1.2.3. **Energy-momentum tensor for a massive scalar field**

Consider the massive scalar field from ch. 0 §2.5:

$$\mathcal{L} = \frac{1}{2}\,(\partial_\mu \varphi)\,(\partial^\mu \varphi) - \frac{m^2}{2}\,\varphi^2$$

and the tensor field ${H_\mu}^\nu$ defined analogously to Problem 1.2.1:

$${H_\mu}^\nu = (\partial_\mu \varphi)\,\frac{\partial \mathcal{L}}{\partial\,(\partial_\nu \varphi)} - {\delta_\mu}^\nu\,\mathcal{L}$$

Determine ${H_\mu}^\nu$ explicitly and show that

$$\partial_\nu {H_\mu}^\nu = 0$$

hint: Use the Euler-Lagrange equation determined in ch. 0 §2.5.

(2 points)

1.2.3.) $$\underline{\mathcal{H}_\mu{}^\nu} = (\partial_\mu\varphi)\frac{\partial\mathcal{L}}{\partial(\partial_\nu\varphi)} - \delta_\mu{}^\nu\mathcal{L}$$

$$= \underline{(\partial_\mu\varphi)(\partial^\nu\varphi) - \delta_\mu{}^\nu\tfrac{1}{2}(\partial_\alpha\varphi)(\partial^\alpha\varphi) + \delta_\mu{}^\nu\tfrac{m^2}{2}\varphi^2}$$

(1)

$$\rightarrow \underline{\underline{\partial_\nu\mathcal{H}_\mu{}^\nu}} = (\partial_\nu\partial_\mu\varphi)(\partial^\nu\varphi) + (\partial_\mu\varphi)(\partial_\nu\partial^\nu\varphi) - (\partial_\alpha\varphi)(\partial_\mu\partial^\alpha\varphi) + m^2\varphi\partial_\mu\varphi$$

$$= \underbrace{(\partial_\nu\partial_\mu\varphi)(\partial^\nu\varphi) - (\partial_\mu\partial_\alpha\varphi)(\partial^\alpha\varphi)}_{=0} + (\partial_\mu\varphi)(\partial_\nu\partial^\nu\varphi + m^2\varphi)$$

(1)

$$= (\partial_\mu\varphi)\underbrace{(\partial_\nu\partial^\nu + m^2)\varphi}_{=0} = \underline{\underline{0}}$$ by the Klein-Gordon eq., cf. 0 p 2.5