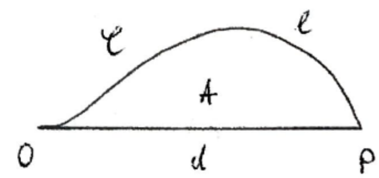
0.2.2 **Dido's problem**

An area A in the x-y-plane is enclosed by straight line between two points O and P that are a distance$d$ apart, and a path $\mathfrak{C}$ with end points O and P and length $\ell > d$. Find the path $\mathfrak{C}$ that maximizes A.



(6 points)

0.2.2.) cf. O §2.4 ex (5) $\rightsquigarrow$ $A = \frac{1}{2}\oint dt\,[x(t)\dot{y}(t) - y(t)\dot{x}(t)]$

$$\ell = \oint dt \sqrt{\dot{x}^2(t) + \dot{y}^2(t)}$$

1 cf. O §2.4 thm $\rightsquigarrow$ $L = \frac{1}{2}(x\dot{y} - y\dot{x}) + \frac{1}{2}\lambda\sqrt{\dot{x}^2+\dot{y}^2}$

EL eqs: $\frac{1}{2}\dot{y} = \frac{\partial L}{\partial x} = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = -\frac{1}{2}\dot{y} + \frac{1}{2}\lambda \frac{d}{dt}\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}}$

$$\rightsquigarrow \quad y - y_0 = \lambda \frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} \qquad (1)$$

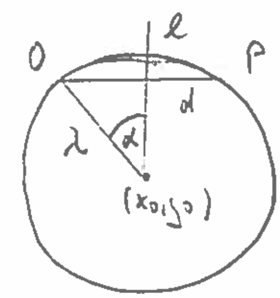
$-\frac{1}{2}\dot{x} = \frac{\partial L}{\partial y} = \frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = \frac{1}{2}\dot{x} + \frac{1}{2}\lambda \frac{d}{dt}\frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}}$

1 $$\rightsquigarrow \quad x - x_0 = -\lambda \frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}} \qquad (2)$$

1 $\rightsquigarrow$ $\boxed{(x-x_0)^2 + (y-y_0)^2 = \lambda^2}$ circle with center (x_0, y_0), radius λ

Now $\overline{OP} = d = 2\lambda \sin\alpha$

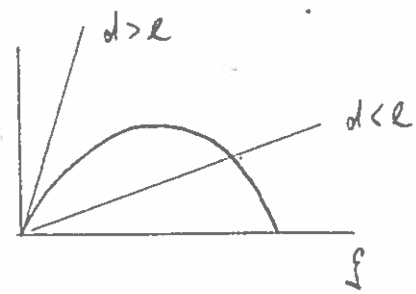
and $\ell = 2\lambda\alpha$,



1 $\rightsquigarrow$ $\boxed{\frac{d}{2\lambda} = \sin\frac{\ell}{2\lambda}}$ determines λ

Define $\xi := \ell/2\lambda$

1 $\rightsquigarrow$ $\boxed{\sin\xi = \frac{d}{\ell}\,\xi}$.



Graphic solution $\rightsquigarrow$

1st case: $d \geq \ell$ no solution

$d < \ell$ exactly one solution, which obviously is

1 a maximum