# Physics 661

Particle Physics Phenomenology

November 6, 2003

# Invariance Principles and Conservation Laws

- Outline
  - Translation and rotation ✓
  - Parity ✓
  - Charge Conjugation ✓
  - Charge Conservation and Gauge Invariance ✓
  - Baryon and lepton conservation ✓
  - CPT Theorem
  - CP violation and T violation
  - Isospin symmetry

# CPT Invariance

- Very general assumptions in quantum field theory:
  - spin-statistics relation: integer and half-integer spin particles obey Bose and Fermi statistics
  - Lorentz invariance

lead to the CPT Theorem:

  - all interactions are invariant under the successive operation of C (=charge conjugation), P (=parity operation), and T (= time reversal)

- Masses, lifetimes, moments, etc. of particles and antiparticles must be identical

# CPT Invariance

- Tests of the CPT Theorem

| | | |
|---|---|---|
| $(m_{W^+} - m_{W^-}) \,/\, m_{\text{average}}$ | | $-0.002 \pm 0.007$ |
| $(m_{e^+} - m_{e^-}) \,/\, m_{\text{average}}$ | | $<8 \times 10^{-9}$, CL $= 90\%$ |
| $\lvert q_{e^+} + q_{e^-} \rvert / e$ | | $<4 \times 10^{-8}$ |
| $(g_{e^+} - g_{e^-}) \,/\, g_{\text{average}}$ | | $(-0.5 \pm 2.1) \times 10^{-12}$ |
| $(\tau_{\mu^+} - \tau_{\mu^-}) \,/\, \tau_{\text{average}}$ | | $(2 \pm 8) \times 10^{-5}$ |
| $(g_{\mu^+} - g_{\mu^-}) \,/\, g_{\text{average}}$ | | $(-2.6 \pm 1.6) \times 10^{-8}$ |
| $(m_{\pi^+} - m_{\pi^-}) \,/\, m_{\text{average}}$ | | $(2 \pm 5) \times 10^{-4}$ |
| $(\tau_{\pi^+} - \tau_{\pi^-}) \,/\, \tau_{\text{average}}$ | | $(6 \pm 7) \times 10^{-4}$ |
| $(m_{K^+} - m_{K^-}) \,/\, m_{\text{average}}$ | | $(-0.6 \pm 1.8) \times 10^{-4}$ |
| $(\tau_{K^+} - \tau_{K^-}) \,/\, \tau_{\text{average}}$ | | $(0.11 \pm 0.09)\%$ (S $= 1.2$ |
| $K^{\pm} \rightarrow \mu^{\pm}\nu_{\mu}$ rate difference/average | | $(-0.5 \pm 0.4)\%$ |
| $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}$ rate difference/average | [f] | $(0.8 \pm 1.2)\%$ |

# CPT Invariance (cont.)

| | | |
|---|---|---|
| $\lvert m_{K^0} - m_{\overline{K}^0}\rvert \,/\, m_{\text{average}}$ | [g] | $<10^{-18}$ |
| $CPT$-violation parameters in $K^0$-$\overline{K}^0$ mixing | | |
| real part of $\Delta$ | | $(2.9 \pm 2.7) \times 10^{-4}$ |
| imaginary part of $\Delta$ | | $(-0.8 \pm 3.1) \times 10^{-3}$ |
| phase difference $\phi_{00} - \phi_{+-}$ | | $(-0.1 \pm 0.8)^\circ$ |
| $\lvert m_p - m_{\overline{p}}\rvert / m_p$ | [h] | $<5 \times 10^{-7}$ |
| $(\lvert \frac{q_{\overline{p}}}{m_{\overline{p}}}\rvert - \frac{q_p}{m_p}) / \frac{q_p}{m_p}$ | | $(-9 \pm 9) \times 10^{-11}$ |
| $\lvert q_p + q_{\overline{p}}\rvert / e$ | [h] | $<5 \times 10^{-7}$ |
| $(\mu_p + \mu_{\overline{p}}) \,/\, \mu_p$ | | $(-2.6 \pm 2.9) \times 10^{-3}$ |
| $(m_n - m_{\overline{n}}) /\, m_n$ | | $(9 \pm 5) \times 10^{-5}$ |
| $(m_\Lambda - m_{\overline{\Lambda}}) \,/\, m_\Lambda$ | | $(-0.1 \pm 1.1) \times 10^{-5}$ (S |
| $(\tau_\Lambda - \tau_{\overline{\Lambda}}) \,/\, \tau_\Lambda$ | | $0.04 \pm 0.09$ |
| $(\tau_{\Sigma^+} - \tau_{\overline{\Sigma}^-}) \,/\, \tau_{\Sigma^+}$ | | $(-0.6 \pm 1.2) \times 10^{-3}$ |
| $(\mu_{\Sigma^+} + \mu_{\overline{\Sigma}^-}) \,/\, \mu_{\Sigma^+}$ | | $0.014 \pm 0.015$ |

