WARTHOG 2017

The hypertoric enveloping algebra I: Higgs description

Exercises

1. Recall that we defined $h_i^+ := x_i\partial_i$, $h_i^- := \partial_i x_i = h_i^+ + 1$, and

$$[h_i]^k := \begin{cases} 1 & \text{if } k = 0 \\ x_i^k \partial_i^k = (h_i^- - 1)\cdots(h_i^- - k) & \text{if } k > 0 \\ \partial_i^{-k} x_i^{-k} = (h_i^+ + 1)\cdots(h_i^+ - k) & \text{if } k < 0. \end{cases}$$

We also defined $m^z := x^{z+}\partial^{z-}$ for all $z \in \mathbb{Z}^n$, and we asserted that we have the following formula:

$$m^z \cdot m^w = \left(\prod_{\substack{z_i w_i < 0 \\ |z_i| \le |w_i|}} [h_i]^{z_i} \right) m^{z+w} \left(\prod_{\substack{z_i w_i < 0 \\ |z_i| > |w_i|}} [h_i]^{-w_i} \right).$$

Let $n = 8$, $z = (1, 5, -2, -3, -4, 6, 0, 3)$, and $w = (-2, -3, 4, 1, -7, 2, 5, -3)$. Write down m^z and m^w, and verify the formula in this case. (This is a situation where working out a single example is more illuminating than writing down a general proof.)

2. Let $T = \mathbb{C}^\times$, and consider the homomorphism $(\mathbb{C}^\times)^3 \to T$ taking (t_1, t_2, t_3) to $t_1 t_2 t_3$. Let $K \subset (\mathbb{C}^\times)^3$ be the kernel of this homomorphism. Let $U := D^K$ be the corresponding hypertoric enveloping algebra.

i) For each $k \in \mathbb{Z} \cong \mathfrak{t}^*_{\mathbb{Z}} \subset \mathbb{Z}^3$, write down the element $m^k \in D^K = U$.

ii) Compute $m^3 \cdot m^{-4}$.

iii) What is the center of U?