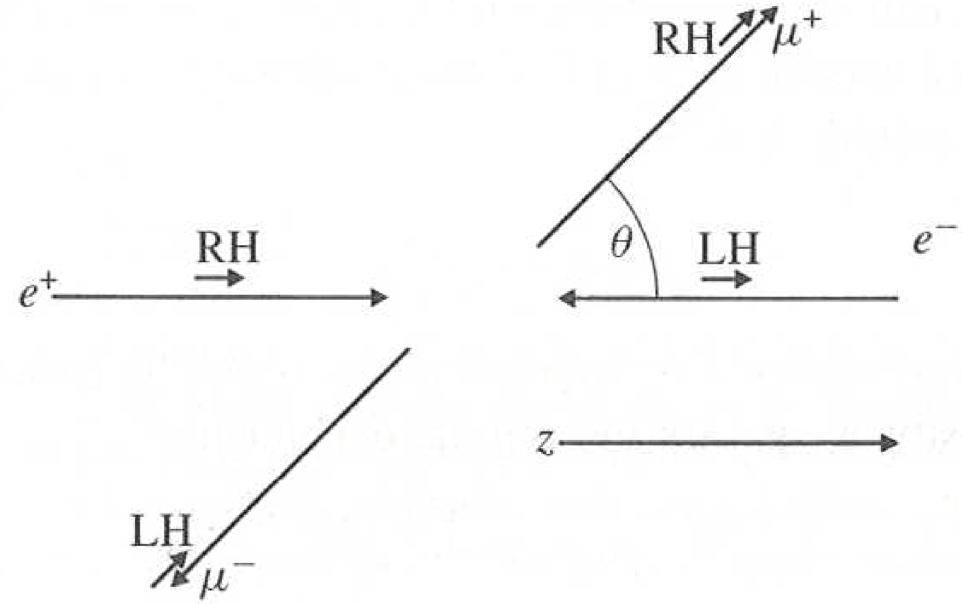
t channel



s channel



$e^+ e^- \to \mu^+ \mu^-$



- Amplitude is $d^J_{mm'}(\theta) = d^1_{1,1}(\theta) = (1+\cos\theta)/2$
  - if RL $\to$ RL
- Amplitude is $d^J_{mm'}(\theta) = d^1_{1,-1}(\theta) = (1-\cos\theta)/2$
  - if RL $\to$ LR
- $M^2 \sim [(1+\cos\theta)/2]^2 + [(1-\cos\theta)/2]^2 = (1+\cos^2\theta)/2$

$e^+ e^- \to \mu^+ \mu^-$

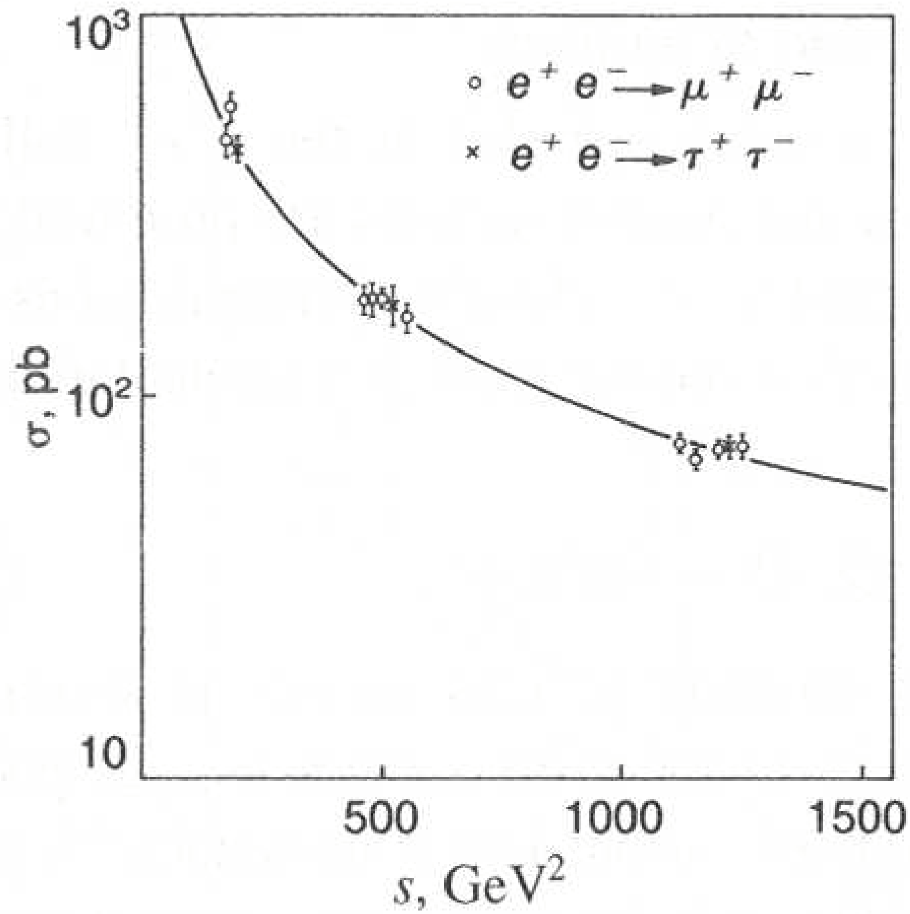
- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