# CPT Invariance (cont.)

| | |
|---|---|
| $(m_{\Xi^-} - m_{\overline{\Xi}^+}) \,/\, m_{\Xi^-}$ | $(1.1 \pm 2.7) \times 10^{-4}$ |
| $(\tau_{\Xi^-} - \tau_{\overline{\Xi}^+}) \,/\, \tau_{\Xi^-}$ | $0.02 \pm 0.18$ |
| $(\mu_{\Xi^-} + \mu_{\overline{\Xi}^+}) \,/\, \lvert\mu_{\Xi^-}\rvert$ | $+0.01 \pm 0.05$ |
| $(m_{\Omega^-} - m_{\overline{\Omega}^+}) \,/\, m_{\Omega^-}$ | $(-1 \pm 8) \times 10^{-5}$ |
| $(\tau_{\Omega^-} - \tau_{\overline{\Omega}^+}) \,/\, \tau_{\Omega^-}$ | $-0.002 \pm 0.040$ |

# CP Violation

- Until 1964 it was believed that CP symmetry was respected in the weak interaction
  - P and C were known to be separately violated
- A small team led by Cronin and Fitch studying the long-lived neutral kaon discovered that it violated CP with a frequency of $2 \times 10^{-3}$
  - $K_L^0 \rightarrow \pi\ \pi\ \pi$ ( > 99 % )
    $CP = -1 \quad \rightarrow \quad CP = -1$
  - $K_L^0 \rightarrow \pi\ \pi$ (0. 2 %)
    $CP = -1 \quad \rightarrow \quad CP = +1$

# CP Violation

- The Standard Model of CP violation explains its origin from the CP-violating phase in the mixing matrix of the six quark flavors
- Experiments at PEPII (BaBar) at SLAC and KEKB (Belle) at KEK have confirmed the Standard Model explanation with measurements of CP violation in B decays

# T Violation

- The CPT Theorem dictates that if CP is violated, T must also be violated so that CPT can be invariant

| | | Effect of T | Effect of P |
|---|---|---|---|
| • position | r | r | –r |
| • momentum | p | –p | –p |
| • spin | $\sigma$ | $-\sigma$ | $\sigma$ |
| | = (r x p) | | |
| • electric field | E (= $-\nabla$V) | E | –E |
| • magnetic field | B | –B | B |
| • mag. dip. mom. | $\sigma \cdot B$ | $\sigma \cdot B$ | $\sigma \cdot B$ |
| • el. dipl. mom. | $\sigma \cdot E$ | $-\sigma \cdot E$ | $-\sigma \cdot E$ |
| • long. pol. | $\sigma \cdot p$ | $\sigma \cdot p$ | $\sigma \cdot p$ |
| • trans. pol. | $\sigma \cdot (p_1 \times p_2)$ | $-\sigma \cdot (p_1 \times p_2)$ | $\sigma \cdot (p_1 \times p_2)$ |

# T Violation

- Transverse polarization

| | | Effect of T | Effect of P |
|---|---|---|---|
| • trans. pol. | $\sigma \bullet (p_1 \times p_2)$ | $-\sigma \bullet (p_1 \times p_2)$ | $\sigma \bullet (p_1 \times p_2)$ |

- $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$
- $n \rightarrow p + e^- + \overline{\nu}_e$
  - T-violating contributions below $10^{-3}$

