1.2.4. **Coulomb gauge**

Consider the 4-vector potential $A^\mu(x) = (\varphi(x), \boldsymbol{A}(x))$. Show that one can always find a gauge transformation such that

$$\boldsymbol{\nabla} \cdot \boldsymbol{A}(x) = 0$$

This choice is called *Coulomb gauge*.

(2 points)

1.2.4.) Gauge trafo: $A^\mu \to A^\mu - \partial^\mu \chi$

$$\to \vec{A} \to \vec{A} - \vec{\nabla}\chi$$

$$\to \vec{\nabla}\cdot\vec{A} \to \vec{\nabla}\cdot\vec{A} - \vec{\nabla}^2\chi$$

(1)

Now choose χ as any solution of the Poisson eq.

$$\vec{\nabla}^2\chi(x) = \vec{\nabla}\cdot\vec{A}(x)$$

Then the transformed $\vec{A}$ has the property

$$\underline{\underline{\vec{\nabla}\cdot\vec{A}'(x)}} = \vec{\nabla}\cdot\vec{A}(x) - \vec{\nabla}^2\chi(x) = \underline{\underline{0}}$$

(1)