- $\sigma(e^+ e^- \to \mu^+ \mu^-)$

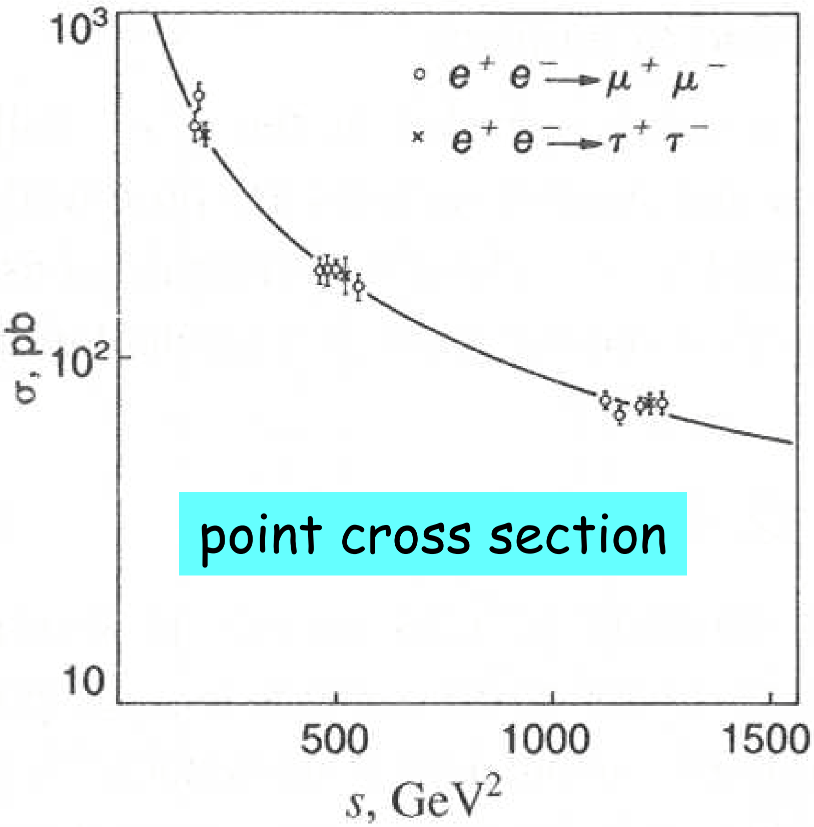
  $= 4\pi\alpha^2 / 3s$

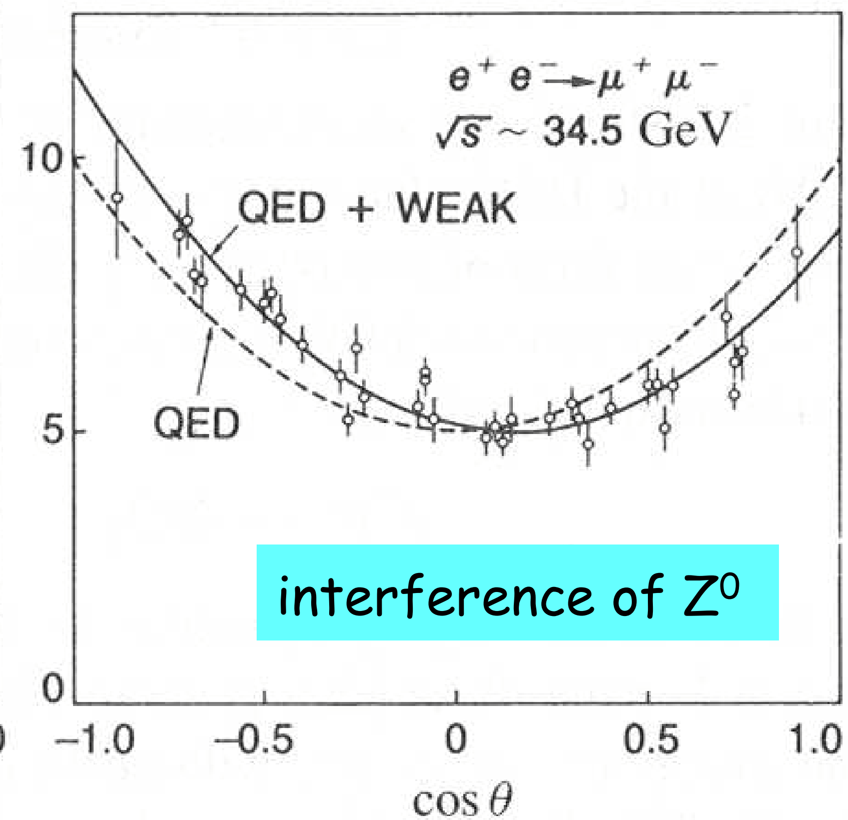
  $= 87\text{nb} / s(\text{GeV}^2)$

  (point cross section)