# T Violation

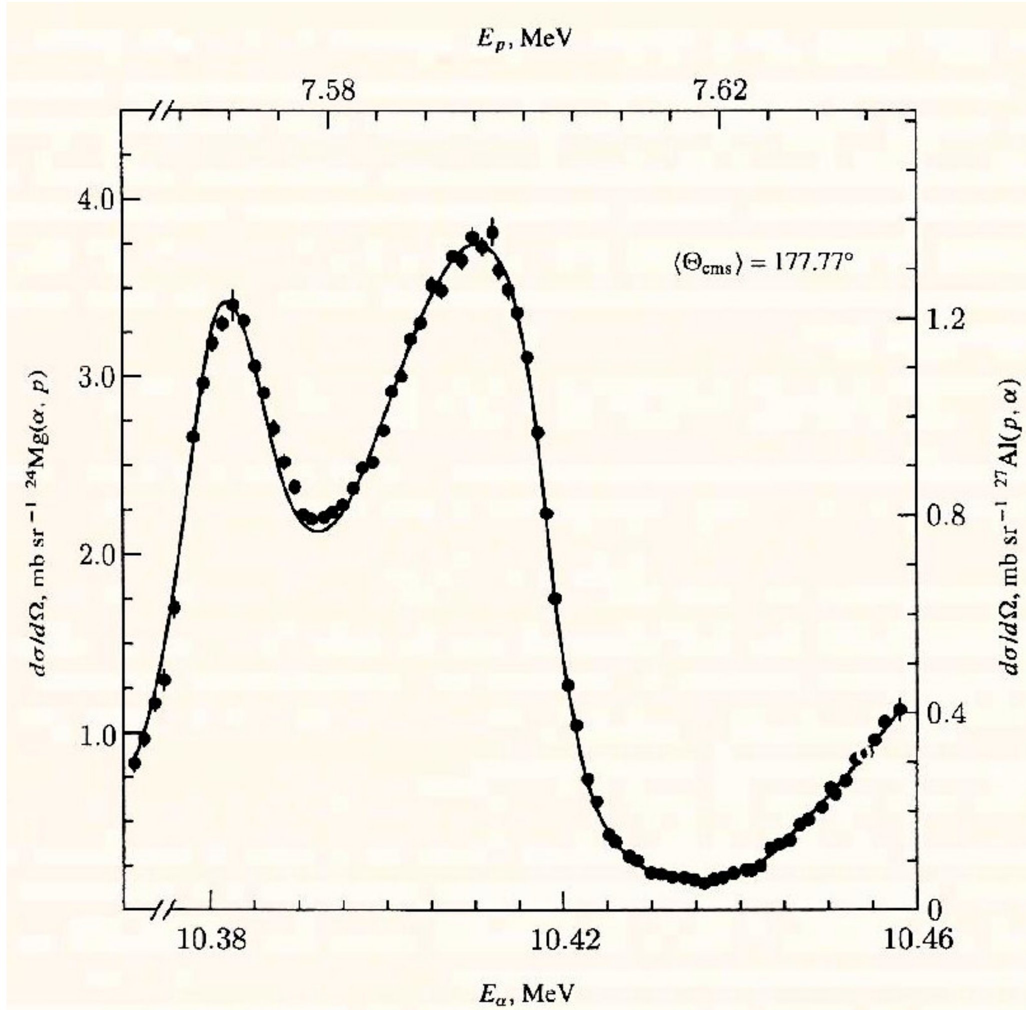
- Detailed Balance

$p + {}^{27}Al \leftrightarrow \alpha + {}^{24}Mg$

curve and points

T violation

$< 5 \times 10^{-4}$



# Neutron Electric Dipole Moment

- Most sensitive tests of T violation come from searches for the neutron and electron dipole moments

  - electron $0.18 \pm 0.16 \times 10^{-26}$ e-cm
  - neutron $< 0.63 \times 10^{-25}$ e-cm

- Note, the existence of these electric dipole moments would also violate parity

# Neutron Electric Dipole Moment

- How big might we expect the EDM to be?
  - EDM = charge x length x T-violating parameter

    length ~ 1/Mass

    $\sim\ e^2 M_N / M_W^2\ \sim\ 4\pi/137\ (0.94\ \text{GeV}) / (80\text{GeV})^2$

    $\sim\ 10^{-5}\ \text{GeV}^{-1}$ [200 MeV-fm]

