1.2.2. **Energy-momentum conservation in the presence of matter**

Prove the corollary of ch. 1 §2.3: In the presence of matter, the energy-momentum tensor obeys the continuity equation

$$\partial_\nu T_\mu{}^\nu(x) = \frac{-1}{c}\, F_\mu{}^\nu(x)\, J_\nu(x)$$

(2 points)

1.22.) Generalizing the proof of the proposition in §1 §2.3:

The only difference is that now the EL eq. reads

$$\partial_\nu F^\nu{}_\alpha = \frac{4\pi}{c} j_\alpha \qquad (1)$$

$$\leadsto \underline{\partial_\nu T_\mu{}^\nu} \overset{§2.3}{=} \frac{1}{4\pi}\left[-(\partial_\nu F_\mu{}^\alpha) F^\nu{}_\alpha - F_\mu{}^\alpha \partial_\nu F^\nu{}_\alpha + \frac{1}{4}\partial_\mu F_{\alpha\beta} F^{\alpha\beta}\right]$$

$$= -\frac{1}{c} F_\mu{}^\alpha j_\alpha + \underbrace{\frac{1}{4\pi}\left[-(\partial_\nu F_\mu{}^\alpha) F^\nu{}_\alpha + \frac{1}{2}(\partial_\mu F_{\alpha\beta}) F^{\alpha\beta}\right]}_{=0 \text{ by } §2.3}$$

$$= \underline{\underline{-\frac{1}{c} F_\mu{}^\nu j_\nu}} \qquad (1)$$