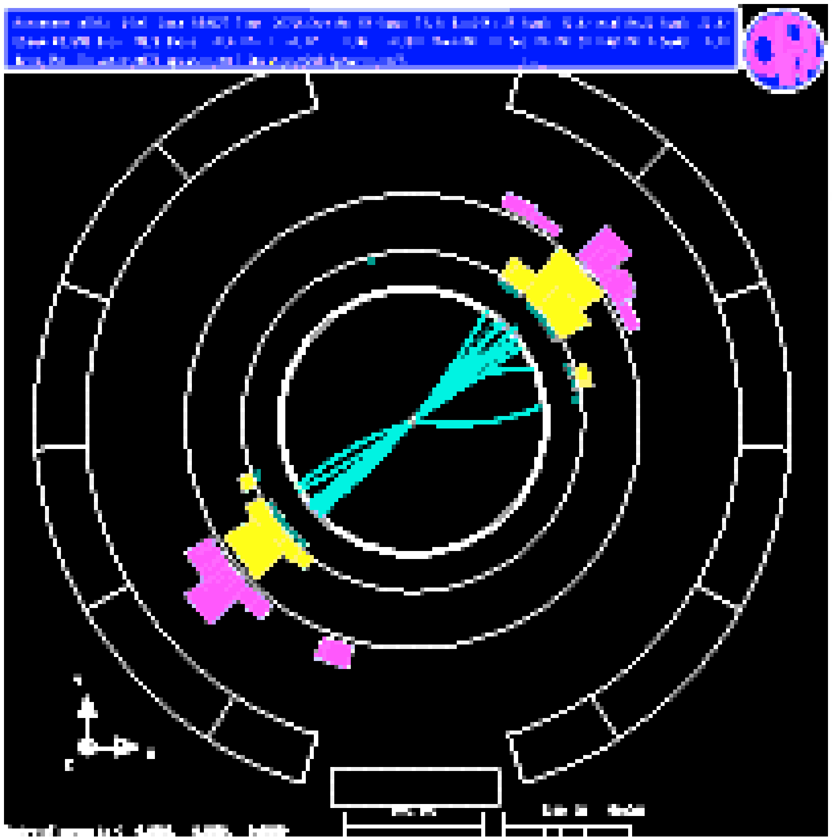
$e^+ e^- \to \mu^+ \mu^-$

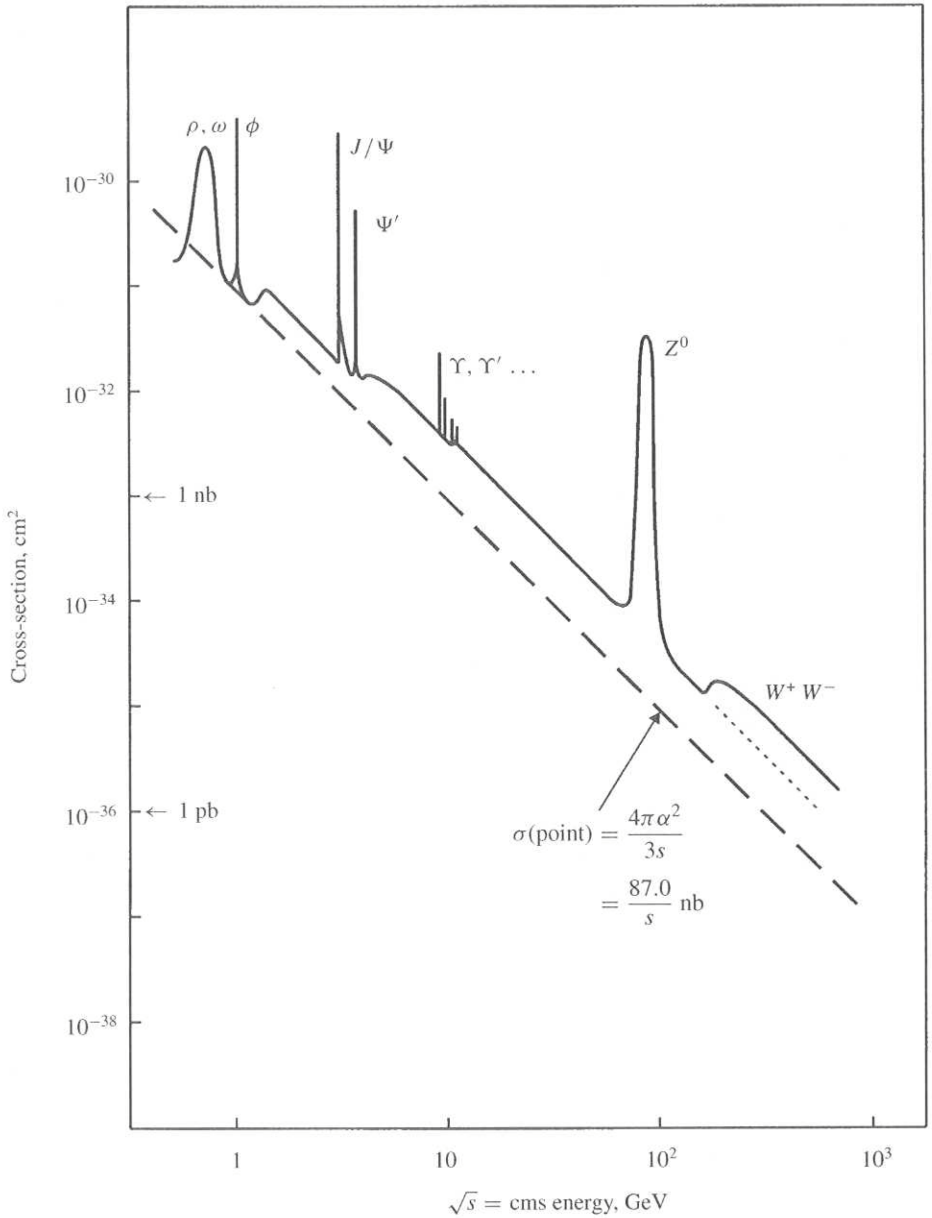




$f \sim a_{wk} a_{em} / a_{em}^2 \sim Gs/(4\pi\alpha) \sim 10^{-4} s$ (interference)

