WARTHOG 2017

The hypertoric enveloping algebra II: Coulomb description

Exercises

Given a torus K and the Coulomb branch for it, the set of cocharacters $\mathfrak{k}_{\mathbb{Z}}$ naturally decompose into cones where for any χ, ν in the cone, we have $r_\chi r_\nu = r_{\chi+\nu}$.

1. Calculate these cones in the case where $K = \mathrm{diag}(a_1, a_2, a_3)$ with $a_1 a_2 a_3 = 1$.
2. Give a general description of these cones.

As discussed in the talk, the Coulomb branch ($\hbar$-deformed or not) has a grading such that

$$\deg r_\chi = \dim V_{\mathcal{O}}/(V_{\mathcal{O}} \cap t^\chi V_{\mathcal{O}}) + \dim t^\chi V_{\mathcal{O}}/(V_{\mathcal{O}} \cap t^\chi V_{\mathcal{O}})$$

3. Find a purely algebraic description of $\deg r_\chi$ (of course, using the action of χ on V).
4. Show that as a function of χ, $\deg r_\chi$ is piecewise linear.
5. Use these calculations to find the Hilbert series of the Coulomb branch for the K discussed above.

The quantum Coulomb branch calculated from my talk differed from Nick's because $\hbar$'s showed up where he had 1's. If $A \supset A_1 \supset A_0 = \mathbb{C}$ is a filtered $\mathbb{C}$-algebra, then recall that the Rees algebra is the $\mathbb{C}[t]$ algebra defined by $R(A) = \bigoplus \hbar^n A_n$

6. Show that $R(A)$ specialized at $\hbar = 1$ is the original algebra A, and that the specialization at $\hbar = 0$ is the associated graded of A.
7. Show that if we take the usual filtration on the universal enveloping algebra $U(\mathfrak{g})$, then the Rees algebra is generated by $X \in \mathfrak{g}$ modulo the relations $XY - YX = \hbar[X, Y]$.
8. Can you describe the Rees algebra of the differential operators in terms of a presentation?