    $\sim\ 10^{-6}\ \text{fm}\ \sim 10^{-19}\ \text{cm}$

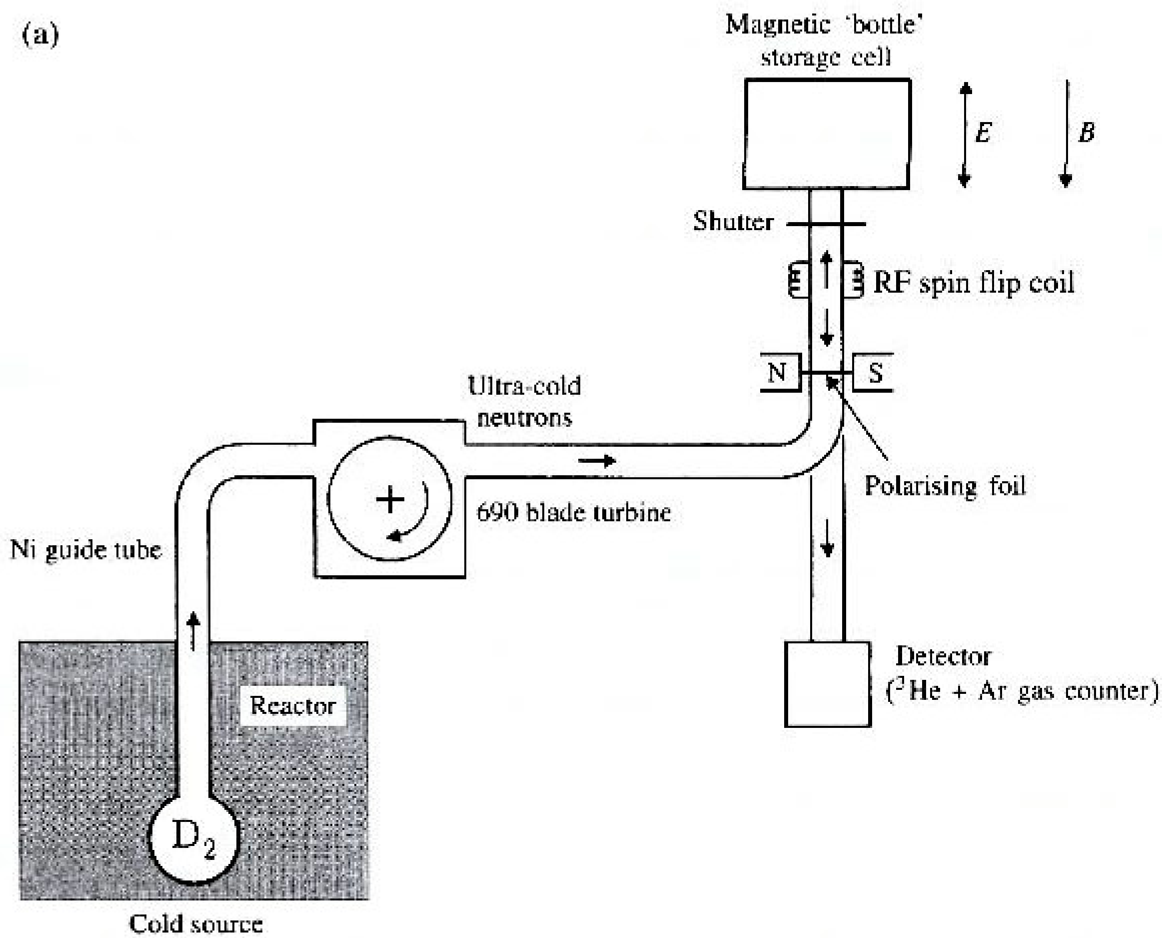
    T-violating parameter ~ $10^{-3}$ (like $K^0$)

  Then:

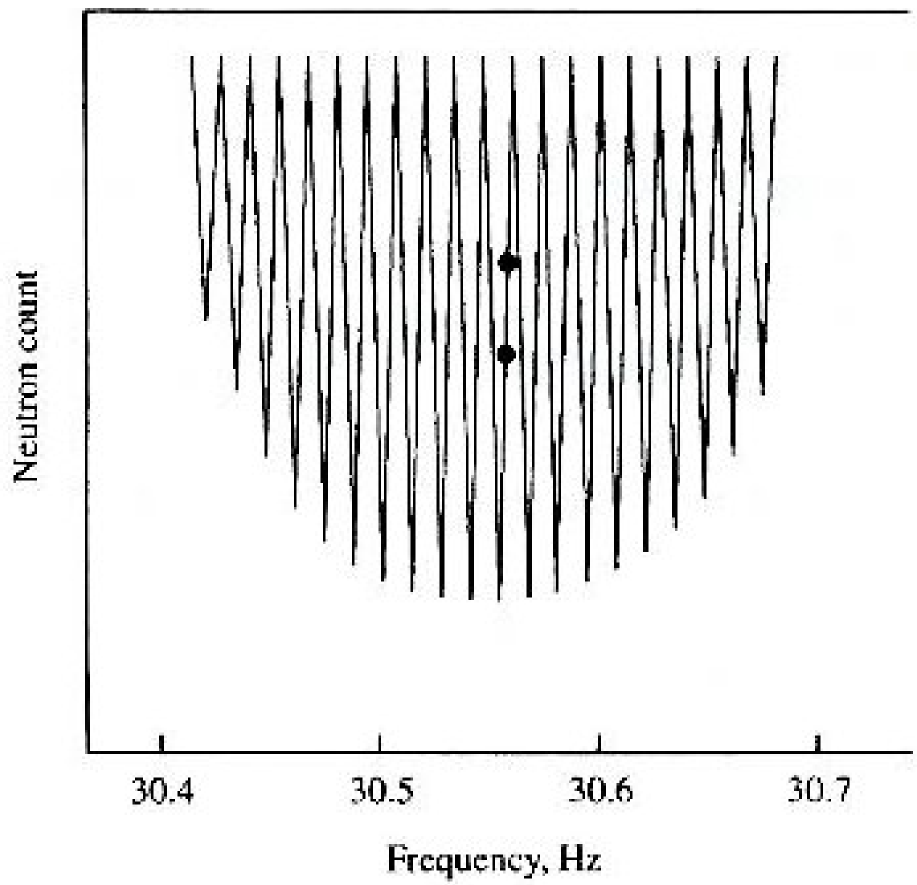
  EDM ~ e ($10^{-19}$ cm)($10^{-3}$)

