Exercises

Exercise 10.1. We have seen that if $f_1, \ldots, f_r$ is a regular sequence then $f_1, \ldots, f_r$ is algebraically independent. Given an example of $f_1, \ldots, f_r \in k[x_1, \ldots, x_n]$ that are homogeneous of positive degree and algebraically independent, but not a regular sequence.

Exercise 10.2. Let R be a graded ring (in non-negative degrees) with $R_0 = k$ a field. Let $f_1, \ldots, f_r$ be homogeneous elements of positive degree such that we have $\mathrm{H}_1(K(f_1, \ldots, f_r)) = 0$. Show that $f_1, \ldots, f_r$ is a regular sequence.

Exercise 10.3. Describe the top Koszul homology group $\mathrm{H}_n(K(x_1, \ldots, x_n))$, for arbitrary elements $x_1, \ldots, x_n$ in a ring R.