Exercises

Exercise 14.1. We establish some basic properties of equivariant noetherianity. In what follows, X is a space on which G acts, and all subsets are endowed with the subspace topology.

(a) Suppose H is a subgroup of G and Y is an H-stable subset of X that is H-noetherian. Show that $\bigcup_{g \in G} gY$ is G-noetherian.
(b) Suppose X is G-noetherian and A is a G-stable subset. Show that A is G-noetherian.
(c) Suppose $X = A \cup B$ where A and B are G-stable and G-noetherian. Show that X is G-noetherian.

Exercise 14.2. Let X be a closed $\mathbf{GL}$-subvariety of $\mathbf{A}^{\underline{\lambda}}$. Show that there are finitely many functions $f_1, \ldots, f_r$ on $\mathbf{A}^{\underline{\lambda}}$ such that X is the common zero locus of the $\mathbf{GL}$-orbits of the f_i's.

Exercise 14.3. Let R be a finitely $\mathbf{GL}$-generated $\mathbf{GL}$-algebra. Show that radical $\mathbf{GL}$-ideals in R satisfy ACC.

Additional exercises

Exercise 14.4. Let R be the coordinate ring of $\mathbf{A}^{(1)} \times \mathbf{A}^{(1)}$. Show that $\mathbf{GL}$-ideals in R satisfy ACC.

Exercise 14.5. Let $\mathfrak{S}$ be the infinite symmetric group (whichever version you prefer), let $\mathfrak{S}$ act on $R = \mathbf{C}[x_1, x_2, \ldots]$ by permuting variables, and let $X = \operatorname{Spec}(R)$. Show that X is $\mathfrak{S}$-noetherian. [Warning: I do not know an easy proof!]