  ~ $10^{-22}$ e-cm

# Neutron Electric Dipole Moment



# Neutron Electric Dipole Moment




- Neutron EDM < $0.63 \times 10^{-25}$ e-cm

# T Violation

## TIME REVERSAL ($T$) INVARIANCE

| | |
|---|---|
| $e$ electric dipole moment | $(0.18 \pm 0.16) \times 10^{-26}$ $e\,$cm |
| $\mu$ electric dipole moment | $(3.7 \pm 3.4) \times 10^{-19}$ $e\,$cm |
| $\tau$ electric dipole moment ($d_\tau$) | $> -3.1$ and $< 3.1 \times 10^{-16}$ $e\,$cm, CL 95% |
| $p$ electric dipole moment | $(-4 \pm 6) \times 10^{-23}$ $e\,$cm |
| $n$ electric dipole moment $d_n$ | $<0.63 \times 10^{-25}$ $e\,$cm, CL = 90% |
| $\Lambda$ electric dipole moment | $<1.5 \times 10^{-16}$ $e\,$cm, CL = 95% |

# T Violation

| | | |
|---|---|---|
| $\mu$ decay parameters | | |
| transverse $e^+$ polarization normal to plane of $\mu$ spin, $e^+$ momentum | | $0.007 \pm 0.023$ |
| $\alpha'/A$ | | $(0 \pm 4) \times 10^{-3}$ |
| $\beta'/A$ | | $(2 \pm 6) \times 10^{-3}$ |
| $\mathrm{Im}(\xi)$ in $K^{\pm}_{\mu 3}$ decay (from transverse $\mu$ pol.) | | $-0.014 \pm 0.014$ |
| asymmetry $A_T$ in $K^0$-$\overline{K}^0$ mixing | | $(6.6 \pm 1.6) \times 10^{-3}$ |
| $\mathrm{Im}(\xi)$ in $K^0_{\mu 3}$ decay (from transverse $\mu$ pol.) | | $-0.007 \pm 0.026$ |
| $n \to p e^- \nu$ decay parameters | | |
| $\phi_{AV}$, phase of $g_A$ relative to $g_V$ | [b] | $(180.07 \pm 0.18)^\circ$ |
| triple correlation coefficient $D$ | | $(-0.5 \pm 1.4) \times 10^{-3}$ |
| triple correlation coefficient $D$ for $\Sigma^- \to n e^- \overline{\nu}_e$ | | $0.11 \pm 0.10$ |

# Isospin Symmetry

- The nuclear force of the neutron is nearly the same as the nuclear force of the proton
- 1932 - Heisenberg - think of the proton and the neutron as two charge states of the nucleon
- In analogy to spin, the nucleon has isospin 1/2

  $I_3 = 1/2$ proton

  $I_3 = -1/2$ neutron

  $$Q = (I_3 + 1/2)\, e$$

- Isospin turns out to be a conserved quantity of the strong interaction

# Isospin Symmetry

- The notion that the neutron and the proton might be two different states of the same particle (the nucleon) came from the near equality of the n-p, n-n, and p-p nuclear forces (once Coulomb effects were subtracted)