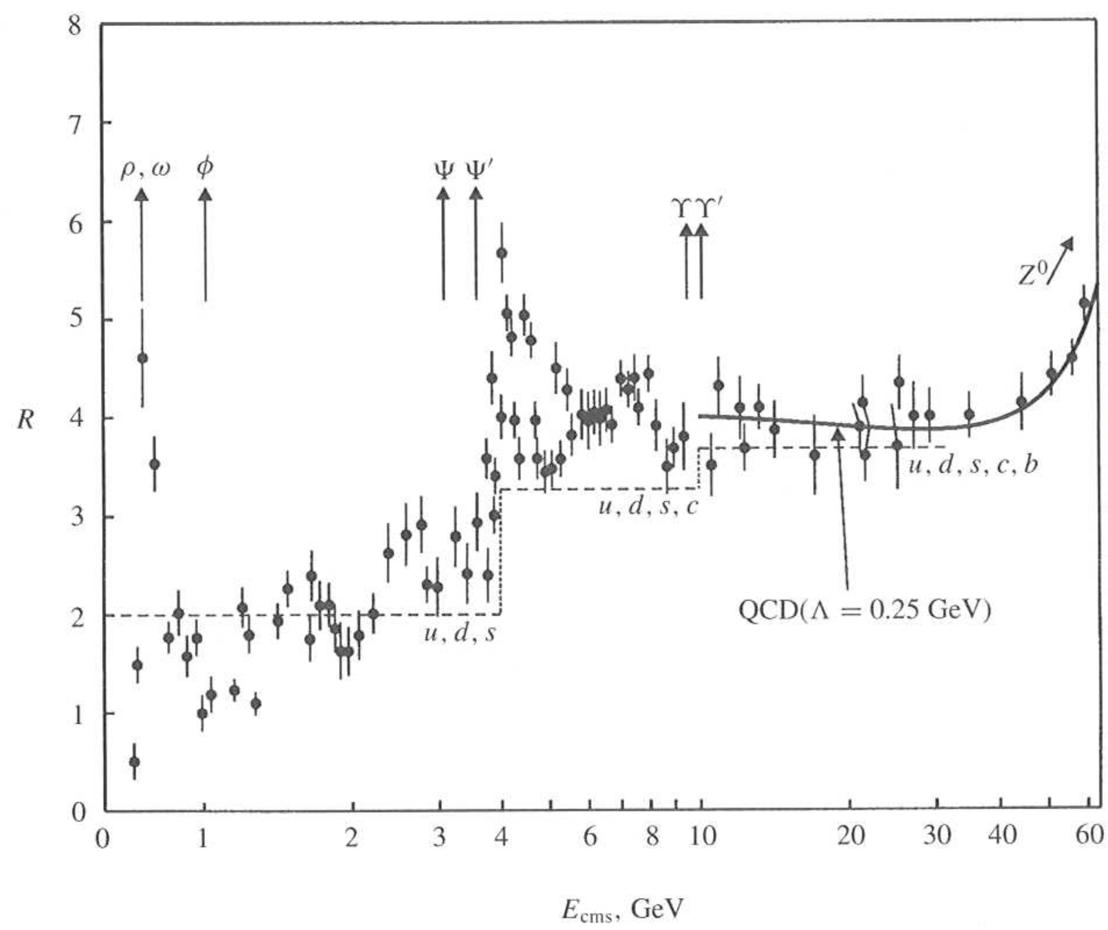
$e^+ e^-$ annihilation to hadrons





$e^+ e^-$ annihilation to hadrons

- $R = \sigma\,(e^+ e^- \rightarrow \text{hadrons}) / \sigma(\text{point})$



$e^+ e^-$ annihilation to hadrons

- $R = \sigma\,(e^+ e^- \rightarrow \text{hadrons}) / \sigma(\text{point})$
  - consider $e^+ e^- \rightarrow$ hadrons as $e^+ e^- \rightarrow Q\bar{Q}$, summed over all quarks

