WARTHOG 2017

The Farkas Lemma

Exercises

Let $I = \{1, 2, 3, 4\}$, and let $\mathcal{V} = (V, \eta, \xi)$ be the polarized arrangement where

$$V := \{(y, y, x, x + y) \mid (x, y) \in \mathbb{R}^2\},$$

η is the image of $(-1, 0, 0, -2)$ in $\mathbb{R}^4/V$, and ξ is the image of $(2, 0, 1, 0) \in (\mathbb{R}^4)^*$ in $V^* = (\mathbb{R}^4)^*/V^\perp$. In terms of the (x, y) coordinates on V_η, the inequalities defining the positive sides of the four hyperplanes are $y \geq 1$, $y \geq 0$, $x \geq 0$, and $x + y \geq 2$. The functional ξ, up to an additive constant, is $\xi(x, y) = x + 2y$. See Figure 1, where we label all of the feasible regions with the appropriate sign vectors. The bounded feasible regions are shaded. Note that besides the five unbounded feasible regions pictured, there is one more unbounded sign vector, namely $+ - ++$, which is infeasible.

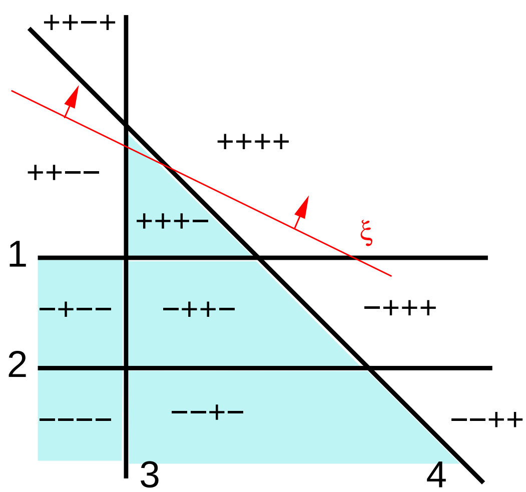

Figure 1: Example of bounded and feasible chambers

Draw a similar picture for the Gale dual $\mathcal{V}^!$. (To check your answer, see Example 2.5 of the paper "Gale duality and Koszul duality", arXiv:0806.3256.)