- Within the quark model, we can think of this symmetry as being a symmetry between the u and d quarks:

  n = d d u

  p = d u u

# Isospin Symmetry

- Example from the meson sector:
  - the pion (an isospin triplet, I=1)

$\pi^+ = u\bar{d}$ $(I_3 = +1)$

$\pi^- = d\bar{u}$ $(I_3 = -1)$

$\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$ $(I_3 = 0)$

The masses of the pions are similar,

$M(\pi^\pm) = 140$ MeV $M(\pi^0) = 135$ MeV,

reflecting the similar masses of the u and d quark

# Isospin in Two-nucleon System

- Consider the possible two nucleon systems
  - pp $I_3 = 1 \Rightarrow I = 1$
  - pn $I_3 = 0 \Rightarrow I = 0$ or $1$
  - nn $I_3 = -1 \Rightarrow I = 1$

- This is completely analogous to the combination of two spin 1/2 states
  - p is $I_3 = 1/2$
  - n is $I_3 = -1/2$

# Isospin in Two-nucleon System

- Combining these doublets yields a triplet plus a singlet

  $2 \otimes 2 = 3 \oplus 1$

- Total wavefunction for the two-nucleon state:

  $\psi(\text{total}) = \phi(\text{space})\ \alpha(\text{spin})\ \chi(\text{isospin})$

- Deuteron = pn

  spin = 1 $\Rightarrow$ $\alpha$ is symmetric

  L = 0 $\Rightarrow$ $\phi$ is symmetric $[(-1)^L]$

  Therefore, Fermi statistics required $\chi$ be antisymmetric

  I = 0, singlet state without related pp or nn states

# Isospin in Two-nucleon System

- Consider an application of isospin conservation
  - (i) $p + p \rightarrow d + \pi^{+}$ (isospin of the final state is 1)
  - (ii) $p + n \rightarrow d + \pi^{0}$ (isospin of the final state is 1)

  - initial state:
    - (i) $I = 1$
    - (ii) $I = 0$ or $1$ (*CG* coeff say 50%, 50%)

  - Therefore $\sigma(ii)/\sigma(i) = 1/2$
  - which is experimentally confirmed

# Isospin in Pion-nucleon System

- Consider pion-nucleon scattering:
  - three reactions are of interest for the contributions of the I=1/2 and I=3/2 amplitudes

    (a) $\pi^+ p \to \pi^+ p$ (elastic scattering)

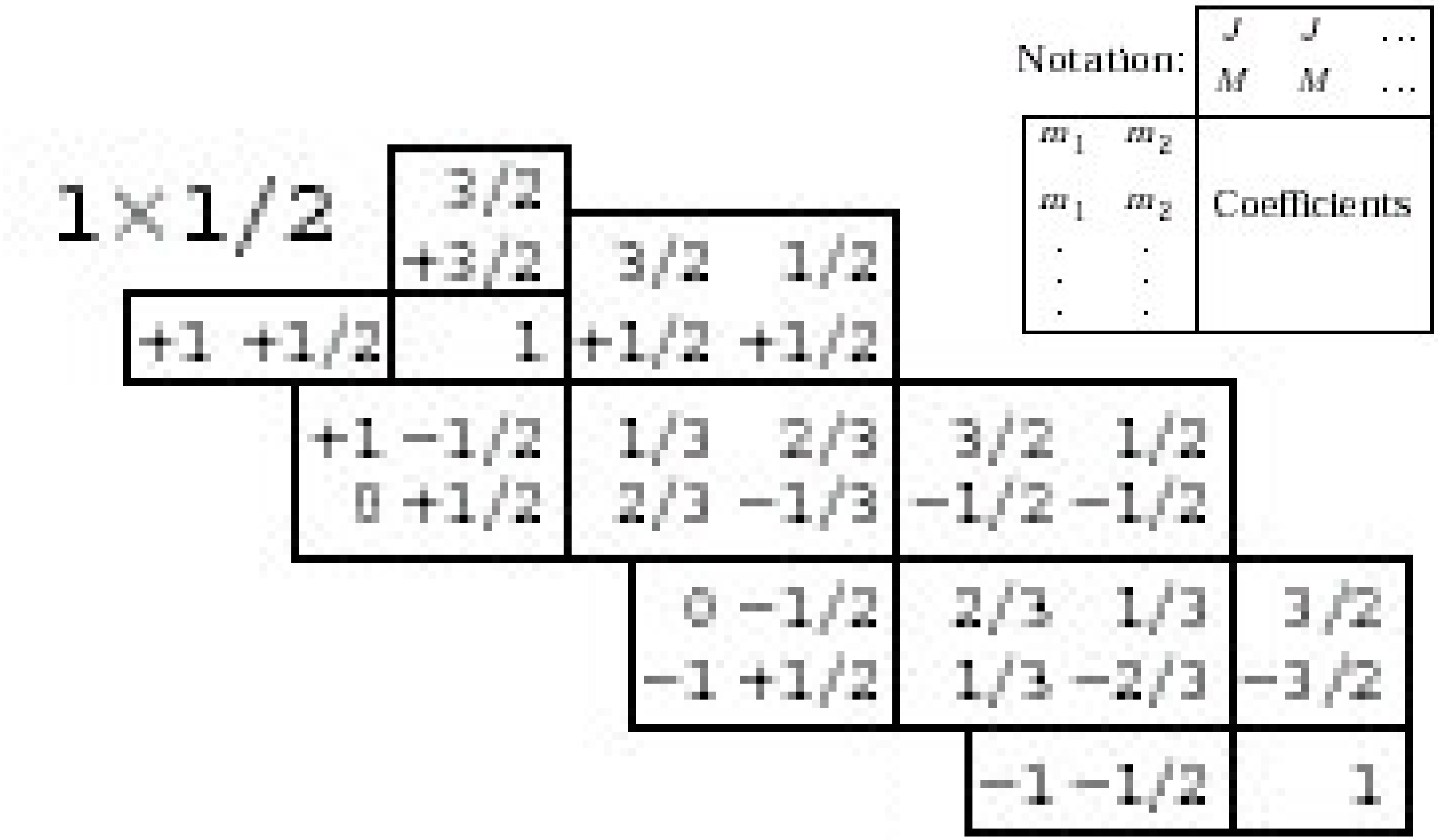
    (b) $\pi^- p \to \pi^- p$ (elastic scattering)