- $R = \Sigma\, e_i^2 / e^2$

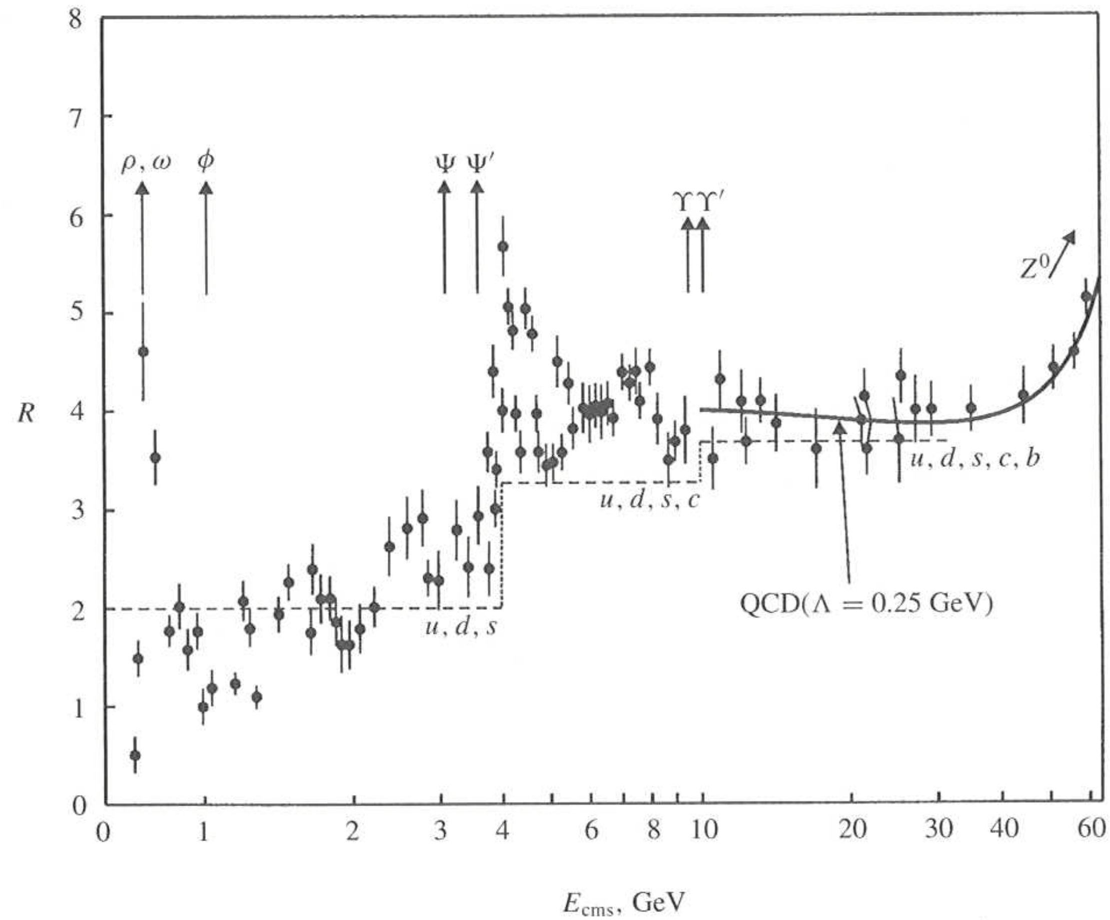
$$= N_c \left(\underset{d}{(1/3)^2} + \underset{u}{(2/3)^2} + \underset{s}{(1/3)^2} + \underset{c}{(2/3)^2} + \underset{b}{(1/3)^2} + \ldots \right)$$

  - $N_c = 3$

- Therefore, R should increase by a well defined value as each flavor threshold is crossed

