1.3.2. **Energy density**

Show that the argument from ch. 1 §3.6 remark (3) for $u(\boldsymbol{x}, t)$ being the energy density of the electromagnetic field still holds if the field is coupled to N relativistic particles rather than one nonrelativistic one.

(3 points)

1.3.2) Consider one relativistic particle. Then the kinetic energy is

$$E_{kin} = \vec{v}\cdot\frac{\partial L_0}{\partial \vec{v}} - L_0 = \frac{mv^2}{\sqrt{1-v^2/c^2}} + mc^2\sqrt{1-v^2/c^2} = \frac{mc^2}{\sqrt{1-v^2/c^2}}$$

Now consider

$$\underline{\vec{v}\cdot\dot{\vec{p}}} = \vec{v}\cdot\frac{d}{dt}\frac{\partial L_0}{\partial \vec{v}} = \vec{v}\cdot\frac{d}{dt}\frac{m\vec{v}}{\sqrt{1-v^2/c^2}} = \frac{m\vec{v}\cdot\dot{\vec{v}}}{\sqrt{1-v^2/c^2}} + mv^2\frac{\vec{v}\cdot\dot{\vec{v}}/c^2}{(1-v^2/c^2)^{3/2}}$$

$$= \frac{m\vec{v}\cdot\dot{\vec{v}}}{(1-v^2/c^2)^{3/2}} = \underline{\frac{d}{dt}E_{kin}}$$

1

But from the eq. of motion we have

$$\dot{\vec{p}} = \vec{F} = e\vec{E} + \frac{e}{c}\vec{v}\times\vec{B}$$

$$\rightarrow \underline{\frac{d}{dt}E_{kin}} = \vec{v}\cdot\dot{\vec{p}} = \underline{e\vec{v}\cdot\vec{E}} \qquad \text{since } \vec{v}\cdot(\vec{v}\times\vec{B}) = 0$$

This is the same expression as in ch 1 §1.6 remark (3), and Poynting's theorem again yields

$$\frac{d}{dt}(U + E_{kin}) = 0 \qquad \text{where } U = \int d\vec{x}\, u(\vec{x},t)$$

1 $\rightarrow$ <u>U is the field energy</u>

For N particles with charges e_i $(i=1,\dots,N)$, consider

$$\int d\vec{x}\; \vec{j}\cdot\vec{E} = \sum_{i=1}^{N} e_i\vec{v}_i\cdot\vec{E}$$

$$\rightarrow \frac{d}{dt}E_{kin} = -\sum_{i=1}^{N} e_i\vec{v}_i\cdot\vec{E}$$

1 $\rightarrow$ <u>same result as for</u> <u>$N=1$</u>