    (c) $\pi^- p \to \pi^0 n$ (charge exchange)

$$\sigma \propto \langle \psi_f | H | \psi_i \rangle^2 = M_{if}^2$$

$$M_1 = \langle \psi_f(1/2) | H_1 | \psi_i(1/2) \rangle$$
$$M_3 = \langle \psi_f(3/2) | H_3 | \psi_i(3/2) \rangle$$

# Clebsch-Gordon Coefficients



# Isospin in Pion-nucleon System

(a) $\pi^+ p \rightarrow \pi^+ p$ (elastic scattering)

(b) $\pi^- p \rightarrow \pi^- p$ (elastic scattering)

(c) $\pi^- p \rightarrow \pi^0 n$ (charge exchange)

- (a) is purely I=3/2

  $\sigma_a = K\,|M_3|^2$

- (b) is a mixture of I = 1/2 and 3/2

  $|\psi_i\rangle = |\psi_f\rangle = \sqrt{1/3}\,|\chi(3/2,-1/2)\rangle - \sqrt{2/3}\,|\chi(1/2,-1/2)\rangle$

  $$\sigma_b = K\,|\langle\psi_f|H_1+H_3|\psi_i\rangle|^2$$
  $$= K\,|(1/3)M_3 + (2/3)M_1|^2$$

# Isospin in Pion-nucleon System

(a) $\pi^+ p \rightarrow \pi^+ p$ (elastic scattering)
(b) $\pi^- p \rightarrow \pi^- p$ (elastic scattering)
(c) $\pi^- p \rightarrow \pi^0 n$ (charge exchange)

- (c) is a mixture of I = 1/2 and 3/2

$$|\psi_i\rangle = \sqrt{1/3}\, |\chi(3/2,-1/2)\rangle - \sqrt{2/3}\, |\chi(1/2,-1/2)\rangle$$

$$|\psi_f\rangle = \sqrt{2/3}\, |\chi(3/2,-1/2)\rangle + \sqrt{1/3}\, |\chi(1/2,-1/2)\rangle$$

$$\sigma_c = K\, |\langle\psi_f | H_1 + H_3 | \psi_i\rangle|^2$$

$$= K\, |\sqrt{(2/9)}M_3 - \sqrt{(2/9)}M_1|^2$$

Therefore:

$$\sigma_a : \sigma_b : \sigma_c = |M_3|^2 : (1/9)|M_3 + 2M_1|^2 : (2/9)|M_3 - M_1|^2$$