$e^+ e^-$ annihilation to hadrons

Threshold	R
below c	2
charm	3 1/3
bottom	3 2/3

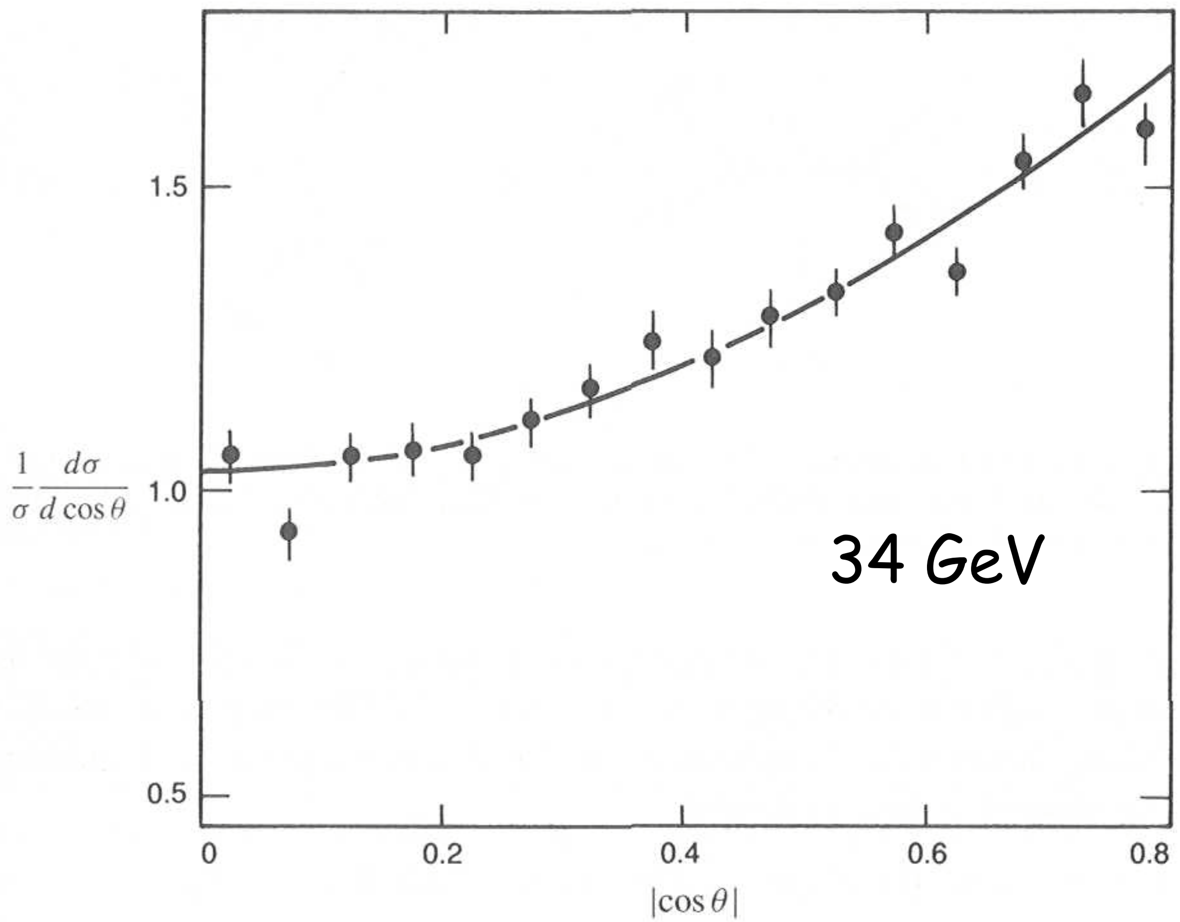


$e^+ e^-$ annihilation to hadrons

$e^+ e^- \to Q\overline{Q}$

If Q is spin 1/2

$(1+\cos^2\theta)$

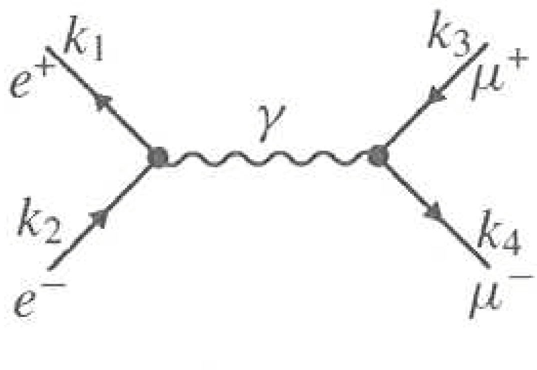


$e^+ e^-$ annihilation to hadrons

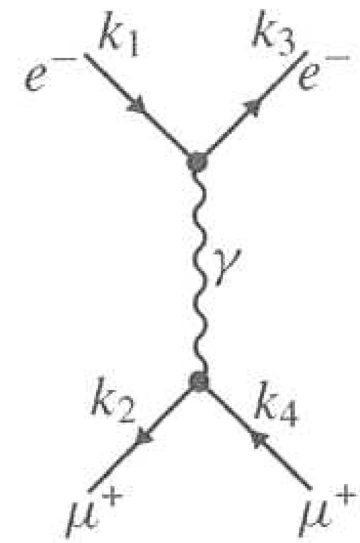
- Constancy of R indicates pointlike constituents
- Angular distributions of hadron jets prove spin 1/2 partons
- Values of R consistent with partons of quarks expected charge and color quantum number

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

s channel



t channel



- Mandelstam variables:

$$s = -(k_1 + k_2)^2 = -(k_3 + k_4)^2 = -2k_1k_2 = -2k_3k_4$$

$$t = q^2 = (k_1 - k_3)^2 = (k_2 - k_4)^2 = -2k_1k_3 = -2k_2k_4$$

$$u = (k_2 - k_3)^2 = (k_1 - k_4)^2 = -2k_2k_3 = -2k_1k_4$$

for $m = 0$

