

WARTHOG 2017

Weight functors and representing projectives

Exercises

Consider the hypertoric enveloping algebra for $K = \mathbb{C}^*$ acting diagonally on $\mathbb{C}^2$.

1. Describe the universal module with a vector where h_1^+ acts with eigenvalue 1 and h_2^+ acts with weight -1 (honestly, not generalized). In particular, what is its composition series?
2. How would this change as we change the eigenvalues?
3. This hypertoric algebra is a quotient of $\mathfrak{sl}_2$ sending $E \mapsto m^1$ and $F \mapsto m^{-1}$; what is the relationship of the modules we considered above to the usual finite dimensional modules, Vermas and dual Vermas of $\mathfrak{sl}_2$.

Consider K to be the subtorus $(a_1, \dots, a_4)$ satisfying $a_1 a_2 a_3 = 1$ and $a_3 = a_4$.

4. For the different cosets of $\mathfrak{t}^*$ (that is, different maximal ideals of the center $U(\mathfrak{k})$), describe the number of finite dimensional modules over this algebra in that coset.
5. In whatever level of generality you like, calculate the dimensions of these simples.

(If you haven't done it before) Prove that in a finite dimensional F -algebra A , any element a is the sum of two unique elements $a_{ss}, a_n \in A$ such that the action by left multiplication of a_{ss} on A is semi-simple and the action of a_n on A is nilpotent.