# Isospin in Pion-nucleon System

$\sigma_a : \sigma_b : \sigma_c = |M_3|^2 : (1/9)|M_3 + 2M_1|^2 : (2/9)|M_3 - M_1|^2$

Limiting situations:

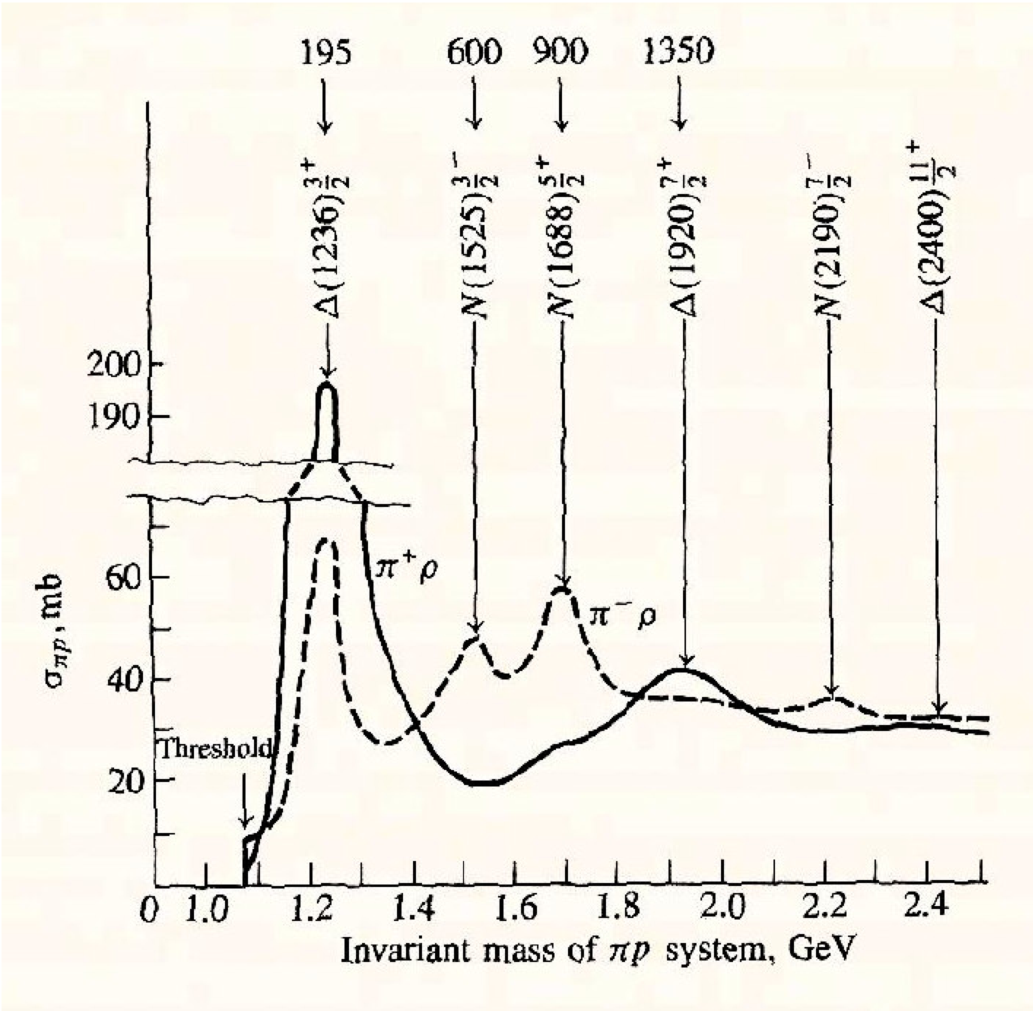
$M_3 >> M_1$

$\sigma_a : \sigma_b : \sigma_c = 9 : 1 : 2$

$M_1 >> M_3$

$\sigma_a : \sigma_b : \sigma_c = 0 : 2 : 1$

# Isospin in Pion-nucleon System



# Isospin Strangeness and Hypercharge

- $Q = [I_3 + (B+S)/2]\, e = (I_3 + Y/2)e$
  - B = baryon number
  - S = strangeness
  - Y = B+S = hypercharge

- Example, u and d quark

  u) $I_3 = 1/2$, $B = 1/3$, $S = 0$, $Q = [1/2 + 1/6]e = 2/3\ e$

  d) $I_3 = -1/2$, $B = 1/3$, $S = 0$, $Q = [-1/2 + 1/6]e = -1/3\ e$