$k = (p_x, p_y, p_z, iE)$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- Mandelstam variables:

$s = -(k_1 + k_2)^2 = -(k_3 + k_4)^2 = -2k_1k_2 = -2k_3k_4$

suppose $k_1 = (p, iE)$ $\quad k_2 = (-p, iE)$

$s = -(p-p)^2 - (2iE)^2 = 4E^2$ $\qquad$ for $m = 0$

$t = q^2 = (k_1 - k_3)^2 = (k_2 - k_4)^2 = -2k_1k_3 = -2k_2k_4$

suppose $k_3 = (p \cos\theta, p \sin\theta, 0, iE)$

$t = (p - p\cos\theta)^2 + (-p\sin\theta)^2 + (E - E)^2 = 2p^2(1-\cos\theta)$

$\quad = 4p^2 \sin^2 \theta/2$

$u = (k_2 - k_3)^2 = (k_1 - k_4)^2 = -2k_2k_3 = -2k_1k_4 = 4p^2\cos^2\theta/2$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- Annihilation cross-section ($e^+e^- \rightarrow \mu^+\mu^-$)
- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2}\left(\frac{t^2+u^2}{s^2}\right) = \frac{\alpha^2}{8p^2}\left[\sin^4(\theta/2)+\cos^4(\theta/2)\right]$$

$$= \frac{\alpha^2}{4s}\left[1+\cos^2\theta\right]$$

$2\sin^2(\theta/2) = 1-\cos\theta$
$2\cos^2(\theta/2) = 1+\cos\theta$

$s=4p^2$

- Now consider the crossed channel, $e^- \mu^+ \rightarrow e^- \mu^+$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

crossed channel ($t \leftrightarrow u$)

- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2}\left(\frac{s^2 + u^2}{t^2}\right)$$

$$= \frac{\alpha^2}{8p^2 \sin^4(\theta/2)}\left[1 + \cos^4(\theta/2)\right]$$

So far, all of these expressions are for the center-of-momentum system

What about the laboratory frame?

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- Suppose the muon is a target at rest in the laboratory
  (Note: this is not practical since the lifetime of the muon is 2.2 microseconds)
- γ is the boost from cms to lab
- p and θ are the projectile parameters in the cms

- In the lab:

$$E_\mu = \gamma\,(p - \beta\, p \cos\theta) \qquad \text{(scattered muon)}$$
$$= \gamma\, p\,(1 - \cos\theta)$$
$$E_e = \gamma\,(p + \beta\, p) = 2\gamma\, p \quad \text{(incident electron)}$$
$$y = E_\mu / E_e = (1 - \cos\theta)\,/2$$

$$\text{so } \cos^2\theta/2 = (1+\cos\theta)/2 = 1-y$$
$$d\,\Omega = 2\pi\, d(\cos\theta) = 4\pi\, dy$$

Electron-muon scattering, $e^- \mu^+ \to e^- \mu^+$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2\sin^4(\theta/2)}\,[1 + \cos^4(\theta/2)]$$

$d\,\Omega = 4\pi\, dy$
$\cos^2\theta/2 = 1-y$
$q^2 = t = 4p^2\sin^2\theta/2$

$$\frac{d\sigma}{4\pi dy} = \frac{\alpha^2}{q^4/2p^2}\,[1 + (1-y)^2]$$

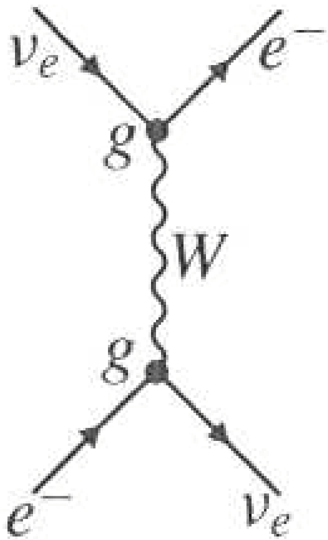
same helicity scattering

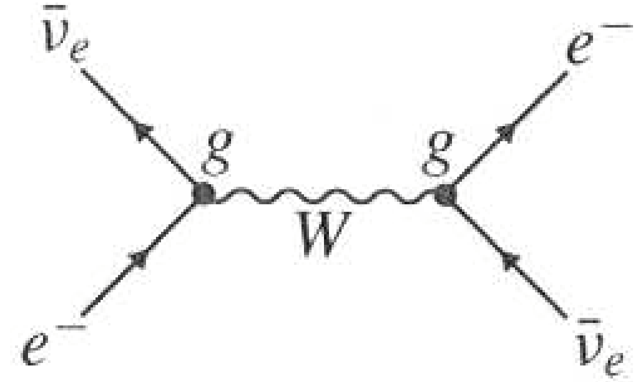
$$\frac{d\sigma}{dy} = \frac{2\pi\,\alpha^2 s}{q^4}\,[1 + (1-y)^2]$$

opposite helicity scattering

As y -> 0, we recover the Rutherford formula

Neutrino-electron scattering, $\nu_e\, e^- \to \nu_e\, e^-$





Consider only the charged-current reaction

$$M(\nu_e\, e^- \to \nu_e\, e^-) = (g/\sqrt{2})^2 \,/\, (q^2 + M_W^2)$$

$$= g^2 \,/\, 2M_W^2 \quad \text{for } q^2 \ll M_W^2$$